EE133 - Prelab 1
Amplitude Modulation and Demodulation

1 Introduction

This week we will be taking a look at the amplitude modulation (AM) scheme. In this process, the amplitude of the carrier signal is varied so that it is proportional to the instantaneous amplitude of the modulating signal. With the invention of oscillators, this scheme quickly supplanted the first spark-gap transmitters. They did so because AM supported the easy transmission of voice while allowing more people to transmit without interfering with each other.

For an oscillating electrical signal to be transmitted through the air, it must first be converted into electromagnetic radiation. To maximize efficiency of transmission, the physical dimensions of the transmitting and receiving antennas must be on the same order of magnitude as the wavelength of the transmitted signal (often a quarter wavelength). For audio signals, whose frequencies range from 20 Hz to 20 kHz, this corresponds to wavelengths from 15 to 15000 km. As you can probably guess, an antenna of this length would be absurd for general applications.

AM solves this problem by using a 'slow' moving audio signal to modulate a fast moving, radio signal. The fast signal is called the carrier signal, and the audio frequency signal is typically called the modulating signal. Typical carrier frequencies range from 30 kHz to upwards of 30 GHz.

By using a carrier frequency, users can 'tune' into a particular transmitter. With the old spark-gap transmitters, every user heard all transmitters that were within range. With interference problems greatly reduced, the radio became something for the masses.

In this prelab, we will examine the mathematical basis for AM signals, both in the time domain and the frequency domain. Using that knowledge we'll design a simple peak detector to decode AM signals. At the same time we'll be using HSPICE to investigate the difference between two-quad and four-quad multipliers. These multiplier chips are the modern day equivalent of the bulky mechanical contraptions that radio pioneers used to create AM signals. What's more amazing is that we'll use those same chips to demodulate our signal too!

At this point, all this might sound very disconnected from the FM circuit that we're supposed to be working on. Lest you think that you're falling behind some master schedule, we've actually chosen a circuit design where the FM modulation and demodulation also requires the use of the same multiplier chips required for this lab (See Figure 1). So we'll be soldering and testing properties of our multipliers on our FM boards. Later on, we'll come back and explore how useful they are for FM multiplication.

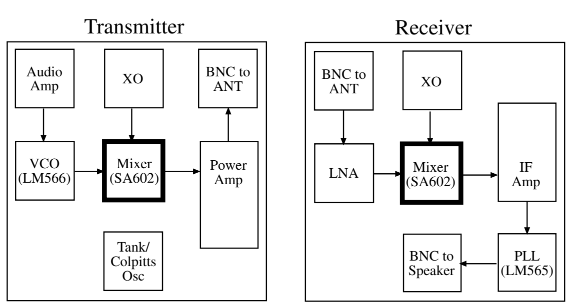

Figure 1: Lab 1 Roadmap

2 Warm-up Exercises

2.1 Units for Power Calculations

The spectrum/network analyzers we have in lab can display values in a number of different units - mV, mW, dBm, dBV, dB, etc. - so it will be important to know what all of these mean.

The decibel scale is convenient, because it allows us to express a quantity that varies over many orders of magnitude in a concise way. Because the argument of a logarithm must be a unitless ratio, however, we must measure quantities like power and voltage relative to some reference value (how is this different from computing a gain in dB?). In communication systems, power is usually referenced to 1mW and is measured in dBmW or dBm. 1dBm is equal to $10\log[power$ (in units of mW)]. For reference, $0dBm = 1mW$, $10dBm = 10mW$, $20dBm = 100mW$, etc ... Similarly you can define $1dBV = 20log[voltage$ (referenced to 1V)]. When deriving voltage from power, however, you must take into account the impedance of the measured component. The input/output impedance of almost all high-frequency equipment is 50Ω, so when you read a power value in dBm on the spectrum analyzer, this is the power into a 50Ω load, where setting the power equal to $V^2/50$ will yield the correct voltage across the load of the analyzer (note that the spectrum analyzers measure rms voltage, so you needn't divide the power by 2 to find the average power). Note that if the node you are measuring has a different impedance than the load of your measurement equipment, you must equate the power read on the spectrum analyzer with the power (V^2/R) into your load in order to find the actual voltage across your load. Let's do some quick calculations to make sure we've got things straight.

EXERCISES:

1. Calculate the power in mW into a 50Ω load if the measured value is $-15dBm$.What voltage value does this correspond to?

2. Calculate the power in dBm if $.2V$ is measured across a 75Ω resistor.

3. Suppose we have an amplifier with an input impedance of 70Ω and a load impedance of 40Ω. If $.001Vrms$ is measured at the input, and $.002Vrms$ is measured across the load, what is the input power and the power delivered to the load in dBm? What is the power 'gain' of this circuit (notice you can subtract the input power in dBm from the output power in dBm to obtain a gain in dB)? This is known as the 'operating' gain of the circuit. There are a number of other definitions of gain for a given circuit, which we will get into later.

4. Assume you have an amplifier with a 500Ω load impedance. Using a probe that does not load the output, you measure a power of -5dBm at the output. What is the voltage at this point?

2.2 High-Frequency Components

We will be working with high-frequency signals this quarter, and though it won't be crucial for this lab, it will be important to be aware of what kinds of components can be used at different frequencies. Read the handout, 'Capacitors Demystified' for a detailed description of different kinds of capacitors and their uses. Also see section 6.5 in Radmanesh for a discussion about component non-idealities. When in doubt, you can always check component values with the LCR Meter in Lab.

3 A First Look at AM Modulation

If we use an RF signal of the form

$$v_c = V_c \cos(\omega_c t + \phi)\,. \tag{1}$$

we can vary V_c, ω_c, or ϕ to create three distinct modulation schemes:

- Varying V_c gives us Amplitude Modulation (AM)
- Varying ω_c gives us Frequency Modulation (FM)
- Varying ϕ gives us Phase Modulation (PM)

So let us work through the equations that describe an AM signal. To start, we take a carrier of the form:

$$v_c = V_c \cos(\omega_c t). \tag{2}$$

with a modulating signal of the form:

$$v_m = V_m \cos(\omega_m t). \tag{3}$$

When we multiply the two signals, the result is:

$$v_{mod} = (V_m \cos(\omega_m t))\, V_c \cos(\omega_c t). \tag{4}$$

1. **Using MATLAB to plot an AM signal:** Using MATLAB, obtain plot of a modulated signal. Use a carrier of freqeuncy 30 kHz and amplitude 1V, and a modulating signal of frequency 1 kHz and amplitude 1V. Include a plot in your lab notebook.

2. **Visualizing the Frequency Domain:** What is happening in terms of the frequency components of the modulated waveform? (Hint: Multiply the two signals and use a trigonometric identity). Draw the frequency spectrum of the modulated signal next to your MATLAB plot.

   But this is an incomplete mathematical model. We know from lecture that in amplitude modulation, a term at the carrier frequency is often present. To model this in Matlab, we can introduce a d.c. offset in the modulating signal.

   In lecture, we used the following signal description:

   $$v_m = V_C\left[1 + m\cos(\omega_m t)\right]. \tag{5}$$

   However, for lab, we'll be using a slightly different, but equivalent formulation. In this case, the modulating signal is defined as:

   $$v_m = V_{DC} + V_m \cos(\omega_m t). \tag{6}$$

   Here we have multiplied V_C in, and redefined it as V_{DC} so that you can visualize this signal a little better. The value m is now separately calculated, but it is clear that there is a DC component and an AC one.

3. **Modeling the DC Offset:** Work out the new expression for the modulated signal. Verify that the modulation index $m = \frac{V_m}{V_{DC}}$. As you may recall from lecture, this is a simple figure of merit used to quickly describe what the waveform looks.

   So long as the modulation index remains below 1, the modulating signal will never drop below 0, which means that the envelope of the modulated signal will be a reproduction of the modulating signal.

4. **The Relation between m and Sideband vs. Carrier Energy:** For m=1, what is the relationship between the energy in the sidebands and the energy in the carrier.

5. **The relationship between V_{DC} and m:** Using MATLAB, plot the modulated signal for two periods, using the same parameters as above. Plot the signal for values of V_{DC} = 0, 0.5, 1.0 & 2.0V. Comment on the qualitative differences.

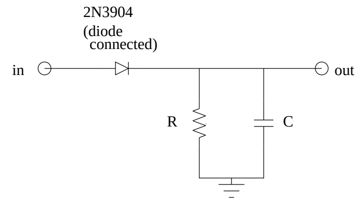

Figure 2: Peak Detector

4 The Peak Detector

In this section, you will be investigating the peak detector (See Figure 2) as an AM demodulator. You will design and build a peak detector to demodulate an AM signal with fc=30kHz, fm=1kHz, and m=0.5. The input voltage looks as follows:

$$V_{in} = \cos(2\pi(30kHz)t) + \frac{1}{4}\cos(2\pi(29kHz)t - 90) + \frac{1}{4}\cos(2\pi(31kHz)t + 90)$$

1. **Designing our Low Pass Filter:** Choose R and C values for your low-pass filter which pass the wanted audio signal but filter out any higher order harmonics. Make sure your filter passes the whole audio band (up to 20kHz), not just the 1kHz tone! You should have from class an expression for the optimal value for C as a function of V_{in} and R, which can assist you in your choice of C. Please use standard component values for R and C which are available on the web page at: `http://www.stanford.edu/class/ee133`.

2. **HSPICE AC Analysis:** Using HSPICE, obtain plots for the input and output voltages in the time domain as well as the frequency domain. (A sample HSPICE deck, with critical values omitted, is available on the web page.) Does the time-domain waveform look as you expected?

3. **HSPICE Distortion Analysis:** What is the Total Harmonic Distortion (THD) for your demodulator? Replace R by a resistor that is 10 times too large and resimulate. Indicate what type of distortion is present as well as the THD. Now replace R by a resistor 10 times too small. Again, indicate the type of distortion and THD that you find. As will always be the case, include the input HSPICE deck and the output plots in your lab notebook.

   Note: You can tell HSPICE to calculate THD by using the output command:

   ```
   .four <fundamental freq.> <node>
   ```

   These results can then be found in the *.lis file.

   For your information, HSPICE calculates the THD using the following equation:

   $$THD(\%) = 100 * \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \ldots + V_n^2}}{V_{fund}}$$

   Where V_i is the voltage of the ith harmonic which is an integer multiple of the fundamental frequency. V_1 is the signal of interest. Keep this in mind, because some machines have quirks where they won't always calculate it properly for you.

5 The Two-Quadrant Multiplier

After all this modeling, you would probably guess that there has to be a circuit that performs multiplication. And you would be right, Figure 3 is one of the simplest multipliers: a variable transconductance, emitter-coupled-pair multiplier.

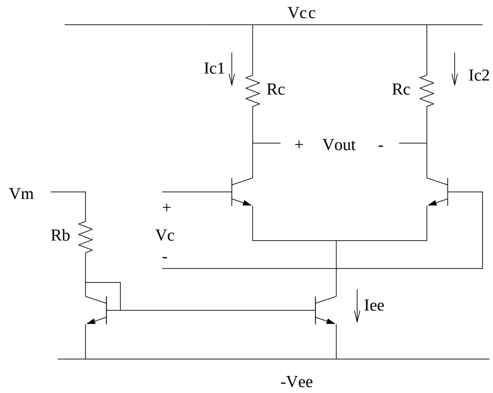

Figure 3: Two-Quadrant Multiplier

This basic multiplier has a few limitations. The circuit functions as a multiplier only under the assumptions that V_c is small compared to V_T, and that V_m is greater than V_{BE} on. This latter restriction means that the multiplier only functions in two quadrants of the $V_c - V_m$ plane, and hence the circuit is called a two-quadrant multiplier.

1. **HSPICE Analysis:** Simulate the circuit in Figure 3 using the Spice deck provided on the EE133 web page. Use $R_c = 2.2\text{k}\Omega$, $R_b = 510\Omega$, $RQ1_a = RQ2_a = 27\text{k}\Omega$, and $RQ1_b = RQ2_b = 15\text{k}\Omega$.

2. **Additional HSPICE Simulation:** Repeat your analysis for (a) V_c=10mV; V_m=1V and (b) V_c=2V; V_m=1V, attach the output waveforms. Explain any differences in the results for cases (a) and (b).

3. **Figures of Merit:** One figure of merit that is often used is Conversion Gain, which is the ratio of the usable output signal voltage to the input signal voltage. Using the small-signal approximation, what is the conversion gain for this circuit? $G_{conv} = \frac{V_{sideband}}{V_m}$. We'll go over additional figures of merit in the next section.

6 The Four-Quadrant Multiplier

From your analysis of the two-quad multiplier, you might have noticed that the output voltage of the two-quadrant multipler has a term which is undesirable because it is a function only of the carrier frequency. Arguably, the information you want is stored in the sidebands. In the search for ever more efficient transmitters, engineers have designed all sorts of circuits to eliminate the carrier, and even one of the sidebands.

However, this all requires the use of extra decoding circuitry, and would probably triple the size of this prelab. In this class, we'll just be exploring the elimination of the carrier. To accomplish this, we use a second emitter coupled pair cross-coupled with the first. Figure 4 shows a schematic of a four-quadrant multiplier, the MC1496, with biasing circuitry. It's a relatively simple circuit, so we still use it for SPICE models. In your real circuit, you'll be using the SA612, a more complex, but definitely more effective chip than the MC1496.

Still, they both work on the same principle. To investigate this, first think about what type of signal we are looking for. If we've eliminated the carrier signal, what does the signal look like in the time domain?

Now think about this circuit (Note that your output is still taken just below the two R_c resistors). Think of the bottom pair as a variable current source, that slowly rocks the current back and forth between the left and right upper pairs. Carefully trace how V_c affects the upper outer and inner pair. Think of this part of the circuit performing an inversion on the incoming current signal from below for half of every cycle of V_c. These current changes are then converted into voltages by the resistors labeled R_c.

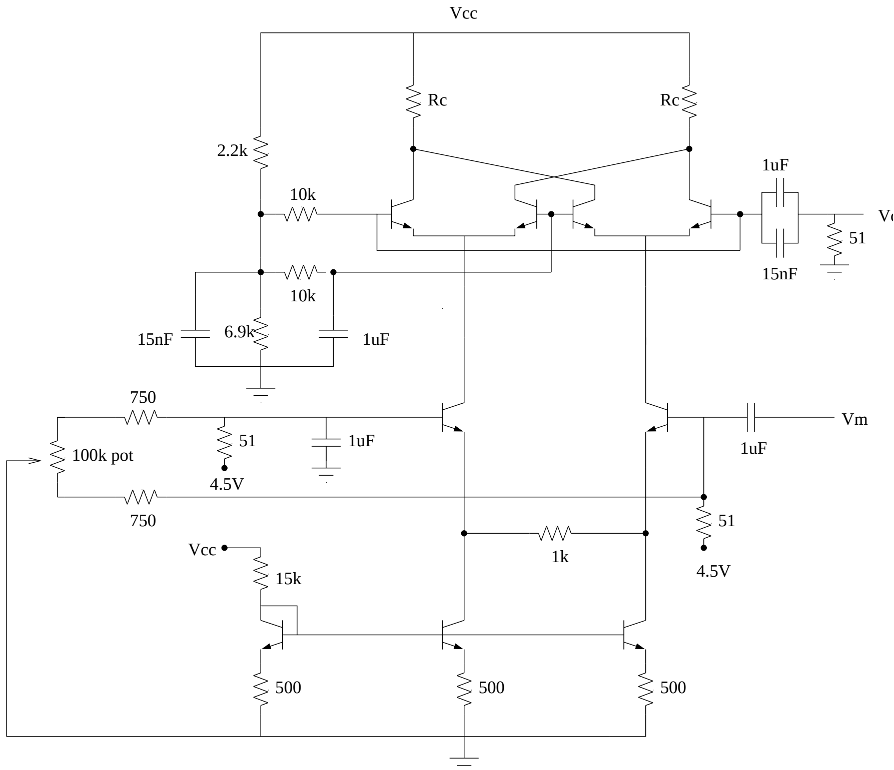

Figure 4: Four-Quadrant Multiplier

Hopefully this will make more sense with the following HSPICE simulations:

1. **Transient HSPICE Analysis:** Using the MC1496 SPICE deck, obtain the transient response for V_c= 2V; V_m = 1V. Include your results in your lab book.

   What is the conversion gain of this multiplier? How does this compare to the conversion gain for the two-quad?

2. **The Advantages of the Four-Quad Multiplier:** How does the output waveform differ from that of the two-quadrant multiplier? Can you use a peak detector with this type of waveform? Why or why not?

   For the lab, we will be using a different, slightly more complex multiplier, the SA612. This multiplier has its own internal biasing circuitry, and all you will need to add is a few DC blocking and supply bypass capacitors.

3. **Configuring the SA612:** Read the SA612 datasheet, which is available on the EE133 web site. Find out how to configure it for single-ended operation where we will use only one side of the output. Draw a schematic in your lab notebook. We have transformers in the lab which can theoretically improve the conversion gain of your circuit by 3dB, so you may consider using one of these in your circuit.

4. **Figures of Merit:** As you dig through the data sheet, please record the following 'Figures of Merit' these will prove useful to refer to as you experiment with the multiplier in lab, and when you later integrate it into your radio:

   - **Power Supply Voltage Range:** Unfortunately, all chips are not designed to run off of the same supply voltages. What is the range for the SA612? How do its characteristics vary over this range?

- **Input Resistance (R_{IN}):** This is important for impedance matching and maximum power or signal transfer.
- **Input Capacitance (C_{IN}):** This defines the imaginary part of the input impedance that you might wish to take into account when designing your impedance matching network between the LNA and the mixer.
- **Output Resistance (R_{OUT}):** This tells you what the source impedance is for your next stage.
- **Conversion Gain (G_{conv}):** As we stated before, this is the useful output voltage compared to the input.
- **Noise Figure (NF):** This is the Signal-to-Noise Ratio (SNR) divided by the SNR at the output port. In other words, this is a measure of how much noise has been added to your signal due to this circuit. Each block of your receiver will add more noise to the system, bu t, as we shall see later, the noise of successive blocks has less effect on the overall performance than the noise added by blocks closer to the input.
- **Third Order Intercept Point ($IIP3$):** We will go into more detail about this later in the quarter, but in a nutshell, this is the input power level at which the third order harmonics (i.e. distortions) at the output are equal to the desired signal at the fundamental frequency. You will want to operate at input powers a good distance below this point.

Now we can explore how these same multiplier chips can perform AM demodulation ...

7 AM Demodulation using Syncronous Detection

Though the simple envelope detector provides a decent demodulation of an AM signal, there are three major problems with the diode peak detector.

- It only works for signals that are less than 100% modulated
- The non-linear components distort the output
- The system attenuates the output signal rather than providing gain.

The synchronous AM detector is a more complex circuit that will provide a less distorted, larger output, and will also allow you to demodulate a signal having an arbitrary amount of modulation.

The theory behind the synchronous detector assumes that we have an AM modulated signal of the form:

$$v(t) = V_{DC}\cos(\omega_c t) + \frac{m}{2}\cos((\omega_c - \omega_m)t) + \frac{m}{2}\cos((\omega_c + \omega_m)t). \quad (7)$$

If we multiply this signal by the original carrier signal we will generate the following signal:

$$v(t) * \cos(\omega_c t) = \frac{V_{DC}}{2} + \frac{m}{2}\cos(\omega_m t) + \frac{1}{2}\cos(2\omega_c t) + \frac{m}{4}\cos((2\omega_c - \omega_m)t) + \frac{m}{4}\cos((2\omega_c + \omega_m)t). \quad (8)$$

The resulting signal will contain a DC term, a term at the modulating frequency, a term at twice the carrier frequency minus the modulating frequency, and a term at twice the carrier frequency plus the modulating frequency. We can use a coupling capacitor to remove the DC term and a low pass filter to eliminate the high frequency components, recover the modulated signal regardless of the incoming AM signal carrier amplitude. The only restriction on the demodulating signal is that it must be of exactly the same frequency as the carrier signal that originally generated the AM signal.

1. **Mathematical Analysis of Synchronous Demodulation:** Why is our last restriction true? Describe what happens when the demodulating signal is of a different frequency than the generating carrier signal.

2. **The Effect of Phase on Signal:** Why does the demodulating signal need to be phase matched? Give a mathematical explanation to justify your answer.

   Since synchronous detection is essentially the multiplication of an AM waveform with a carrier, the circuit necessary for a synchronous detector is (surprise!) a Four-Quadrant Multiplier. To test this circuit in lab, you will use the exact same multiplier from the previous section, the SA612.

3. **MATLAB Analysis:** There is a MATLAB script called on the EE133 website called 'synch_am.m' that produces an AM signal using a specific carrier frequency. The script then multiplies the AM signal by a demodulating signal of the same frequency as the carrier, and plots the result in both the time and frequency domains. It also filters the resulting signal through a low-pass filter to remove the high-frequency components, and plots the signal in both domains.

   Using 'synch_am.m', plot all graphs that are obtained for the following values of V_{DC}:0,0.5,1.0,2.0. Are there any restrictions on V_{DC}? Comment on the differences between this demodulator and the envelope detector from Section 4. Comment on distortion, expense, complexity, etc.

8 A Few Final Comments

We recommend coming into office hours to solder up at least one of your multipliers. That will leave you more time to perform your characterization.

Solder an SA612 to the receiver side of your board before coming to lab. Keep in mind that the amplifier in Lab 2 will eventually feed the RF port (the lower pair), and a crystal oscillator (XO) (AKA the local oscillator, or LO) will feed the upper pair. Orient your SA612 accordingly. Don't forget your AC coupling capacitors!

Once you've finished that, you can solder a second SA612 to the transmitter side of your board. On the transmitter side, the Voltage Controlled Oscillator(VCO) block will feed the RF input port with your slow moving modulating signal. Since this is a low-frequency signal, use a large (22 μF) capacitor for the modulating coupling cap. Electrolytics should be fine here. Remember to use a high-frequency AC coupling capacitor for the LO port (the upper pair). This one, as you can guess, should NOT be an electrolytic cap.

Good luck!