Device-Level Noise Modeling

(excerpts from Gray & Meyer text

Analysis and Design of Analog Integrated Circuits,

John Wiley)

11.3.2 Bipolar Transistor[11]

In a bipolar transistor in the forward-active region, minority carriers diffuse and drift across the base region to be collected at the collector-base junction. Minority carriers entering the collector-base depletion region are accelerated by the field existing there and swept across this region to the collector. The time of arrival at the collector-base junction of the diffusing (or drifting) carriers is a purely random process, and thus the transistor collector current consists of a series of random current pulses. Consequently, collector current I_C shows *full shot noise* as given by (11.2) and this is represented by a shot noise current generator $\overline{i_c^2}$ from collector to emitter as shown in the equivalent circuit of Fig. 11.13.

Base current I_B in a transistor is due to recombination in the base and base-emitter depletion regions and also to carrier injection from the base into the

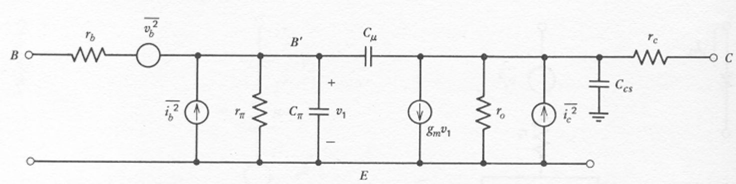


Fig. 11.13 Complete bipolar transistor small-signal equivalent circuit with noise sources.

emitter. All of these are independent random processes, and thus I_B also shows *full shot noise*. This is represented by shot noise current generator $\overline{i_b^2}$ in Fig. 11.13.

Transistor base resistor r_b is a physical resistor and thus has thermal noise. Collector series resistor r_c also shows thermal noise, but since this is in series with the high-impedance collector node, this noise is negligible and is usually not included in the model. Note that resistors r_π and r_o in the model are *fictitious* resistors that are used for modeling purposes only, and they do *not* exhibit thermal noise.

The full small-signal equivalent circuit including noise for the bipolar transistor is shown in Fig. 11.13. Since they arise from separate, independent physical mechanisms, all the noise sources are *independent* of each other and have mean-square values:

$$\overline{v_b^2} = 4kTr_b\,\Delta f \qquad (11.11)$$

"Johnson" noise

$$\overline{i_c^2} = 2qI_C\,\Delta f \qquad (11.12)$$

$$\overline{i_b^2} = \underbrace{2qI_B\,\Delta f}_{\text{Shot noise}} + \underbrace{K_1 \frac{I_B^{\,a}}{f}\,\Delta f}_{\text{Flicker noise}} + \underbrace{K_2 \frac{I_B^{\,c}}{1 + \left(\frac{f}{f_c}\right)^2}\,\Delta f}_{\text{Burst noise}} \qquad (11.13)$$

"shot" noise

(ignore both of these)

This equivalent circuit is valid for both *npn* and *pnp* transistors. For *pnp* devices, the magnitudes of I_B and I_C are used in the above equations.

11.3.5 Capacitors and Inductors

Capacitors are common elements in integrated circuits, either as unwanted parasitics or as elements introduced for a specific purpose. Inductors in general cannot be realized on the silicon die but are sometimes used as external elements, particularly in integrated communication circuits. There are *no sources of noise* in *ideal* capacitors or inductors. In practice, real components have parasitic resistance that *does* display noise as given by the thermal noise formulas of (11.4) and (11.5). In the case of integrated-circuit capacitors, the parasitic resistance usually consists of a small value in series with the capacitor. Parasitic resistance in inductors can be modeled either by series or shunt elements.

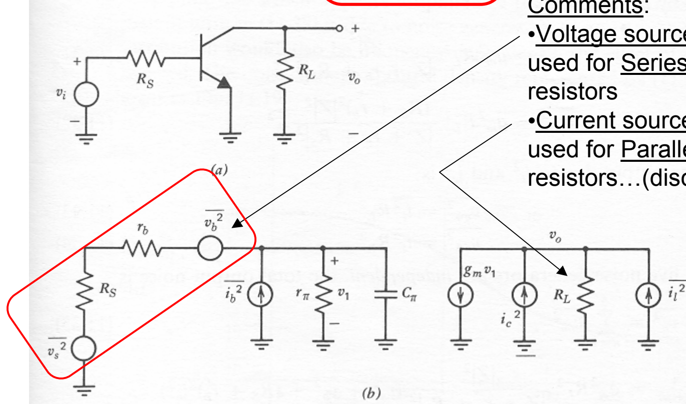
11.4.1 Bipolar Transistor Noise Performance

As an example of the manipulation of noise generators in circuit calculations, consider the noise performance of the simple transistor stage with the ac schematic shown in Fig. 11.18*a*. The small-signal equivalent circuit including noise is shown in Fig. 11.18*b*. (It should be pointed out that, for noise calculations, the equivalent circuit analyzed must be the actual circuit configuration used. That is, Fig. 11.18*a* cannot be used as a half-circuit representation of a differential pair for the purposes of noise calculation because noise sources in each half of a differential pair affect the total output noise.)

In the equivalent circuit of Fig. 11.18*b*, the external input signal v_i has been ignored so that output signal v_o is due to noise generators only. C_μ is assumed small and is neglected. Output resistance r_o is also neglected. The transistor noise generators are as described previously and in addition

$$\overline{v_s^2} = 4kTR_S\,\Delta f \qquad (11.24)$$

$$\overline{i_l^2} = 4kT\frac{1}{R_L}\Delta f \qquad (11.25)$$



Comments:
•Voltage source representation is used for Series-connected resistors
•Current source representation is used for Parallel-connected resistors…(discussion in class!)

Fig. 11.18 (*a*) Simple transistor amplifier ac schematic. (*b*) Small-signal equivalent circuit with noise sources.

Z(frequency)

Substituting expressions for the noise generators we obtain

$$\frac{\overline{v_o^2}}{\Delta f} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2}[4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B] + R_L^2\left(4kT\frac{1}{R_L} + 2qI_C\right) \quad (11.37)$$

Red=Johnson
Blue=Shot

where flicker noise has been assumed small and neglected. Substituting for Z from (11.27) in (11.37) we find

$$\frac{\overline{v_o^2}}{\Delta f} = g_m^2 R_L^2 \frac{r_\pi^2}{(r_\pi + R_S + r_b)^2}\frac{1}{1 + \left(\frac{f}{f_1}\right)^2}[4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B] + R_L^2\left(4kT\frac{1}{R_L} + 2qI_C\right) \quad (11.38)$$

basically, input τ_{RC} of the circuit

where

$$f_1 = \frac{1}{2\pi(r_\pi \| R_S)C_\pi} \quad (11.39)$$

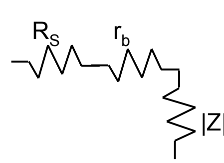
Let's look at just ONE TERM…

$$\frac{\overline{v_o^2}}{\Delta f} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2}[4kT(R_S + r_b)$$

Basic gain expression (SQUARED)

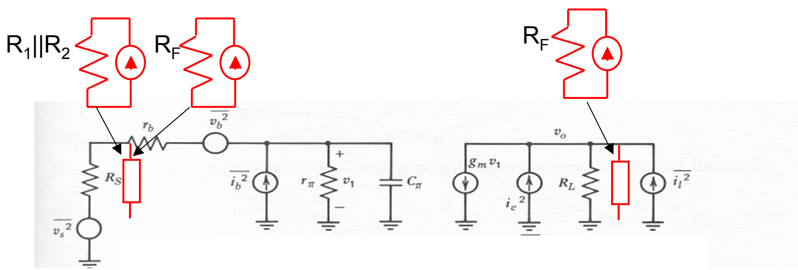
Voltage divider expression at the input (SQUARED)

Transfer Function for Johnson Noise



Johnson Noise for **two resistors** <u>in series</u> (per unit BW)

Additional Noise Sources:
- Bias Resistors (parallel)
- Shunt-Shunt R_F (parallel)



Analysis: One needs to determine the transfer function for each noise source to the output and then SQUARE it in order to get the respective POWER relationship…

EXAMPLE

In order to give an appreciation of the numbers involved, specific values will now be assigned to the parameters of (11.38) and the various terms in the equation will be evaluated. Assume that

Typical numbers…
I_C, low
R's, a bit high…

$$I_C = 100\ \mu\text{A} \qquad \beta = 100 \qquad r_b = 200\ \Omega$$
$$R_S = 500\ \Omega \qquad C_\pi = 10\ \text{pF}$$
$$R_L = 5\ \text{k}\Omega$$

Substituting these values in (11.38) and using $4kT = 1.66 \times 10^{-20}$ V-C gives

$$\frac{\overline{v_o^2}}{\Delta f} = \left[5.82 \times 10^{-18} \frac{1}{1 + \left(\frac{f}{f_1}\right)^2} (700 + 9.4) \right.$$

dominant term… **resistors**, not I_B

$$\left. + 1.66 \times 10^{-20} (5000 + 48{,}080) \right] \text{V}^2/\text{Hz}$$

dominant term… I_C and not resistors

$$= \frac{4.13 \times 10^{-15}}{1 + \left(\frac{f}{f_1}\right)^2} + 0.88 \times 10^{-15}\ \text{V}^2/\text{Hz} \qquad (11.40)$$

If I_C increases **10X**, becomes **8.8x10^-15**

Equation 11.39 gives

$$f_1 = 32.4 \text{ MHz} \tag{11.41}$$

Equation 11.40 shows that the output noise-voltage spectral density is 5.0×10^{-15} V^2/Hz at low frequencies and it approaches 0.88×10^{-15} V^2/Hz at high frequencies. The major contributor to the output noise in this case is the source resistance, R_S, followed by the base resistance of the transistor. The noise spectrum given by (11.40) is plotted in Fig. 11.19.

EXAMPLE

Suppose the amplifier in the above example is followed by later stages that limit the bandwidth to a sharp cutoff at 1 MHz. Since the noise spectrum as shown in Fig. 11.19 does not begin to fall significantly until $f_1 = 32.4$ MHz, the noise spectrum may be assumed constant at 5.0×10^{-15} V^2/Hz over the bandwidth 0 to 1 MHz. Thus the *total* noise voltage at the output of the circuit of Fig. 11.18*a* in a 1-MHz bandwidth is

$$\overline{v_{oT}^2} = 5.0 \times 10^{-15} \times 10^6 \text{ V}^2 = 5.0 \times 10^{-9} \text{ V}^2$$

Determined by **BW!**

and thus

$$v_{oT} = 71\ \mu\text{V rms} \tag{11.42}$$

Now suppose that the amplifier of Fig. 11.18*a* is *not* followed by later stages that limit the bandwidth but is fed directly to a wideband detector (this could be an oscilloscope or a voltmeter). In order to find the total output noise voltage in this case, the contribution from each frequency increment Δf must be summed at the output. This reduces to *integration* across the bandwidth of the detector of the noise-voltage spectral-density curve of Fig. 11.19. For example, if the detector had a 0 to 50-MHz bandwidth with a sharp cutoff then the total output noise would be

$$\overline{v_{oT}^2} = \sum_{f=0}^{50\times10^6} S_o(f)\,\Delta f = \int_0^{50\times10^6} S_o(f)\,df \tag{11.43}$$

where

$$S_o(f) = \frac{\overline{v_o^2}}{\Delta f} \tag{11.44}$$

is the noise spectral density defined by (11.40). In practice, the exact evaluation of such integrals is often difficult and approximate methods are often used. Note that if the integration of (11.43) is done graphically, the noise spectral density versus frequency must be plotted on *linear scales*.

11.4.2 Equivalent Input Noise and the Minimum Detectable Signal

In the previous section, the output noise produced by the circuit of Fig. 11.18 was calculated. The significance of the noise performance of a circuit is, however, the limitation it places on the smallest input signals the circuit can handle before the noise degrades the quality of the output signal. For this reason, the noise performance is usually expressed in terms of an *equivalent input noise signal*, which gives the same output noise as the circuit under consideration. In this way, the equivalent input noise can be compared directly with incoming signals and the effect of the noise on those signals is easily determined. For this purpose, the circuit of Fig. 11.18 can be represented as shown in Fig. 11.20 where $\overline{v_{iN}^2}$ is an input noise-voltage generator that produces the same output noise as all of the original noise generators. All other sources of noise in Fig. 11.20 are considered removed. Using the same equivalent circuit as in Fig. 11.18*b*, we obtain, for the output noise from Fig. 11.20,

$$\overline{v_o^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2} \overline{v_{iN}^2} \tag{11.45}$$

Gain Expression (Transfer Function) SQUARED

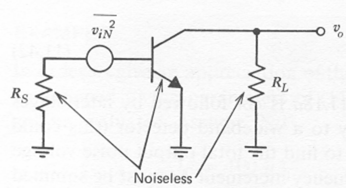
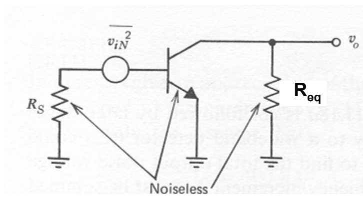


Fig. 11.20 Representation of circuit noise performance by an equivalent input noise voltage.

If this noise expression is equated to $\overline{v_o^2}$ from (11.37), the equivalent input noise voltage for the circuit can be calculated as

$$\frac{\overline{v_{iN}^2}}{\Delta f} = 4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B + \frac{1}{g_m^2 R_L^2} \frac{|Z + r_b + R_S|^2}{|Z|^2} R_L^2 \left(4kT\frac{1}{R_L} + 2qI_C\right) \tag{11.46}$$

Note that the noise-voltage spectral density given by (11.46) *rises* at high frequencies because of the variation of $|Z|$ with frequency. This is due to the fact that as the *gain* of the device falls with frequency, output noise generators $\overline{i_c^2}$ and $\overline{i_l^2}$ have a larger effect when referred back to the input.



<u>Rough Approximation of Open-Loop LNA (Shunt-Shunt Feedback):</u>

At <u>output</u>--$R_{eq}=R_L||R_F$

At <u>input</u>--make ONE equivalent noise resistor ($R_{S\text{-}eq}$) that is in series with r_b

The "transfer function" used to refer output noise to the input DOES NOT CHANGE…basically the parallel resistors ($R_1||R_2||R_F$) are basically now part of **new** "Z^*"

$$\frac{\overline{v_{iN}^2}}{\Delta f} = 4kT(R_{S\text{-}eq}+r_b)$$

$$+ \frac{1}{g_m^2 R^2_{eq}} \frac{|Z^* + r_b + R_S|^2}{|Z^*|^2} R^2_{eq} \left(4kT\frac{1}{R_{eq}} + 2qI_C\right) \qquad (11.46)$$

Modified
(per above discussion)

EXAMPLE

Calculate the *total* input noise voltage, $\overline{v_{iNT}^2}$, for the circuit of Fig. 11.18 in a bandwidth of 0 to 1 MHz.

This could be calculated using (11.46) derived above. Alternatively, since the total output noise voltage, $\overline{v_{oT}^2}$ has already been calculated, this can be used to calculate $\overline{v_{iNT}^2}$ (in a 1-MHz bandwidth) by dividing by the circuit voltage gain squared. If A_v is the low-frequency, small-signal voltage gain of Fig. 11.18, then

$$A_v = \frac{r_\pi}{r_b + r_\pi + R_S} g_m R_L$$

Use of the previously specified data for this circuit gives

$$A_v = \frac{26{,}000}{200 + 26{,}000 + 500}\frac{5000}{260} = 18.7$$

Since the noise spectrum is flat up to 1 MHz, the low-frequency gain can be used to calculate $\overline{v_{iNT}^2}$ as

(see above comment about **SQUARED**)

$$\overline{v_{iNT}^2} = \frac{\overline{v_{oT}^2}}{A_v^2} = \frac{5 \times 10^{-9}}{18.7^2}\text{V}^2 = 14.3 \times 10^{-12}\text{ V}^2$$

Thus we have

$$v_{iNT} = 3.78\ \mu\text{V rms}$$

The above example shows that in a bandwidth of 0 to 1 MHz, the noise in the circuit *appears to come* from a 3.78-μV rms noise-voltage source in series with the input. This noise voltage can be used to estimate the smallest signal that the circuit can effectively amplify, sometimes called the *minimum detectable signal* (MDS). This depends strongly on the nature of the signal and the application. If no special filtering or coding techniques are used, the MDS can be taken as equal to the equivalent input noise voltage in the passband of the amplifier. Thus, in this case,

$$\text{MDS} = 3.78\ \mu\text{V rms}$$

If a sinewave of magnitude 3.78 μV rms were applied to this circuit, and the output in a 1-MHz bandwidth examined on an oscilloscope, the sine wave would be barely detectable in the noise, as shown in Fig. 11.21. The noise waveform in this figure is typical of that produced by shot and thermal noise.

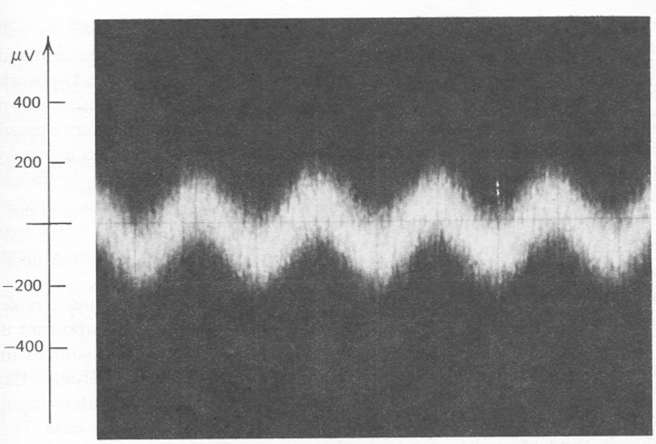
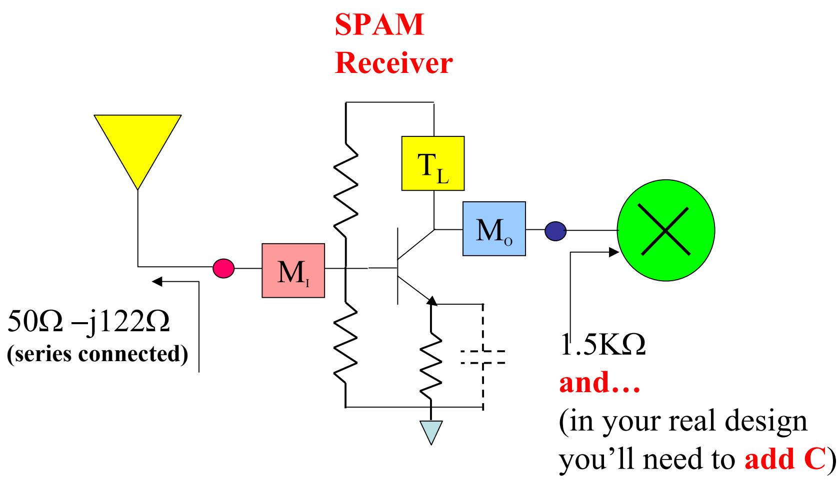
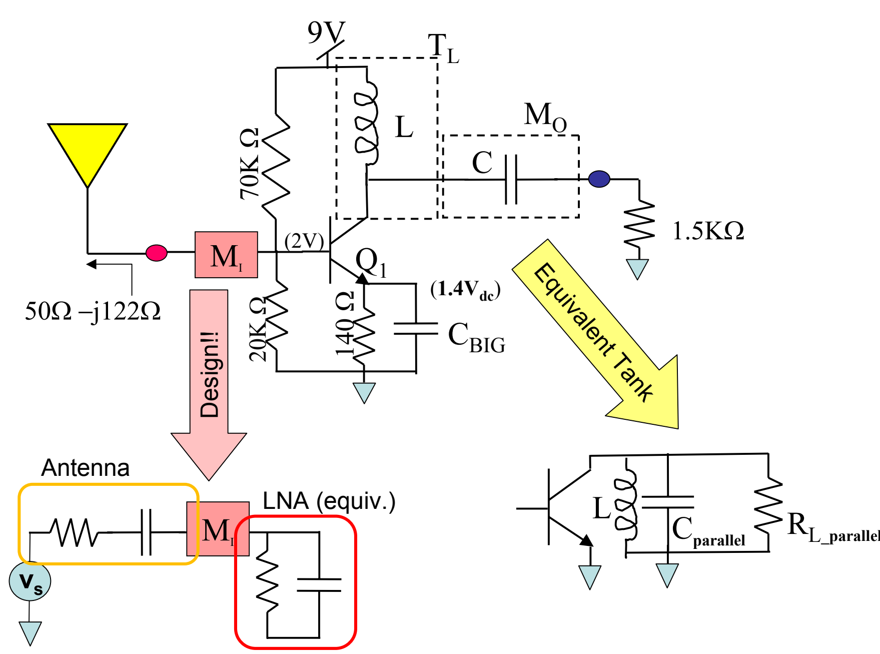


Fig. 11.21 Output voltage waveform of the circuit of Fig. 11.18 with a 3.78-μV rms sinewave applied at the input. The circuit bandwidth is limited to 1 MHz, which gives an equivalent input noise voltage of 3.78 μV rms.

Mission Possible (in 1hr. +??)

(a reminder that Mid-Term will ask you to do "Mission Possible" In-Class and in finite time)





Blocks for SPAM Receiver