About the Smith Chart

The Smith chart is a plot of

$$\Gamma = \frac{Z_N - 1}{Z_N + 1} \qquad (9.2)$$

in the Γ-plane as a function of r and x. Using Equation 9.2 and separating Γ in terms of its real part (U) and imaginary part (V) we obtain:

$$Z_N = r + jx$$

$$\Gamma = \frac{r + jx - 1}{r + jx + 1} = U + jV \qquad (9.3)$$

$$U = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} \qquad (9.4)$$

$$V = \frac{2x}{(r+1)^2 + x^2} \qquad (9.5)$$

At this juncture we note that by using Equations 9.4 and 9.5, we can obtain two families of circles that, when superimposed on each other, will make up the entire Smith chart. The procedure to obtain these two families of circles is described next.

1. Constant-*r* *circles:* The first family of circles is obtained by eliminating x from Equations 9.4 and 9.5, which gives:

$$\left(U - \frac{r}{r+1}\right)^2 + V^2 = \left(\frac{1}{r+1}\right)^2 \qquad (9.6)$$

Equation 9.6 represents a family of circles with a center located at

$$(U_o, V_o) = \left(\frac{r}{r+1}, 0\right), \qquad (9.7a)$$

and with a radius of

$$R = \left(\frac{1}{r+1}\right). \qquad (9.7b)$$

From Equations 9.7, we can observe that all constant-r circles are centered on the real axis with a shrinking size as "r" is increased. In this regard, we note that the $r = 0$ circle is the outermost circle of the Smith chart while the $r = \infty$ circle is reduced to a point at (0,1). Figure 9.1 depicts this concept further.

2. Constant-*x* circle: The second family of circles is obtained by eliminating *r* from Equations 9.4 and 9.5, which gives:

$$(U-1)^2+\left(V-\frac{1}{x}\right)^2=\left(\frac{1}{x}\right)^2 \qquad (9.8\text{a})$$

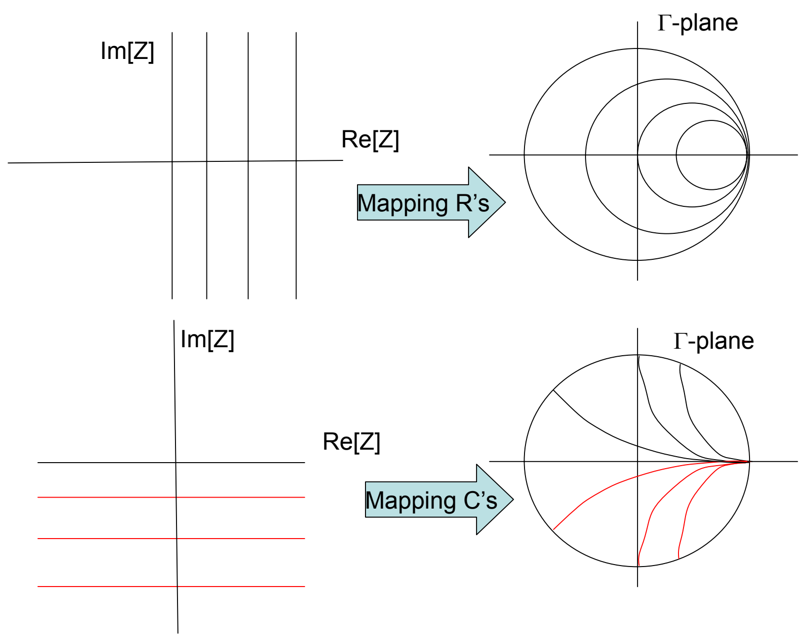
Equation 9.8a represents a family of circles with a center located at

$$(U'_o\,,V'_o)=\left(1,\frac{1}{x}\right) \qquad (9.8\text{b})$$

and with a radius of

$$R'=\frac{1}{|x|} \qquad (9.8\text{c})$$

From Equations 9.8 we can observe that all constant-*x* circles are centered on a shifted line parallel to the imaginary axis (by +1 unit to the right), with a shrinking size as *x* increases. In this regard, we note that the $x=0$ circle is the real axis of the Smith chart while the $x=\pm\infty$ circles are reduced to a point at (1,0). This is shown in Figure 9.1.



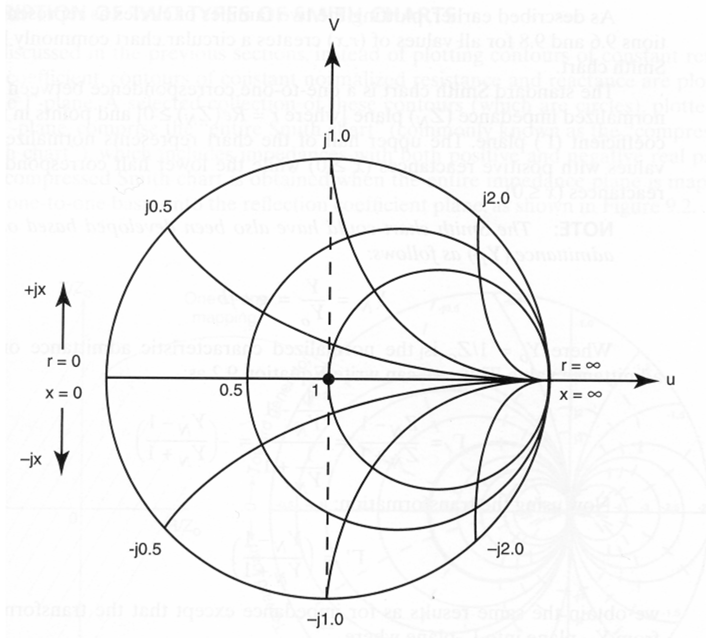


FIGURE 9.1 Construction of a standard Smith chart (for $r \geq 0, -\infty \leq x \leq \infty$).

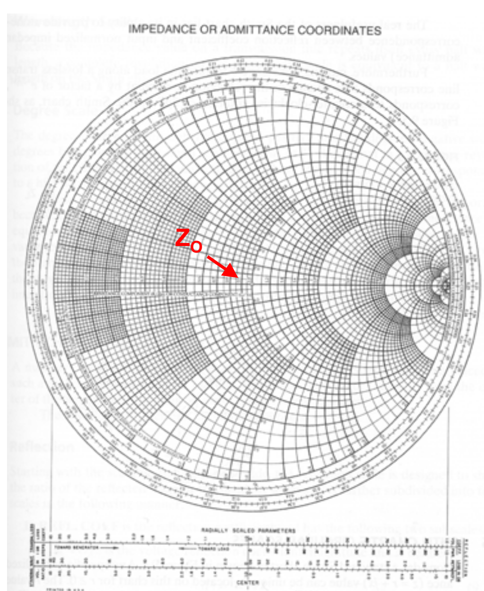
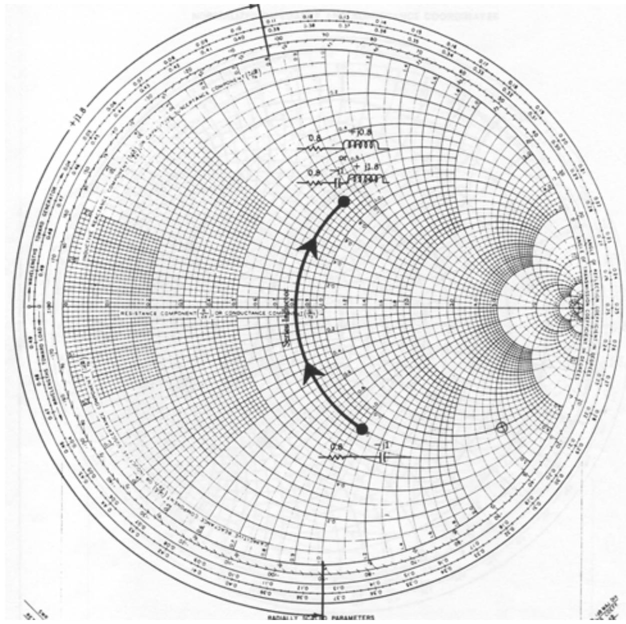
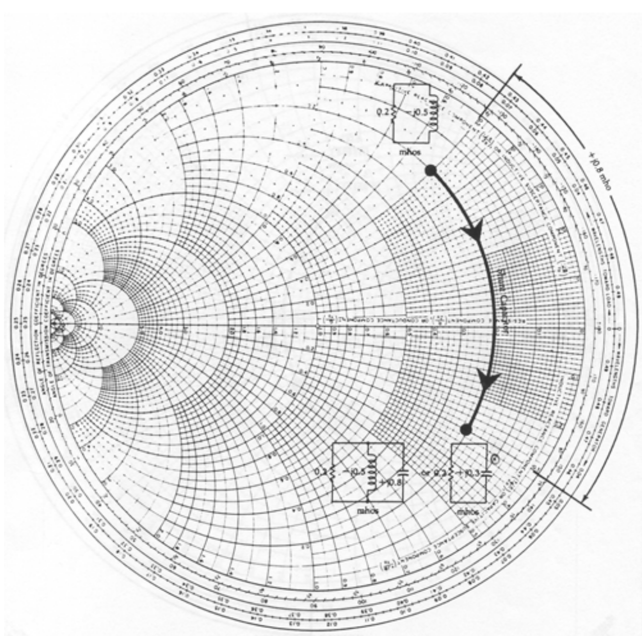
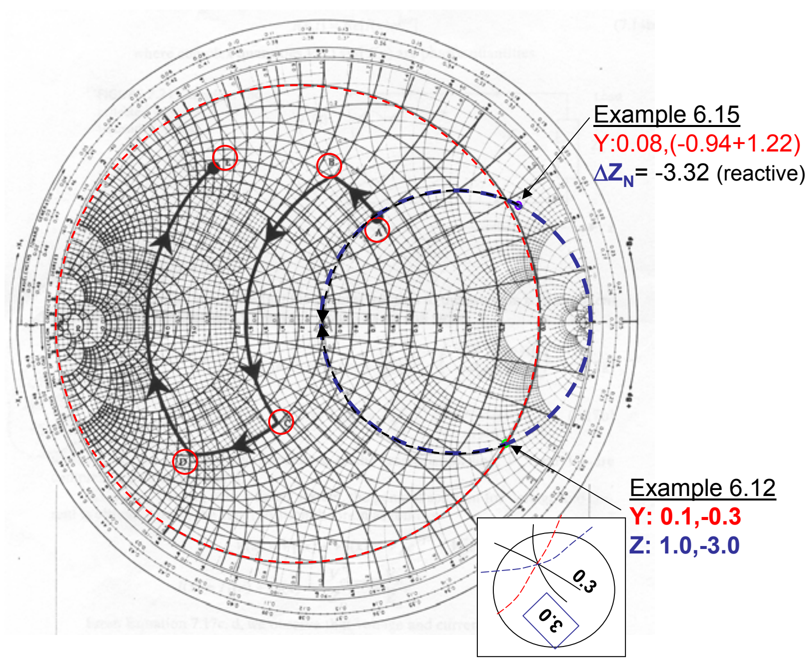


FIGURE 9.4 The standard Smith chart.
(Reproduced with permission of Analog Instruments Co., New Providence, NJ.)


0.8
+j0.8
or
RESISTANCE COMPONENT
RADIALLY SCALED PARAMETERS


0.2
−j0.5
mhos
+j0.8 mho
Shunt Capacitor
+j0.8
mhos



Solution:
From Figure 6.59 we have:

$$Z_1 = 1000 \parallel -j333 = 100 - j300\ \Omega$$

$$Z_2 = j300 + (100 - j300) = 100\ \Omega$$

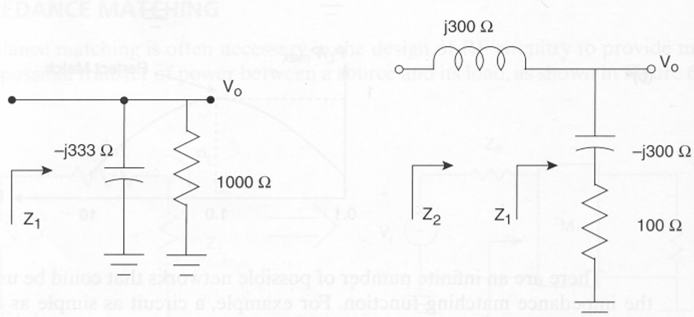


FIGURE 6.59 Circuit analysis for Example 6.12.

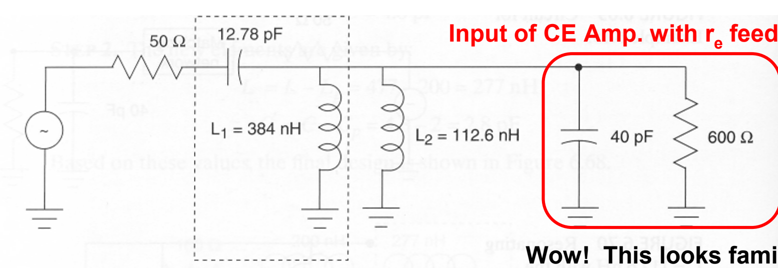
A bit more discussion of Ex. 6.12

Parallel -j333Ω corresponds to 1/jωC
Doing the parallel to series conversion:
$Q_P=R_P/X_P=1000/333=$**3**
Using formula Parallel-to-Series--
$C_S=[(Q_P^2+1)/Q_P^2](1/\omega X_P)=[10/9](1/\omega 333\Omega)$
Therefore..
$1/\omega C_S=X_S=$**300Ω**
Reminder about the R's…
$R_S=R_P/(Q_P^2+1)=1000/(9+1)=$**100Ω**
The above two sets of #s related to **Fig. 6.59**
Looking at all this on the Smith Chart…
The series combination gets normalized by **100Ω**
Thus, (100, 300)-> **(1, -3)**
If we write it in terms of Y_N (corresponding to the parallel values)
We get, Z_P->Y_P-> 1000/100->10->1/10 and
For the capacitive term 333/100->3.3->1/3.3 so that…
Z_P of (1000, 333) goes to $\mathbf{Y_N}$ of **(0.1, -0.3)**

This is sort of messy…

And **YOU** need to do it yourself to have it make sense

Input of CE Amp. with r_e feedback



Wow! This looks familiar :)

FIGURE 6.72 Using L_1 and L_2 to match a capacitive load to 50 Ω.

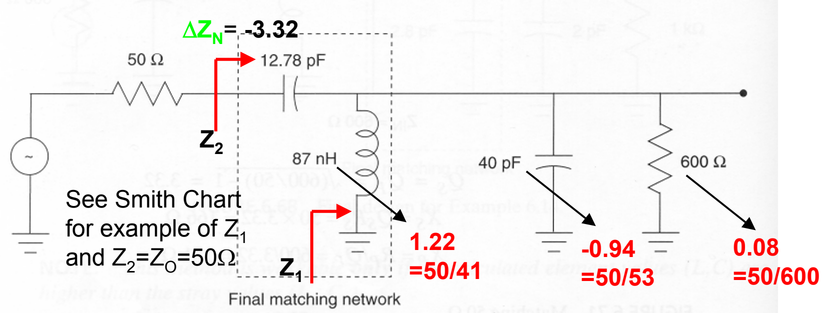


FIGURE 6.73 The final circuit design of Example 6.15.