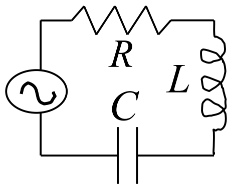
Tank Circuits/Impedances

Series Resonant “Tank” Circuit

$$Z(j\omega) = R\left[1 + jQ_s\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right]$$

where :



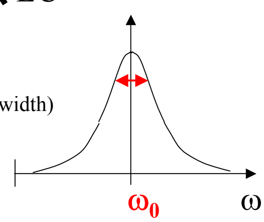
$$Q_s \equiv \frac{\omega_o L}{R} = \left(\frac{X_s}{R_s}\right)$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Per details given below…X_S can either be $\omega_0 L$ or $1/\omega_0 C$… (Basically, the same resistor could be associated with either L or C…)

Another KEY point…

Q is defined as ω_0/ **BW** (Bandwidth)



We need to understand and be able to QUICKLY manipulate parameters in TUNED CIRCUITS.

Here is one of the simplest tuned circuits, the so-called series resonant “tank” circuit.

Simple ac analysis gives us the first equation that relates the impedance (seen by the source) in terms of the R and two defined quantities: 1) the “Q” or quality factor of the circuit and 2) the resonant frequency ω_O

The plot of this impedance versus frequency gives a sharply peaked curve with minimum value equal to R and going to infinite impedance for very low and very high frequencies…

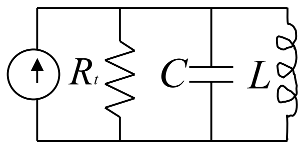
$1/j\omega C$-> infinite at $\omega=0$; $j\omega L$-> infinite as ω->infinite

The discussion of “Q” is fascinating (including the history of how it came to be called “Q”…no, it’s not from a James Bond movie:)

From an energy perspective--it’s the ratio of energy stored to energy lost per cycle (low loss-> high Q)

From a “bandwidth” perspective the Q tells how quickly signals will “fall off” away from the resonant frequency

Parallel Resonant "Tank" Circuit



$$Y(j\omega) = G_t\left[1 + jQ_p\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right]$$

where :

$$Q_p = \frac{\omega_o C}{G_t} = \frac{R_t}{1/\omega_o C} = \left(\frac{R_p}{X_p}\right)$$

Per details given below…X_p can either be $\omega_0 L$ or $1/\omega_0 C$…

<u>About notation and components</u>:

- We'll use the "p" and "s" subscripts (per text) and their definitions in terms of respective R and X values **(where X is <u>either ωL or 1/ωC</u>)**
- Components are NOT ideal!
- Basically, when talking about resonant circuits we want to have equivalent tank circuit (either series or parallel per above two cases)

The "dual" circuit, the parallel resonant tank circuit is shown here.

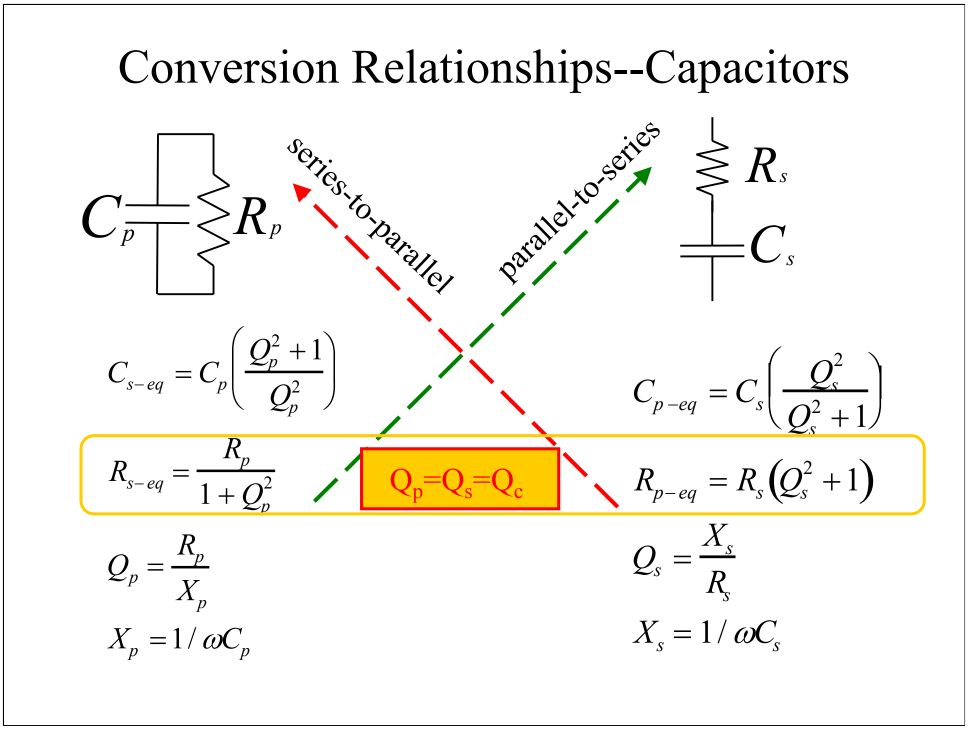
The equations are almost the same; the differences are as follows:

- We now have G instead of R
- There is a "Q" defined here as well--now we will start to pay careful attention to the "p" and "s" subscripts!
- BOTH circuits have the SAME resonant frequency for a given L and C.

There's plenty to discuss here; simple slides in class (even with notes etc.) will not substitute for you READING the assigned materials (now, via a Cadre of 133 Alums:)!

Considering the impedance of this circuit with frequency, again it is a sharply peaked curve.

Its peak value is R at ω_O and it goes to ZERO (impedance) at high and low frequencies…basically the C "shorts out" at the higher frequencies and the L is a short at dc.



Based on computations similar to the one shown for the inductance with series resistance (and resulting conversion to a parallel configuration), the above TRANSFORMATIONS show us how to move from one configuration to the DUAL.

For example, give the parallel version (left side) we know Q_P and X_P

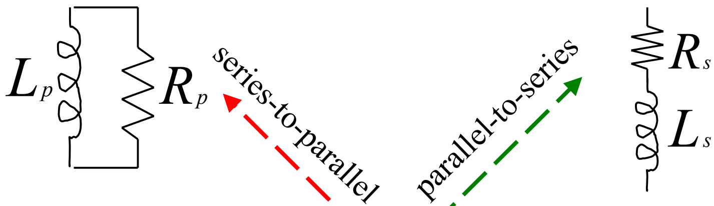
Using the Q_P and C_P values in the equations just below the circuit, we can calculate $C_{S\text{-}eq}$ and $R_{S\text{-}eq}$

Similarly, if we want to go from series to parallel, we start on the right side equations (from the bottom)

Determine Q_S and then use that in the formulae just below the circuit and compute $C_{P\text{-}eq}$ and $R_{P\text{-}eq}$

As you will see from the example presented in a moment as well as others related to lab exercises, we often need to (QUICKLY) do several of these manipulations to combine and simplify a more complex circuit to its minimal TANK equivalent.

Conversion Relationships--Inductors



$$L_{s-eq} = L_p\left(\frac{Q_p^2}{Q_p^2+1}\right) \qquad L_{p-eq} = L_s\left(\frac{Q_s^2+1}{Q_s^2}\right)$$

$$R_{s-eq} = \frac{R_p}{1+Q_p^2} \qquad Q_p=Q_s=Q_c \qquad R_{p-eq} = R_s\left(1+Q_s^2\right)$$

$$Q_p = \frac{R_p}{X_p} \qquad Q_s = \frac{X_s}{R_s}$$

$$X_p = \omega L_p \qquad X_s = \omega L_s$$

This is the same story, now for inductors.

Note, that one of these is EXACTLY what appeared in the supporting equations for page 3.

<u>Parallel-to-Series</u>:

•Determine Q_P

•Compute $L_{S\text{-}eq}$ and $R_{S\text{-}eq}$

<u>Series-to-Parallel</u>:

•Determine Q_S

•Compute $L_{P\text{-}eq}$ and $R_{P\text{-}eq}$

This last computation (series to parallel) was what appeared as part of the results on page 3

A bit more about "Q"

We can think about (and/or measure) the quality factor (Q) in two ways--parallel (sub_p) or series (sub_s).
There is ultimately only one "Q" for the circuit, we'll call it Q_c
The definitions of the parallel and series (component) Q's are as follows:

$Q_p=R_p/X_p$ (parallel or "shunt")

$Q_s=X_s/R_s$ (series)

Based on simple math, comparing the series and parallel cases for either capacitor or inductor:

$R_p=(Q_c^2+1)\ R_s$

and $Q_c=Q_p=Q_s$

We can derive all of the following relationships based on using the above relationships

This is a very interesting discussion and again "screams out"…time for you to READ the notes (and supplemental TEXT) related to this section.

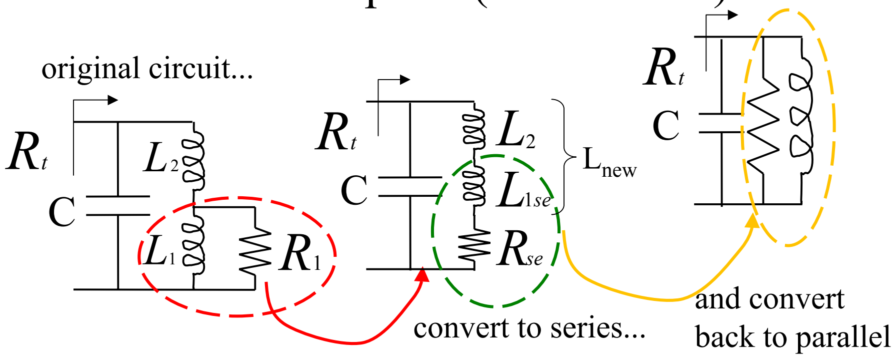
As shown in the previous "series" and "parallel" TANK circuits, there are "Q's" defined.

As the bottom equation says, there is only one Q for the circuit; depending on how we want to think about a given problem WE choose to use a parallel or series form.

There are conversions between the two…this is the first example where the relationship between R_P and R_S is defined in terms of the Q.

One PRACTICAL point to interject here…components such as inductors (for example) have their Q specified in terms of how they were measured (I.e. "Series Resonant Q=##)

An Example…(a.k.a. Lab 2)



<u>Following steps</u>:

- Combine L_2+L_{1se}(series equivalent)=L_{new}
- Convert L(new) & R_{se} back to parallel
- Result gives final equivalent tank circuit

Here’s the example related to lab. Basically, we need to get from a circuit with equivalent resistance R_t to the resistance R_1--for example, a 50Ω measurement equipment impedance.

So, if we TRANSFORM the parallel L_1 and R_1 to their series equivalent.

We can then follow the steps listed on the slide…

The key FINAL step is to determine the complete (simplified) parallel tank circuit values--especially R_t

Obviously, all this could be done in the opposite direction.

For example…

Assume you have a real circuit with an effective R_t and you want to make an “impedance match” to the 50Ω.

You can make an “inductive divider” circuit to achieve this impedance match.

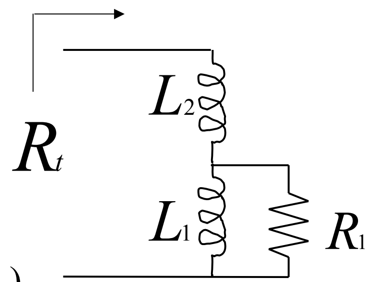
Design-Oriented Example

(including use of transformer concepts*…text (Sect. 6.8.2))

$$N \equiv \frac{L_1 + L_2}{L_1}$$

$$\frac{R_t}{R_1} = N^2$$

$$(per \quad book \quad notation\ )$$



$$N = \frac{n}{n_1}$$

$$L_T = L = L_2 + L_1$$

See p.226-7
Fig. 6.53 (HO#8)

Example:
R_1=50Ω
In order to have R_t=2KΩ,
N^2=40 or N=6.3=L_T/L_1

*caution: this simple relationship works for Q_p>10, otherwise one needs to do the transformations per above example

Here's more details about such an example, as well as some useful ways to think about the transforms (note the caution--there are limits to these simplifications based on Q_p values)

I'll primarily talk about the top set of equations and let you read the book discussion (with slightly different notation)

If we want to "match" 50Ω to a 2KΩ tank environment…

The second equation (left side) tells us that the ratio of the resistors gives us an "N^2" value.

Having determined that N…

We now know what the ratio of L_T/L_1 must be

Pretty COOL, yes?

In readings (TEXT and supplemental notes) the analogy to "transformers" are pushed a bit further.

Reminder about Biasing… **(both CE and CB)**

Analysis-oriented:

$$V_X = V_{CC}\left[\frac{R_2}{R_1 + R_2}\right]$$

$$I_C \approx I_E = \frac{V_X - V_{BE}}{R_E}$$

Design-oriented:

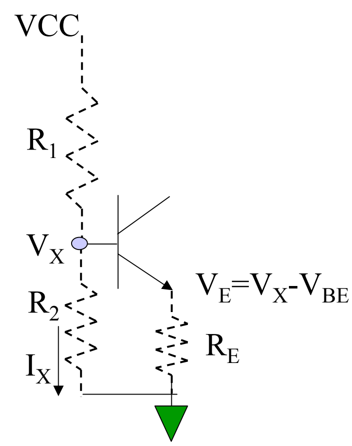
$$V_E = I_E \cdot R_E$$

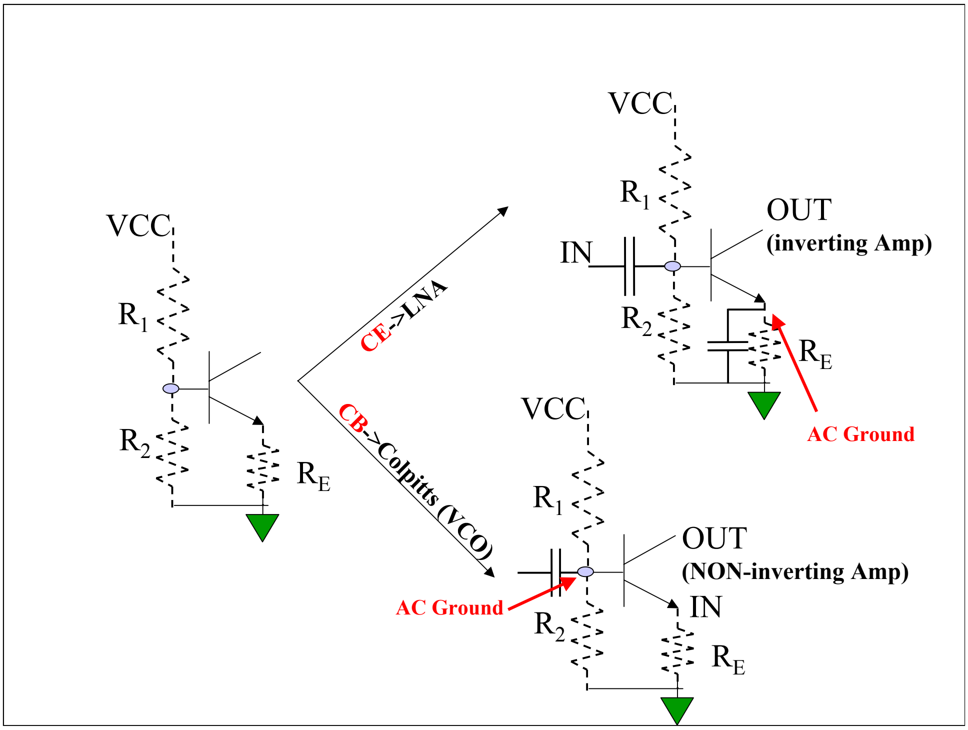
$(assume\ \ reasonable\ \ V_E \approx V_{BE})$

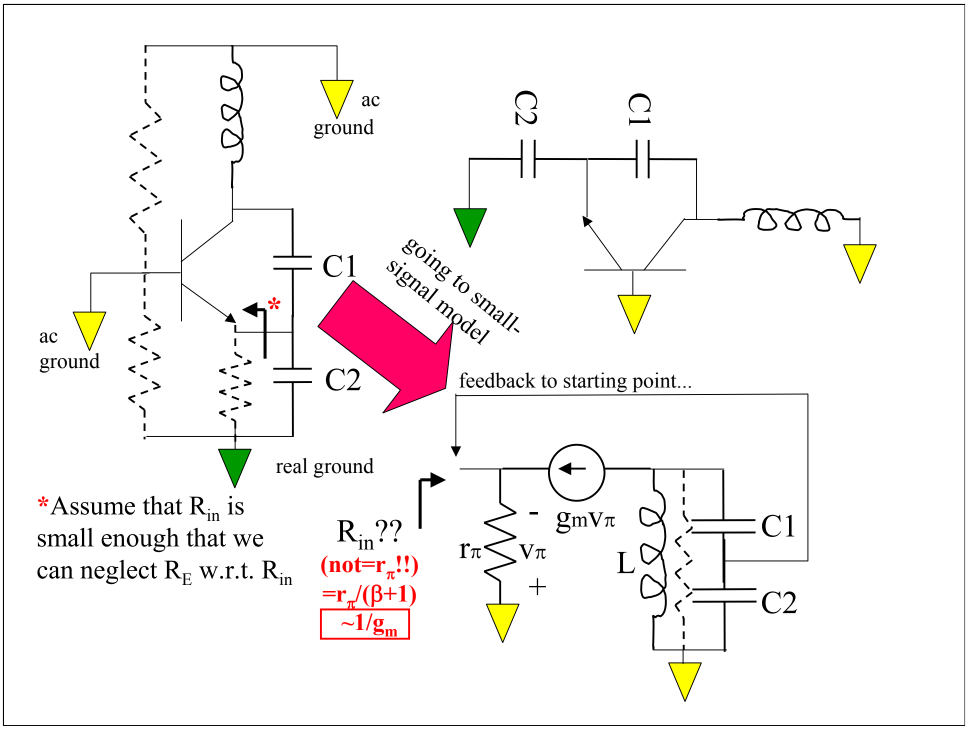
$$V_X = V_E + V_{BE}$$

$Choose\ \ R_1 \,\&\, R_2:$

$(make\ \ I_X\ \ fraction\ \ of\ \ I_C)$




VCC
R1
R2
RE
CE->LNA
CB->Colpitts (VCO)
VCC
R1
IN
OUT
(inverting Amp)
R2
RE
AC Ground
VCC
R1
OUT
(NON-inverting Amp)
AC Ground
R2
IN
RE



OK, let's work through the details…

This figure shows the transformation of the circuit into it's equivalent "small signal" equivalent.

Note that the arrow shows where the "input" to the CB is and viewed as a "voltage" input this appears across the base-emitter junction (and the r_π of the circuit)

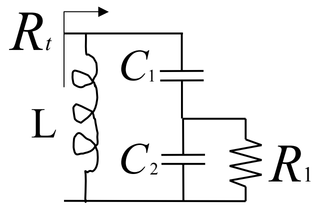
The impedance seen here is NOT r_π! What is it? (remember the reflection rules? It is reduced by $1/(\beta+1)$!)

Now, since we haven't done all the "tuned circuit" part of things yet, we initially will give you the parameters needed for lab in order to get oscillations at the desired frequency.

The following is more of the DETAILS of the analysis and design of the Colpitts. It will make much more sense after we have worked (in detail) on the tuned circuits part…

So, here goes with the analysis (and discussion).

Important Note (not in HO #7)

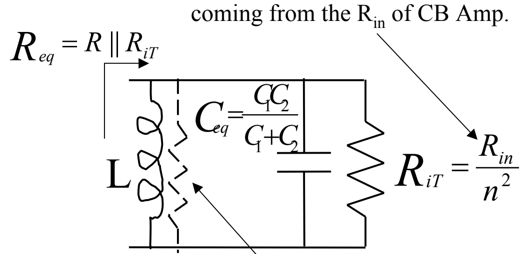


voltage divide ratio

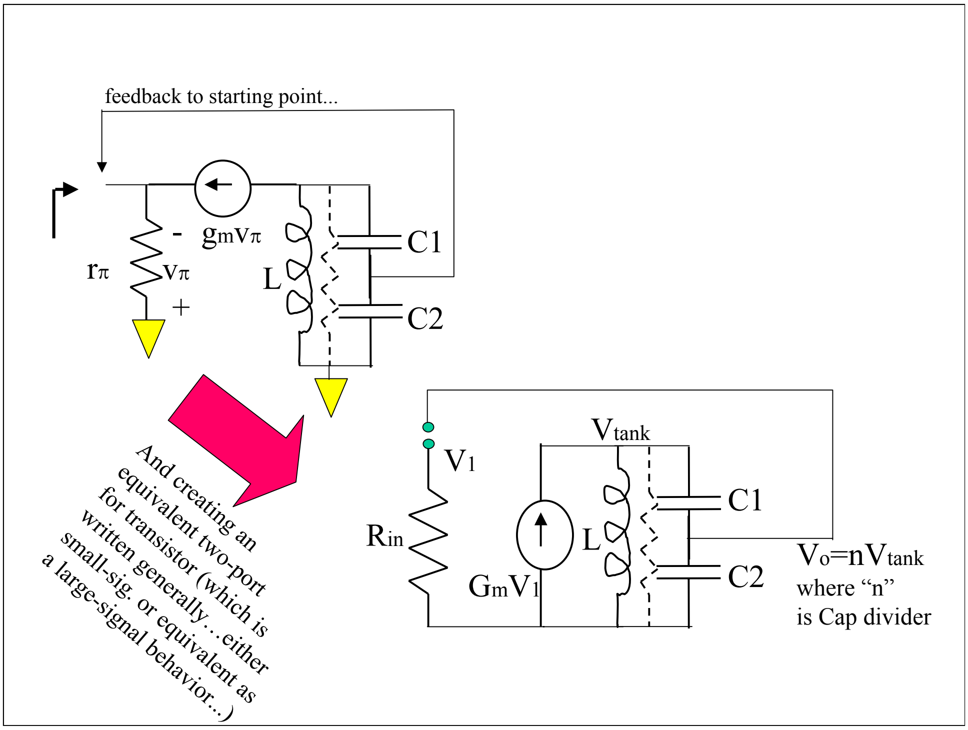
$n=C_1/(C_1+C_2)$

$R_t/R_1=1/n^2$

This is the "transformed R" coming from the R_{in} of CB Amp.



This is the "R" coming from the inductor L



Since we end up doing all the analysis in terms of voltage and for large signal we can not simply use the small-signal g_m, the circuit shown in the bottom right corner is a LARGE SIGNAL equivalent R_{in} and G_m

The input (loop) has been broken and we will compute the "loop gain" by inserting a voltage at V_1 and do the analysis "around the loop" to see what voltage comes back. [Comment on loading at C_2]

Clearly, $G_m V_1$ is the current that "drives" the load. There is an equivalent (complex, frequency selective) impedance seen at V_{tank}

And…

That voltage will be "divided down" by the capacitive divider ratio "n" [$=C_1/(C_1+C_2)$]

Hence, the overall "loop gain" will be:

$$A_{loop-gain} = G_m \cdot Z_{\tan k}(\omega_{resonance}) \cdot n$$

Assume a V1:

$V_{tank} = +G_m V_1 Z_{tank}$

at resonance, $Z_{tank}=R_{eq}=R_{iT} \,||\, R$
(where R is all other resistances* and R_{iT} comes from the **i**mpedance **T**ransform of R_{in} based on C_1 & C_2)

$Vo = n\, V_{tank}= [C_1/(C_1+C_2)]\, V_{tank}$

and, if $V_o = V_1$ we will have condition for oscillations

*Footnote: This notation follows T. Lee (copy from text attached in HO#5).

This slide simply walks through some of the intermediate steps that go with the words shown on the "notes" part of the previous slide.

A KEY (and at the moment a bit mysterious) part of this slide and portions of ones that follow is the "Impedance Transformation" part of things.

Basically, just as the capacitive "divider" divides down the voltage (V_{tank}), it also "transforms" up the impedance that appears across C_2

That is, if the input impedance seen at V_1 is R_{in}, the series C_1-C_2 combination makes that resistance appear much larger as seen at the V_{tank} node…(more to come soon in the next lectures)

Bottom-line:

Even if R_{in} is small (as we expect for the input of a CB stage), it's value gets "Impedance Transformed" up to a larger value so that the "loop gain" can be greater than unity--the condition needed to sustain oscillations.

At the highest level, we can use simple feedback theory to emphasize a couple of points:

Note*: this doesn't Specify where it Comes from...

$a = G_m R_{eq}$
$f = C_1/(C_1+C_2)$
af=1--> denominator is zero (in "feedback equation")
Or..
af>1 and the phase is such that we have POSITIVE feedback (the case for the Colpitts)

*Footnote: It turns out that, as shown in Fig. 16.6 (T. Lee book) the current flow in the device is highly non-linear (spiked in time as V_{BE} turns on) and we really can't use normal small-signal parameters for G_m. How to cope with that problem is discussed in Sect. 16.3.2 of T. Lee text (Ch.16)

This slide simply reiterates what we said at the beginning of lecture in slide one.

If "af" goes to unity and has phase such that it cancels the "1" in the feedback equation, we have the conditions for oscillations.

There are various "pointers" to sections/figures in the Tom Lee book on CMOS RF.

That book and these sections are really awesome in terms of presenting simple conceptual pictures and nice computations that give quantitative details about the oscillations, based on a large signal model that was constructed above. [Form of model is like the small-signal…Comments in class]

Basically, I'll very quickly highlight some of the points from the sections in the CMOS RF book of Prof. Lee.

Good News: Even if you feel "bogged down" in the math and derivation [next pages], the final equations/results give you a quick estimate of how large the expected voltage oscillations will be for the Colpitts and how to control them.

Detailed Equations…

Loading

$$\left(\frac{R_{iT}}{R_i}\right)^{\frac{1}{2}} = \frac{1}{n}$$

$$\therefore R_{iT} = \frac{R_i}{n^2} = \frac{1}{n^2 G_m}$$

$$R_{eq} = R \parallel R_{iT} = R \parallel \frac{1}{n^2 G_m}$$

$at..resonance:$

See T. Lee Book…

This is a highlight summary of the T. Lee discussion, Ch. 16, Section 16.3.4...

Voltage gain

$$\upsilon_{\tan k} = G_m \upsilon_1 \left(R \parallel \frac{1}{n^2 G_m}\right)$$

$$\upsilon_{\tan k} = (2 I_{bias}) \frac{R}{1 + n^2 G_m R}$$

the next not-so-obvious step uses

Eqn (7) Lee

Closing the Loop...

$$G_m = \frac{2 I_{bias}}{\upsilon_1} = \frac{2 I_{bias}}{n \upsilon_{\tan k}}$$

$$\upsilon_{\tan k} = \frac{2 I_{bias} R}{1 + \frac{n^2 \cdot R \cdot 2 I_{bias}}{n \cdot \upsilon_{\tan k}}} \Rightarrow$$

2

1

This slide won't make much/any sense unless you READ the sections from CMOS RF and you are "up to speed" on impedance transformations.

The top [bracketed] set of equations tell us what the equivalent tank impedance will be--the parallel combination of the R that's there due to the inductance and $(1/G_m)(1/n^2)$

This product of TWO terms is R_{in} times the transform. Since n<1, that means that $1/n^2$ will be a rather LARGE number.

To achieve large loop-gain we want $R||R_{iT}$ to still be large.

The middle part about "voltage gain" is rather obvious, with the exception of the $(2I_{tank})$ term (again…read the handout!)

Finally, the "Closing the Loop…" part is some detailed equation hacking. It is not hard but it's not obvious either….

I'll discuss in class a bit as time allows.

$$\upsilon_{tank} + n \cdot R \cdot 2I_{bias} = 2I_{bias}R$$

$$\boxed{\therefore \upsilon_{tank} = 2I_{bias}R \cdot (1-n)}$$

Other..notation :

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2};..\omega = \frac{1}{\sqrt{L\,C_{eq}}};..n = \frac{C_1}{C_1 + C_2}$$

This is the bottom-line result, giving the final tank voltage in terms of the bias current, R and the voltage divider ration n.

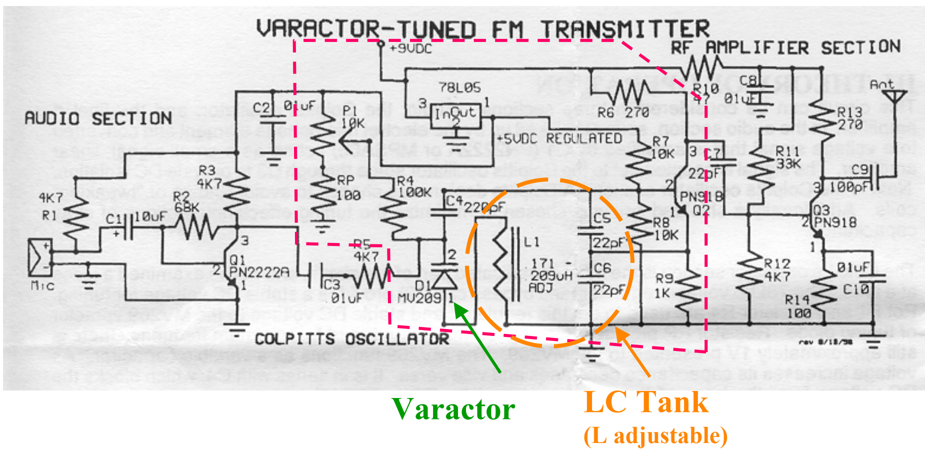
The final (DESIGN) result is the [boxed] equation for v_{tank} in terms of:

1) I_{bias}
2) R (the "load" at resonance provided by the "tank" circuit) and
3) (1-n) which is determined by the capacitive divider ratio

Hence, if you want larger tank voltages you need to increase I_{bias} or R and make "n" much smaller than unity (clearly you can only get so much from the "n" term vis a vis the bias current or the quality of the L-C components (which in turn controls the R)

Again, in thinking about and planning for lab, at this point we will have already "designed" the Colpitts. Your job will be to hook it up and make it oscillate--sometimes frustrating (like fishing, there are elements of both skill and luck involved…listen carefully to your TA/mentors :).

A Colpitts Oscillator Example



In lab you will be implementing the Colpitts and related circuitry for making it a "Voltage Controlled Oscillator" (VCO) by means of a Varactor (Variable Reactor) Diode--basically a voltage-variable capacitance (the C(V) of the diode in reverse bias--one of my favorite uses for diodes from EE111 :)

I'll leave the discussion of this figure to the text from the kit (per the next slide)

Hopefully after the lab and all your hard-won experience, reading this discussion will make LOTS of sense and you can even explain it to your little brother or sister who will certainly want to follow in your footsteps as a gEEk…building such kits to prepare for their future career path.

The Simplified Explanation

The Colpitts oscillator section is the most complicated part of the circuit, and will be examined a piece at a time. The 78L05 voltage regulator (and bypass cap C2) provides a stable DC voltage for tuning. Pot P1 and Resistor R4 are used to get this regulated and stable DC voltage to the MV209 varactor or tuning diode. Resistor RP provides "biasing" so that when tuned for minimum frequency there is still approximately 1V presented to the MV209. The MV209 functions as a variable capacitor. As voltage increases its capacitance decreases and vice versa. It is in series with C4, which blocks the DC voltage from the base of the oscillator transistor Q2 (a PN918). Also in this series is L1, a variable coil (.171-.190 uH) or inductor. Tuning depends on the value of inductor L1 in series with capacitors MV209 (the varactor), C4 and C5. This series capacitance resonates with L1 to determine the oscillator frequency. Taken together, these three capacitances and the inductor L1 form a tank circuit. The audio signal is fed into the tank at the junction of the varactor and C4. This audio signal modulates the resonant frequency of the tank circuit. The rest of the Colpitts consists of Q2 (PN918) and biasing components R6, R7, R8 and R9. Finally, the modulated oscillator signal is coupled through C7 to the RF amplifier stage.

As stated in the last slide, I won't reiterate the above discussion. However, I would like to point out one key feature of this design that may indeed be important (and useful in the future)

There is a single-chip voltage regulator used in this design to keep the Varactor voltage sable.

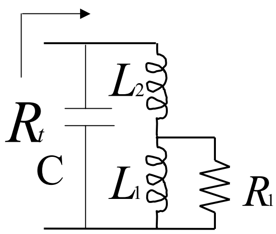
Note in the text that this is considered to be important in achieving "frequency stability"

As you will see as we get into the SPAMMING exercises, the tuning (and drift) of your oscillators is a MAJOR challenge.

[More to come on all this]

Useful (more complete) Design Equations

(per supplemental book by Krauss)



$For\quad Q_c = Q_t \approx \omega_o / BW \ge 10$

$$C \approx \frac{1}{2\pi \cdot BW \cdot R_t}$$

$$L \approx \frac{1}{\omega_o^2 C}$$

$$N = \left(\frac{R_t}{R_1}\right)^{\frac{1}{2}}$$

$$\frac{Q_t}{N} = Q_p \ldots If \quad \frac{Q_t}{N} \ge 10$$

$Q_p<10$ / $Q_p>10$

$For\quad Q_p \lhd 10:$

$$Q_p = \left(\frac{Q_t^2+1}{N^2} - 1\right)^{\frac{1}{2}}$$

$$L_1 = \frac{R_1}{\omega_o Q_p}$$

$$L_{1\_se} = \frac{L_1 Q_p^2}{Q_p^2+1}$$

$$L_2 = L - L_{1\_se}$$

$For\quad Q_p \ge 10:$

$$Q_p = \frac{Q_t}{N}$$

$$L_1 = \frac{L}{N}$$

$$L_2 = (N-1)L_1 = L - L_1$$

This is one more pass on a very similar problem (left over from a year where we used the text by Krauss…also a useful book)

This problem starts from the assumptions that we want to have a particular Bandwidth (BW) about a given resonant frequency ω_O

The next two equations basically specify (completely) the relationships between C, R_t and L (based on the parallel tank formulae)

The bottom half of the slide shows how to determine L_1 and L_2 for the cases where $Q_P<10$ and $Q_P>10$

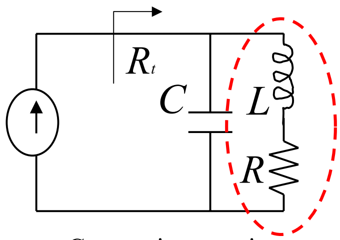
Note: the manipulations on the left-hand equations follow directly from transformations we've looked at on page 6. The one "twist" in all this is that Krauss used Q_t--basically the total TANK circuit value. The relationship between Q_t and Q_P comes from the fact that Q_P is defined only in terms of L_1 whereas Q_t is defined in terms of L_1+L_2 as follows:

$$Q_P \equiv \frac{R_1}{\omega_O L_1}$$

$$Q_t \equiv \frac{R_t}{\omega_O (L_1 + L_2)}$$

Example of non-ideal Inductor

(and transforming back to equivalent tank circuit)



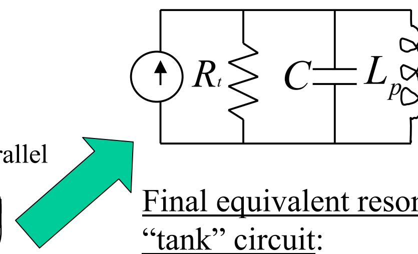
Conversion…series-to-parallel

$$L_p = L\left(\frac{Q_s^2 + 1}{Q_s^2}\right)$$

$$R_t = R\left(Q_s^2 + 1\right)$$

where :

$$Q_s = \frac{\omega_o L}{R}$$



<u>Final equivalent resonant “tank” circuit</u>:

- •transformed L now sets resonant frequency
- •let’s see how these transformations work...

Now we get to the exciting part…real components and how they are used in and affect circuits!

If we tried to make an “ideal” LC filter, both the L and the C have parasitic series resistance--due to contacts on the capacitor and the intrinsic resistance of wire in the inductor.

We’ll start by considering just the inductor…

Now, if we take this circuit and “brute force” solve for the equivalent parallel tank circuit parameters, we find that there will be equivalent R_t and L_P values as shown in the equations.

The Q_P value is the same as what we computed in our initial discussions.

Note: in the definition of the two parallel R and L terms that the Q_S value appears.

This is a result of the math + definition of Q_S

In the following couple of pages we will generalize the two-way transformations that allow us to move backwards and forwards (QUICKLY) from SERIES to PARALLEL circuit topologies…