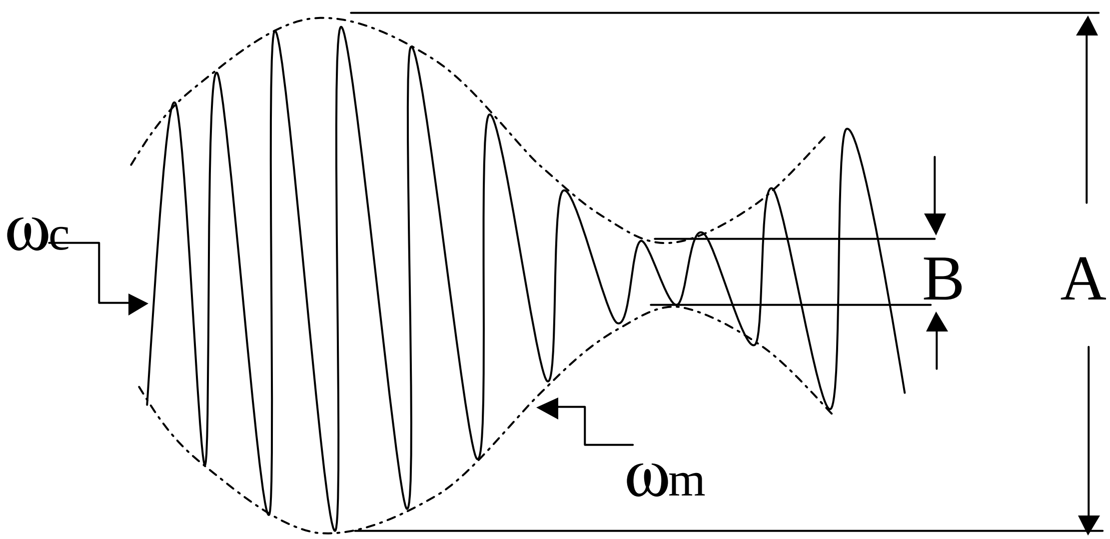
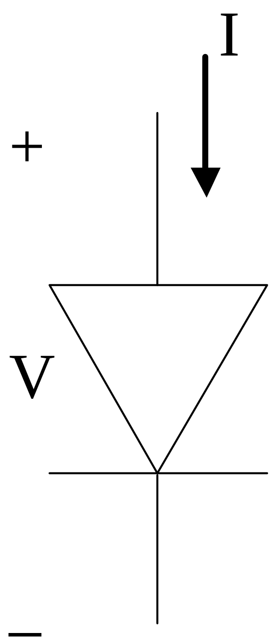
Back to our discussion of AM



$$m \equiv \frac{n_m}{n_c} = \frac{A - B}{A + B}$$

“m” is modulation index, which can be determined graphically using A and B. It corresponds to the vm and vc terms that we will see shortly for real voltages (and currents).

A Trivial Multiplier…



$$I = I_s[e^{\frac{qV}{kT}} - 1] \approx I_s[(1 + x + \frac{x^2}{2} + \frac{x^3}{3!}\ldots) - 1]$$

$$I \approx I_s[x + \frac{x^2}{2} + \frac{x^3}{3!}\ldots]$$

$$where:$$

$$x \equiv \frac{qV}{kT}$$

This is multiplication… but, one only gets x-times-x (assuming that one can filter out the higher and lower frequencies…) and…
All the other terms are higher order distortion. We'll have more serious discussion of this soon.

The most basic multiplier

(called the "transconductance multiplier")

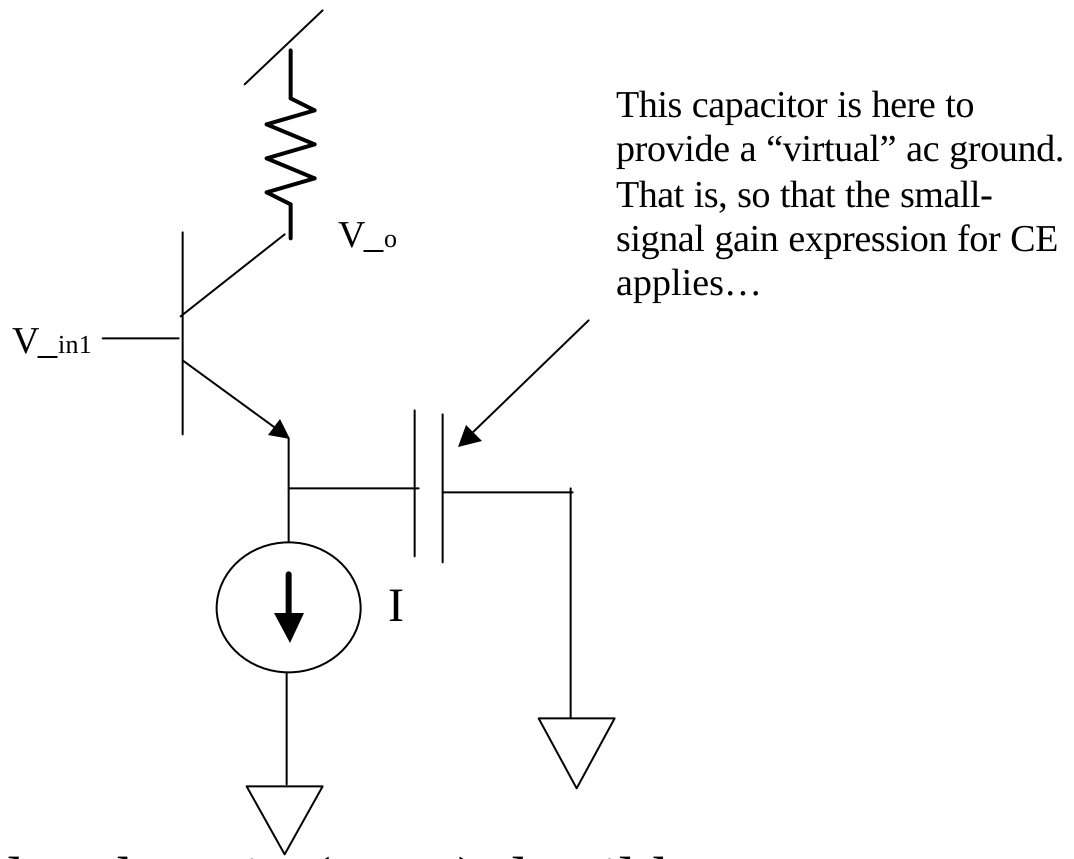
$$v_o = -g_m R_L \cdot v_{in1}$$

$$but...what\_if:$$

$$g_m = I(v_{in2}) \cdot \frac{q}{kT}$$

$$then:$$

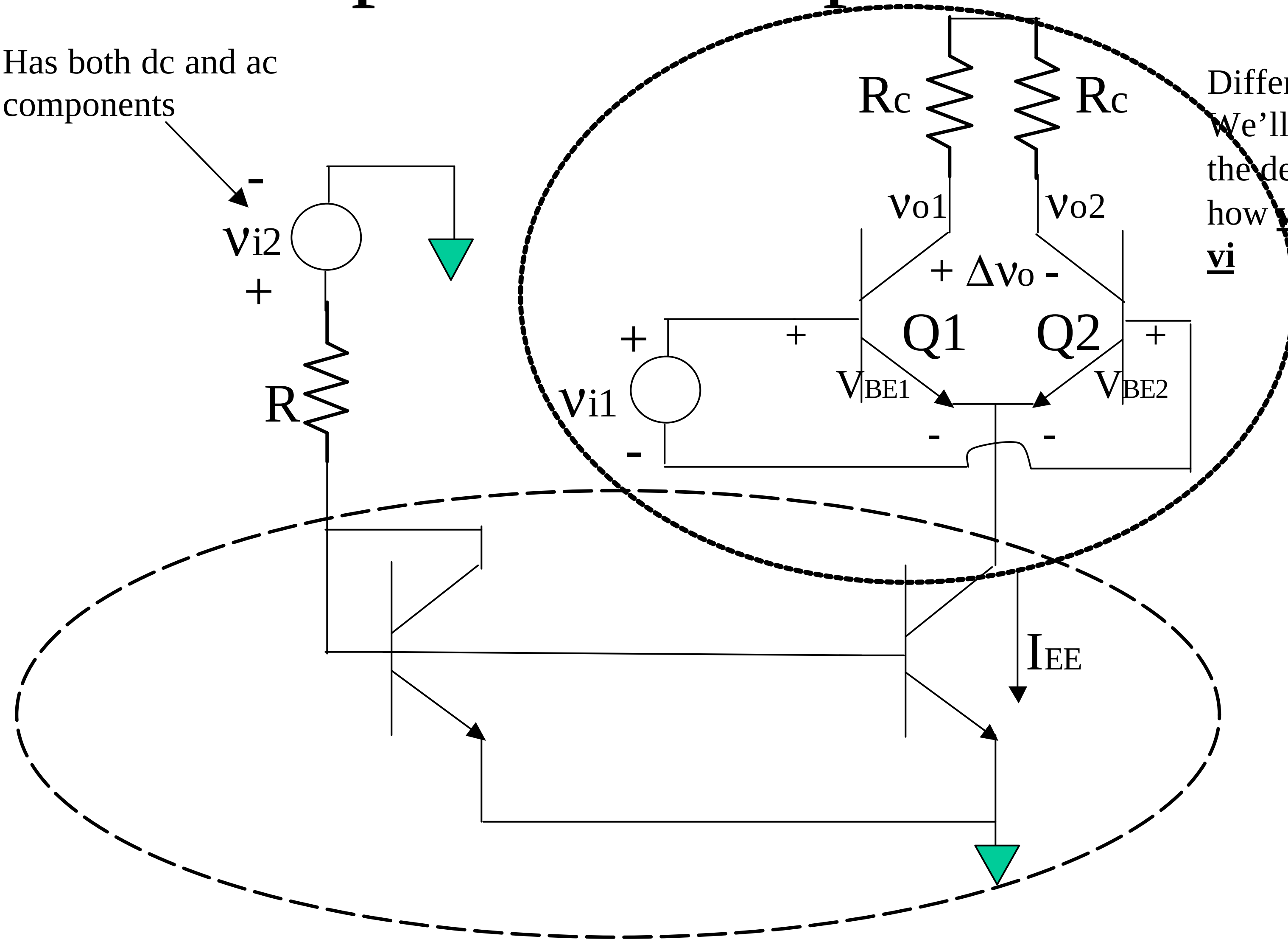
$$v_o = -R_L \frac{q}{kT}[I(v_{in2}) \cdot v_{in1}]$$



The following slides show in (great) detail how we actually realize this simple circuit in practive…

A practical implementation

Has both dc and ac components



Differential Pair: We'll need to revisit the derivation of how **vo** depends on **vi**

Current Mirror where IEE is determined by (v_{i2}-V_{BE})/R

The basic BJT “Laws” and their applications

$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

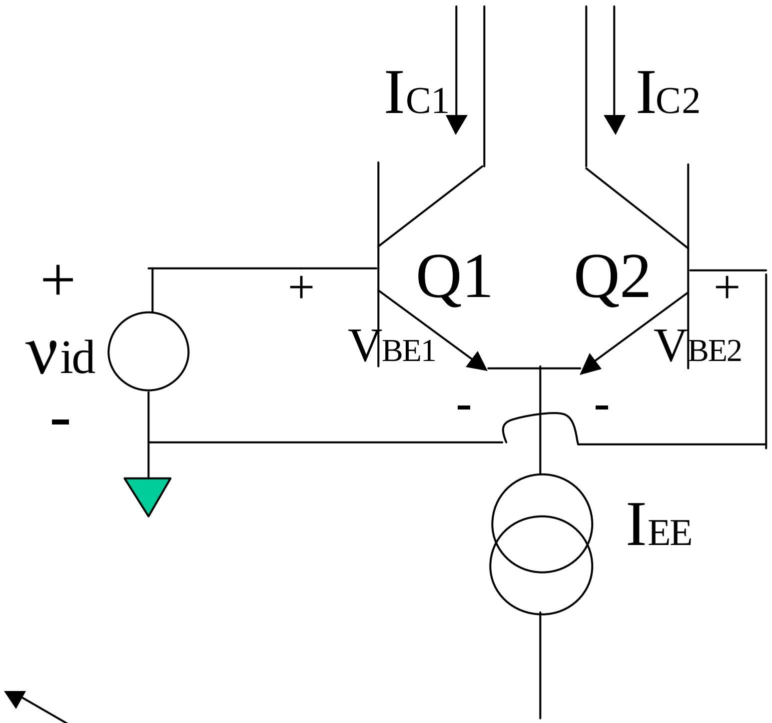
$$V_T = \frac{kT}{q}$$

$$\therefore V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$$

$$and\_for\_each\_Q_x$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right); V_{BE2} = V_T \ln\left(\frac{I_{C2}}{I_S}\right)$$

$$\therefore v_{id} = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right)$$



Each transistor (Q) has it’s V_{BE} determined by the log of the ratio Ic/Is

And the differential input $v_{id} = V_{BE1} - V_{BE2}$ is determined by the log of the ratio I_{c1}/I_{c2}

In terms of the two Ic's and vid…

$$I_{C1} = I_{C2} e^{\frac{+v_{id}}{V_T}} \ldots or \ldots I_{C2} = I_{C1} e^{\frac{-v_{id}}{V_T}}$$

$$I_{C1} + I_{C2} = I_{EE}$$

$$and\_with\_\mathrm{mod}est\_"hacking"$$

$$I_{C1} = I_{EE} - I_{C2} = \frac{I_{EE}}{1 + e^{\frac{-v_{id}}{V_T}}}$$

$$I_{C2} = I_{EE} - I_{C1} = \frac{I_{EE}}{1 + e^{\frac{+v_{id}}{V_T}}}$$

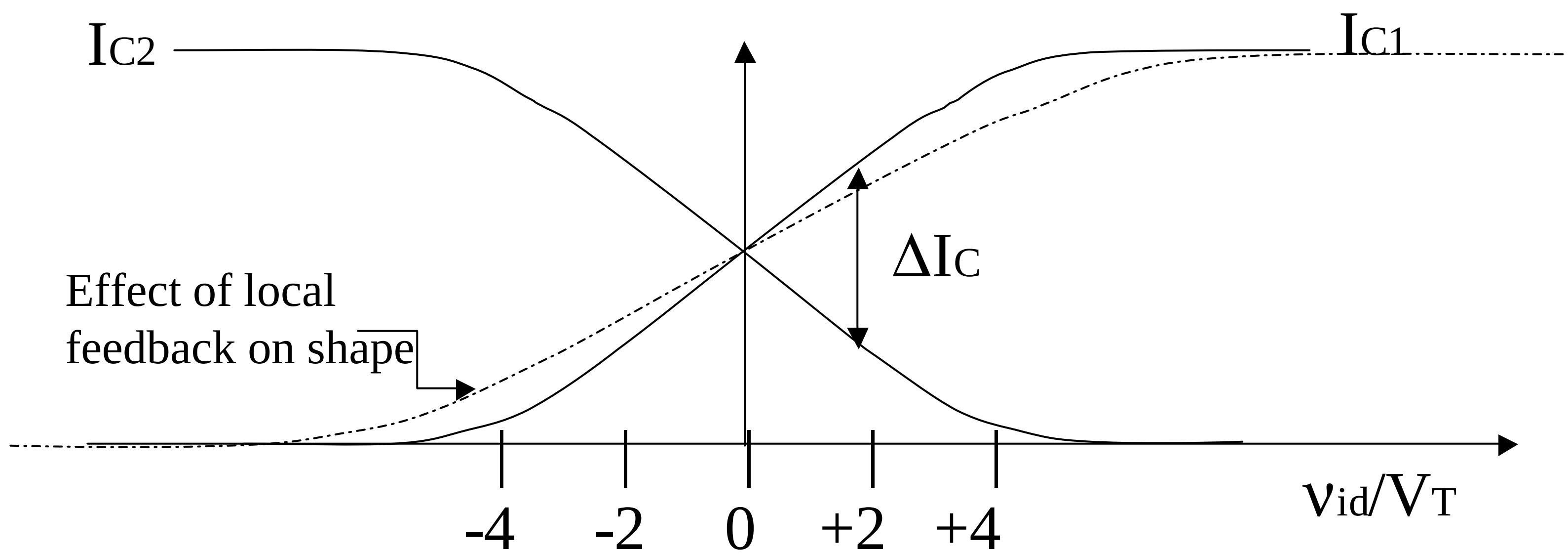
Basically, we express each Ic interms of the total "tail current" IEE and the total input (differential) voltage vid

$$\Delta I_C = I_{C1} - I_{C2} = I_{EE} \left(\frac{1}{1 + e^{\frac{-v_{id}}{V_T}}} - \frac{1}{1 + e^{\frac{+v_{id}}{V_T}}} \right)$$

$$\Delta I_C = I_{EE} \tanh\left(\frac{v_{id}}{2V_T} \right)$$

This is the general expression for the difference of the two collector currents in terms of the input voltage…we will soon see that in small-signal it reduces to gm times vid

What do Collector Currents look like?



$$In\_addition...$$

$$and\_in\_small\_signal\_regime:$$

$$\tanh\left(\frac{v_{id}}{2V_T}\right) \approx \frac{v_{id}}{2V_T}$$

$$\therefore \Delta I_C \approx I_{EE}\frac{v_{id}}{2V_T}$$

$$I_{C1} = I_{C2} = \frac{I_{EE}}{2} \equiv I_C$$

$$g_m \equiv \frac{I_C}{V_T}$$

$$\therefore \Delta I_C = g_m v_{id}$$

And now for the details about I_{EE}

$$I_{EE} = \frac{v_{i2} - V_{BE}}{R} \ldots where \ldots v_{i2} = v_{i2}(dc) + v_{i2}(ac)$$

$$= \frac{v_{i2}(dc) - V_{BE}}{R} + \frac{v_{i2}(ac)}{R}\cos(w_m t)$$

$$I_{EE} = \frac{v_c}{R} + \frac{v_m}{R}\cos(w_m t) \ldots where:$$

$$v_c \equiv v_{i2}(dc) - V_{BE}$$

$$v_m \equiv v_{i2}(ac)$$

Basically, we've achieved the form we initially assumed for the "Amplitude Modulating" part of our AM signal

And using the above results in ΔIC

$$\Delta I_C = I_{EE}\tanh\left(\frac{v_{id}}{2V_T}\right) = \frac{1}{R}(v_c + v_m \cos w_m t)\tanh\left(\frac{v_{id}}{2V_T}\right)$$

$$and\_now\_considering\_R_C\_loading:$$

$$\Delta v_o = -\Delta I_C R_C = \frac{-R_C}{R}(v_c + v_m \cos w_m t)\tanh\left(\frac{v_{id}}{2V_T}\right)$$

$$(and\_for\_small\_signal...)$$

$$\boxed{\Delta v_o = \frac{-R_C}{2V_T R}(v_c + v_m \cos w_m t)\cdot \bar{v}_{id}\cos w_c t}$$

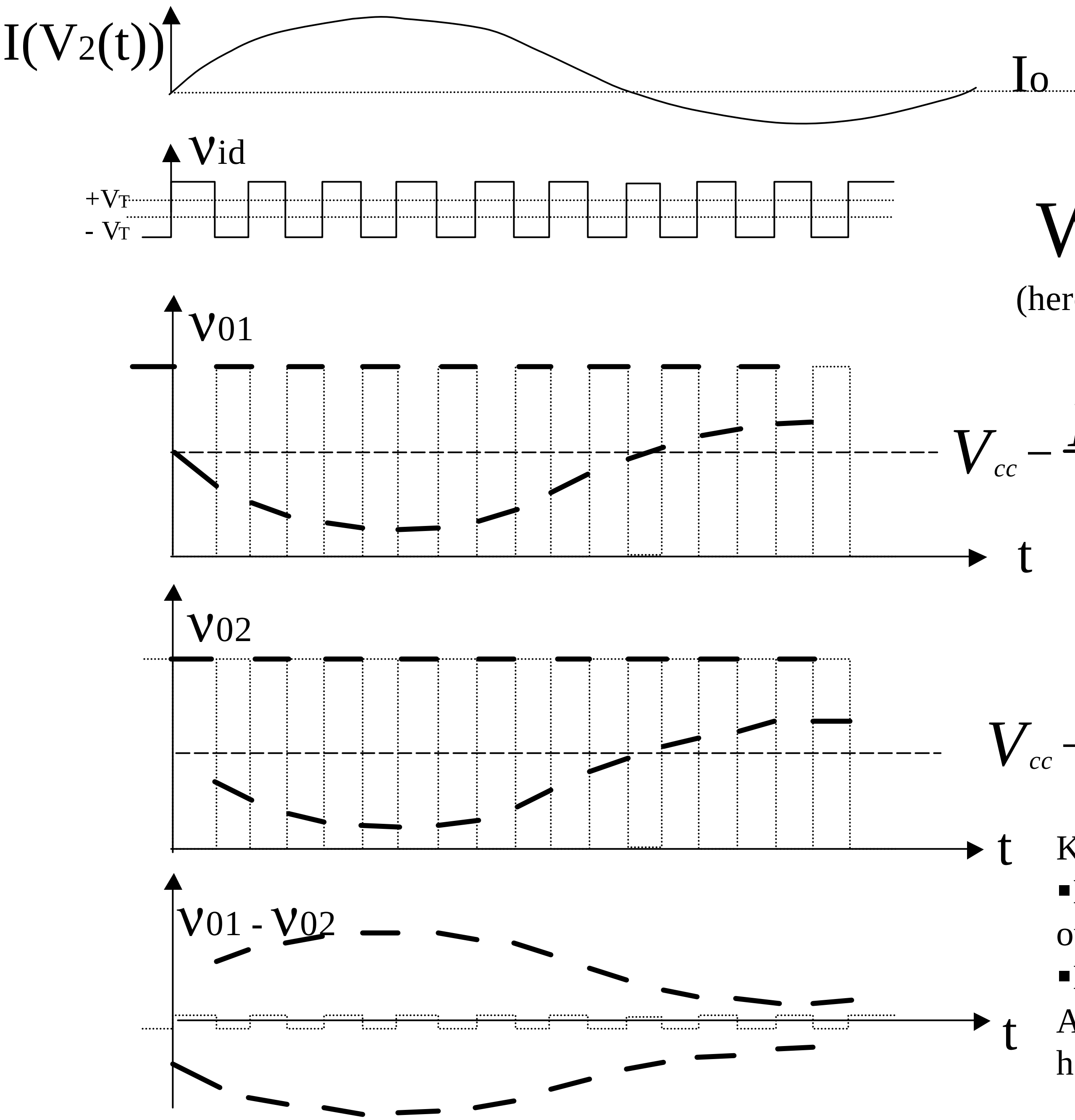
$$where\_v_{id}\_now\_has\_frequency\_w_c$$

Now we've realized the final AM signal based on the two blocks:

- Differential pair to handle the carrier (ωc)
- Current Mirror to provide a "modulating" tail current (ωm)

What about voltgages Wave-Forms?

(here we've "over driven" v_{id} with respect to V_T)



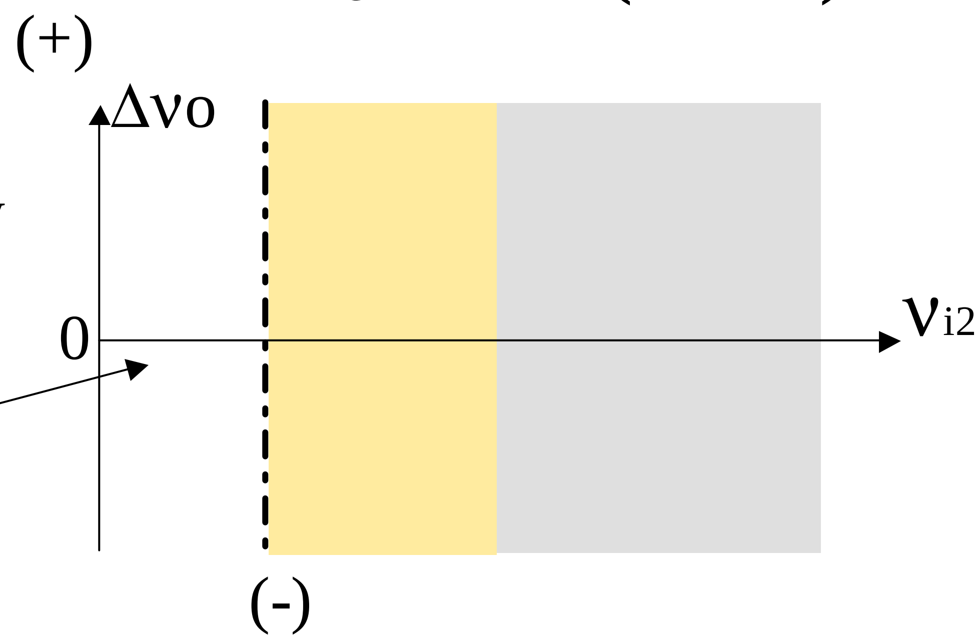
Key points of this slide:

- Multiplier still works with overdrive on the upper pair
- Bottom plot still looks like AM! (but with higher order harmonics)

Two_Quad vs Four_Quad…(huh?)

2Quad:

+ν_{i2} only

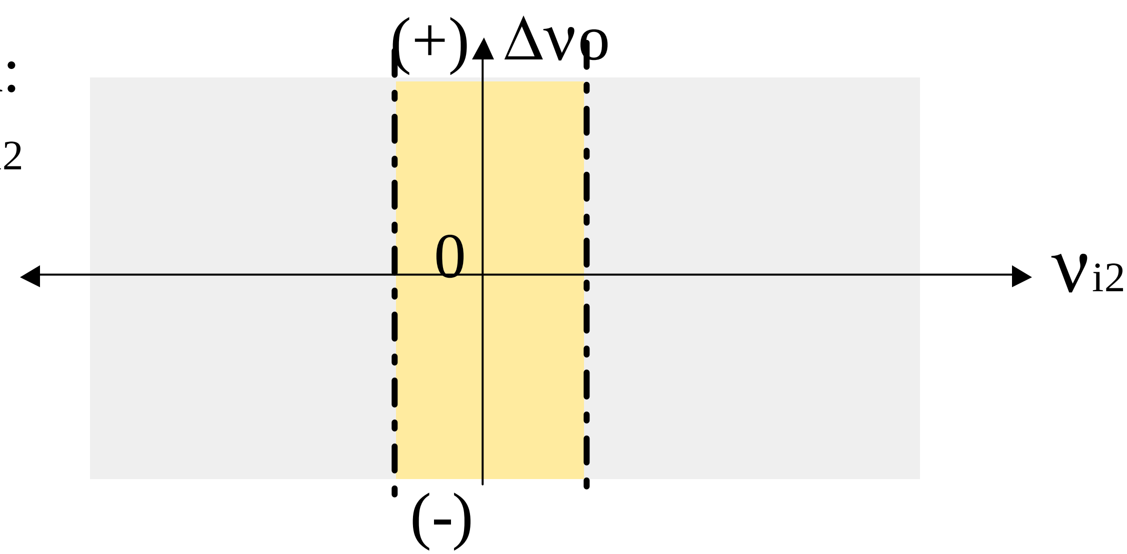


The difference between the 2Q and 4Q multipliers is the range of voltages that can be applied at the vi2 input:

- 2Q requires ALWAYS that $v_{i2}>0$ while for
- 4Q (due to diff. Pair on the bottom part of the circuit) **can have both + and - v_{i2}**…it's fully differential

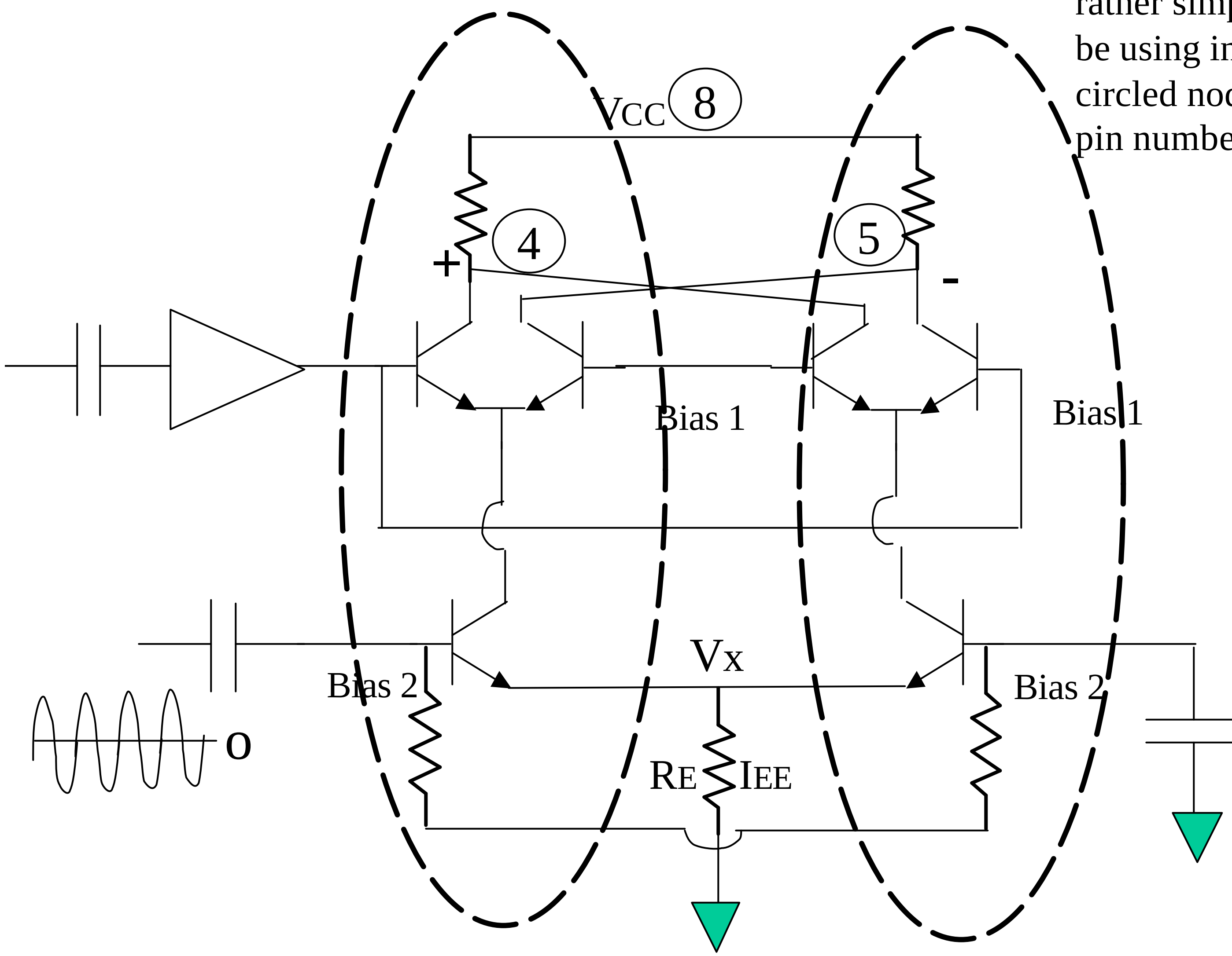
4Quad:

+&- ν_{i2}



What is a Four-Quad Multiplier Anyway?

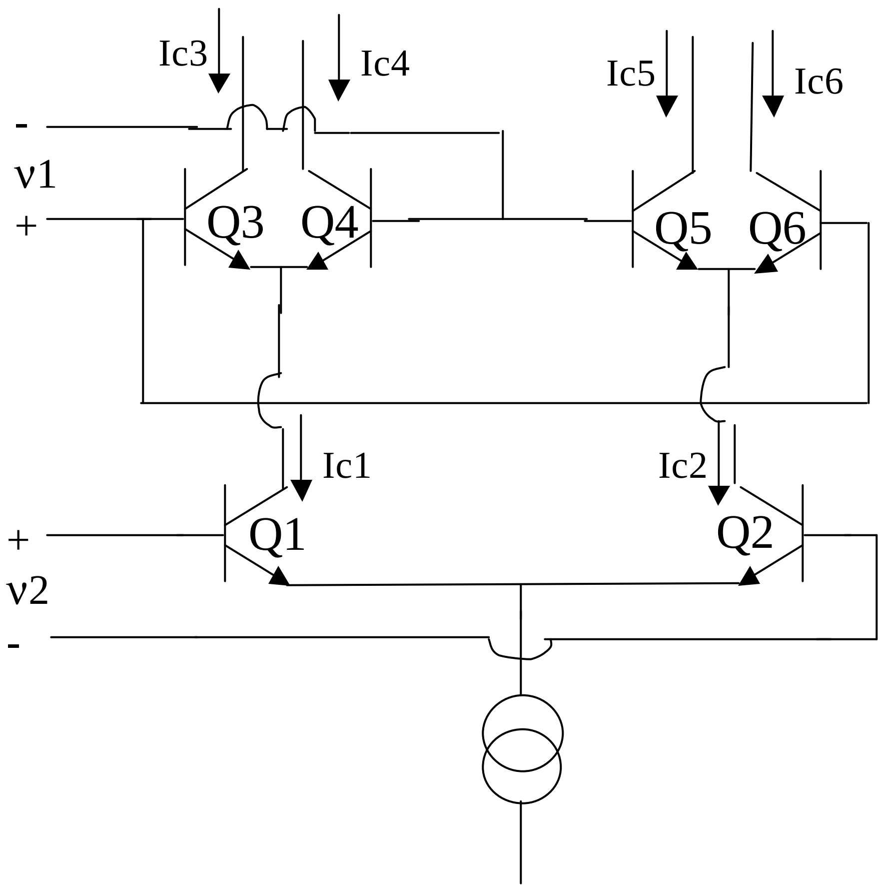
Simple anwer, **two 2Q multipliers** that are cross-coupled. The following is a rather simplified version of what you'll be using in lab--the 612. The three circled node numbers correspond to the pin numbers on the 612.



What's the deal with the cross-coupling at the outputs…two answers:

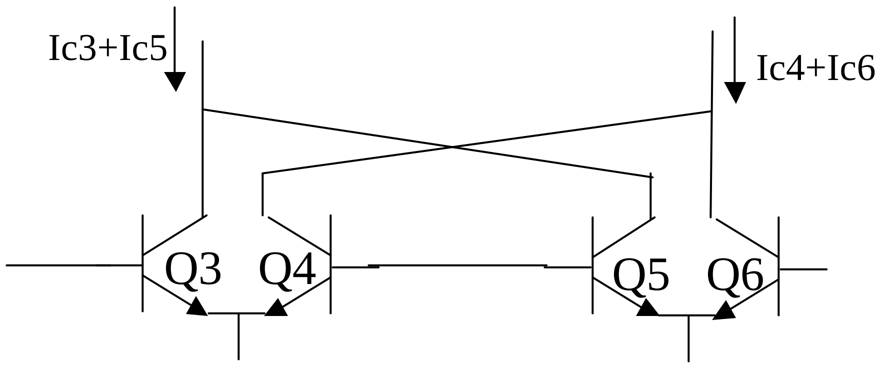
- Doing the math (next) we'll see that by doing the cross-terms as shown we get a beautiful set of two tanh(x)tanh(y) functions
- Looking at it simply we can argue about how inverting and non-inverting signals need to add at the outputs

Derivation of 4Q Results--defining Ic's



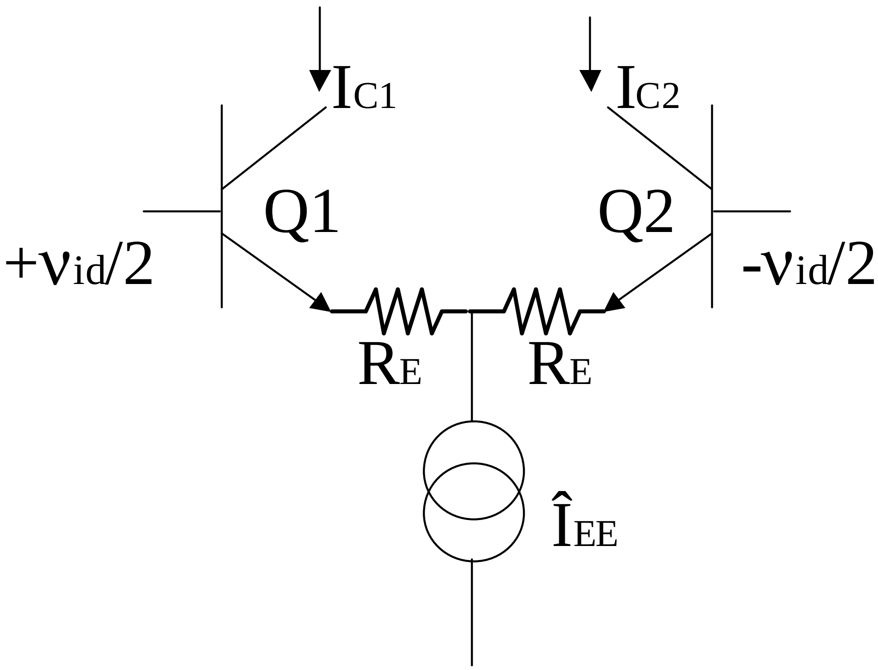
Work in progress…see Tueday's lecture :)

How to get the final “Tanh” at Outputs



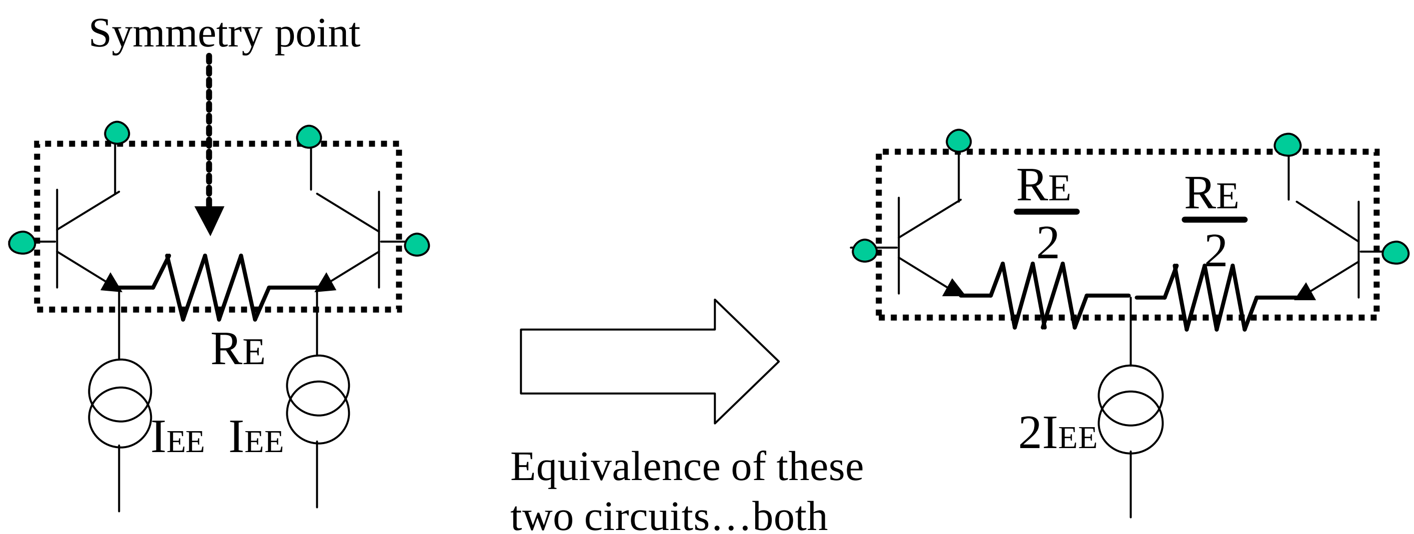
Work in progress…see Tueday’s lecture :)

Way to improve input range for bottom differential pair (versus 612 limits)



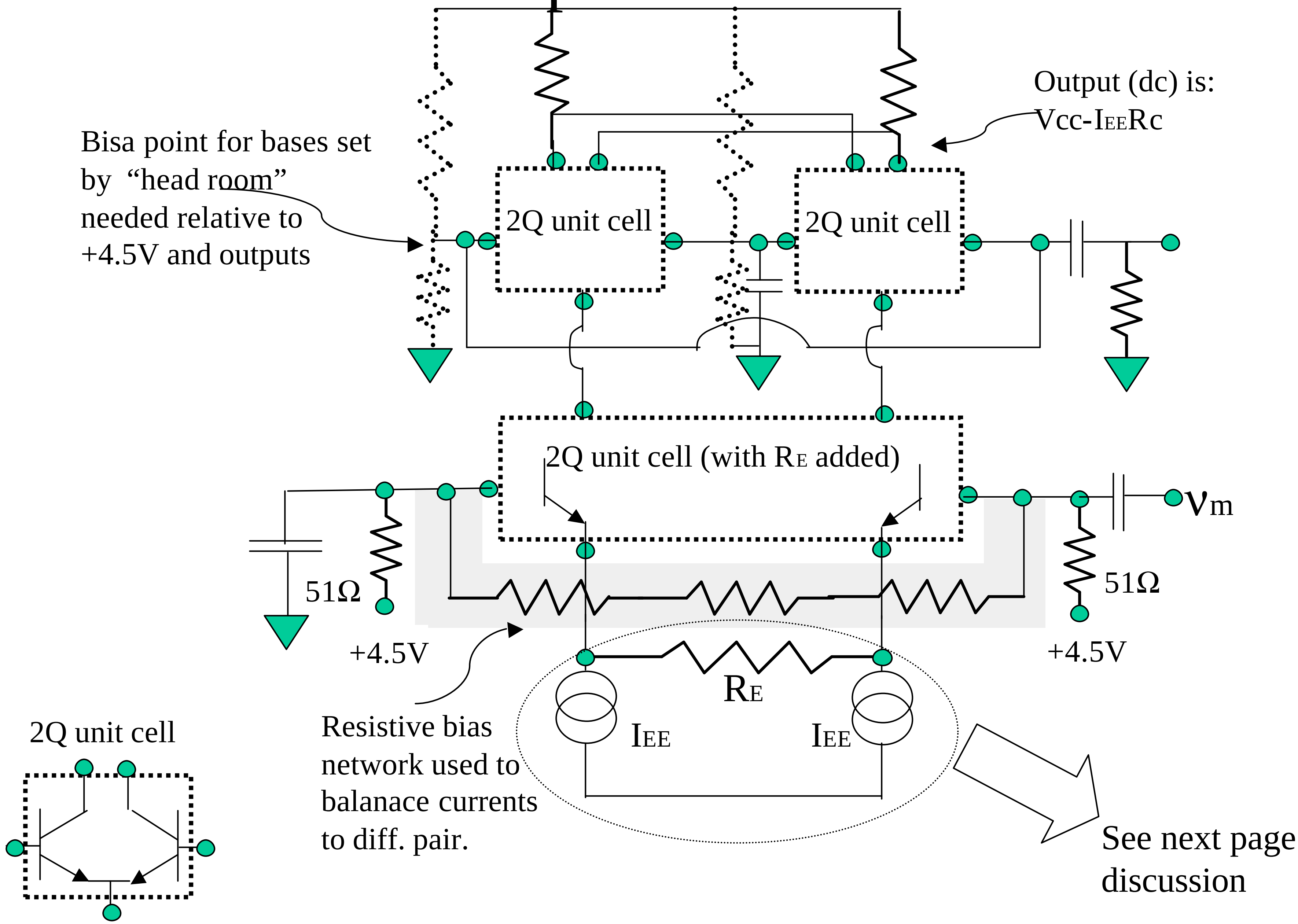
Add RE to each transistor which degrades it's transconductance; gm'=gm/(1+gmRE)

An Equivalent way to do it with single RE

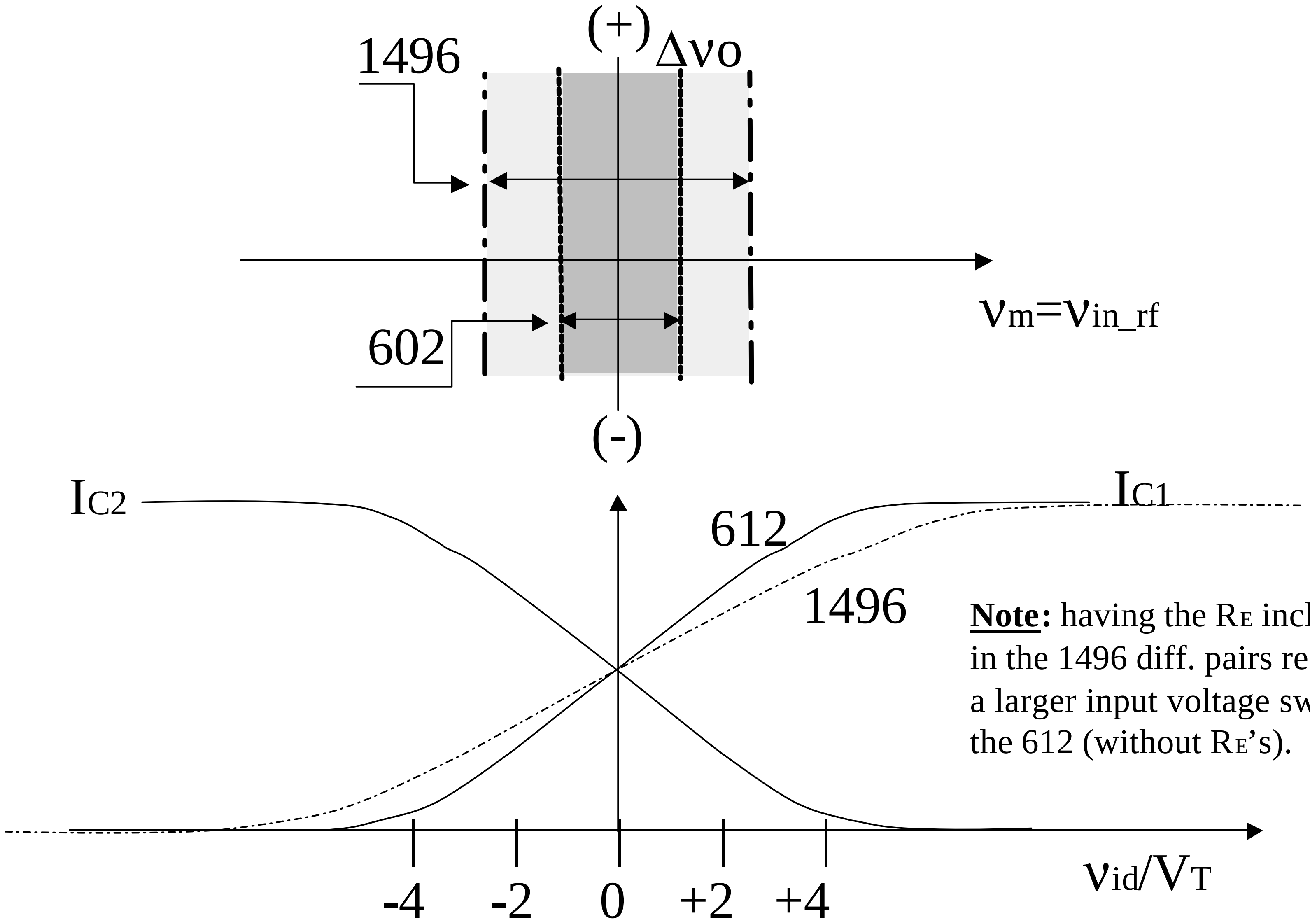


Equivalence of these two circuits…both give same input-output per base & collectors

Practical implementation in 1496



Results comparing range for 612 vs 1496



Note: having the R_E included in the 1496 diff. pairs results in a larger input voltage swing vs. the 612 (without R_E's).