FREQUENCY/PHASE MODULATION

GENERALLY CAN FORMULATE AS ANGLE MODULATION

$$v_o(t) = A\cos\theta(t)$$

$$= A\cos(\omega t + \phi)$$

Frequency　　　Phase

(FM)　　　　　(PM)

Both are important and used in practice, let’s do FM first.

$$\omega \equiv \frac{d\theta}{dt} \qquad (\mathrm{a})$$

Instantaneous frequency is

$$f = f_c + K_o v_m(t) \qquad (\mathrm{b})$$

Equating (a) & (b) above

$$\frac{d\theta}{dt} = 2\pi f_c + 2\pi K_o v_m(t)$$

And then integrating w.r.t. time

$$\theta(t) = 2\pi f_c t + \theta_o + 2\pi K_o \int v_m(t)\,dt$$

$$\text{Assuming } v_m(t) = v_p \cos(2\pi f_m t)$$

$$(\text{and letting } \theta_o + \text{integration const.} \equiv 0)$$

$$\theta(t) = 2\pi f_c t + \frac{\overbrace{K_o v_p}^{\Delta f_c}}{f_m} \sin(2\pi f_m t)$$

$$m_f \equiv \frac{K_o v_p}{f_m}$$

$$v_o(t) = A\cos\left(\underbrace{2\pi f_c t}_{a} + \underbrace{m_f \sin(2\pi f_m t)}_{b} \right)$$

Narrow -- vs. wide - Band FM

<u>NBFM</u> $m_f<0.25$ (quarter radian)

$$A\cos(a+b) \equiv A(\cos b\cos a - \sin b\sin a)$$

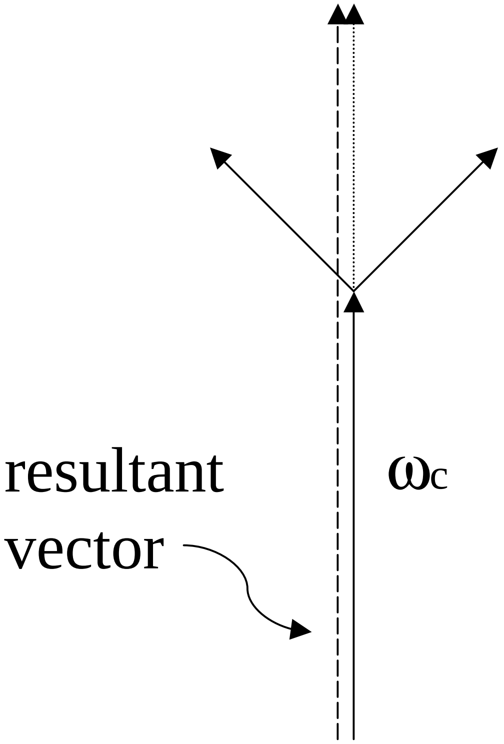
$$v_{out}(t) = A[\cos(m_f \sin\omega_m t)\cos\omega_c t - \sin(m_f \sin\omega_m t)\sin\omega_m t]$$

$$\text{For NBFM using small} < \text{formulae } \cos(\mathrm{x}) \simeq 1, \sin(x) \simeq x$$

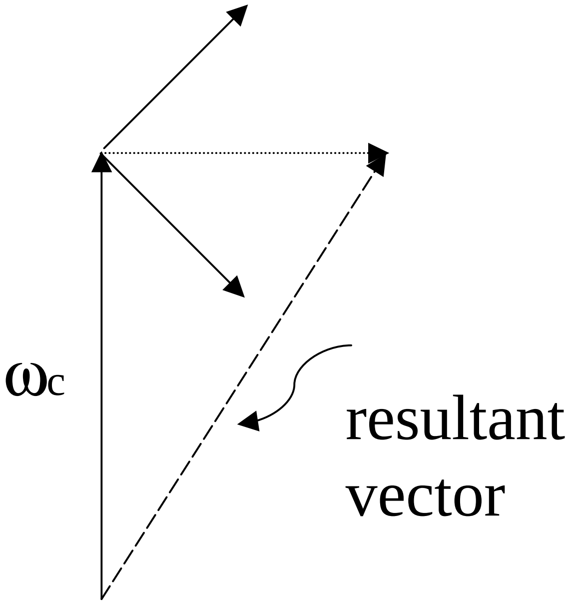
$$v_o(t) = A[\cos\omega_c t - m_f \sin\omega_m t\sin\omega_c t]$$

Wow, looks like AM (…sort of…)

AM



FM

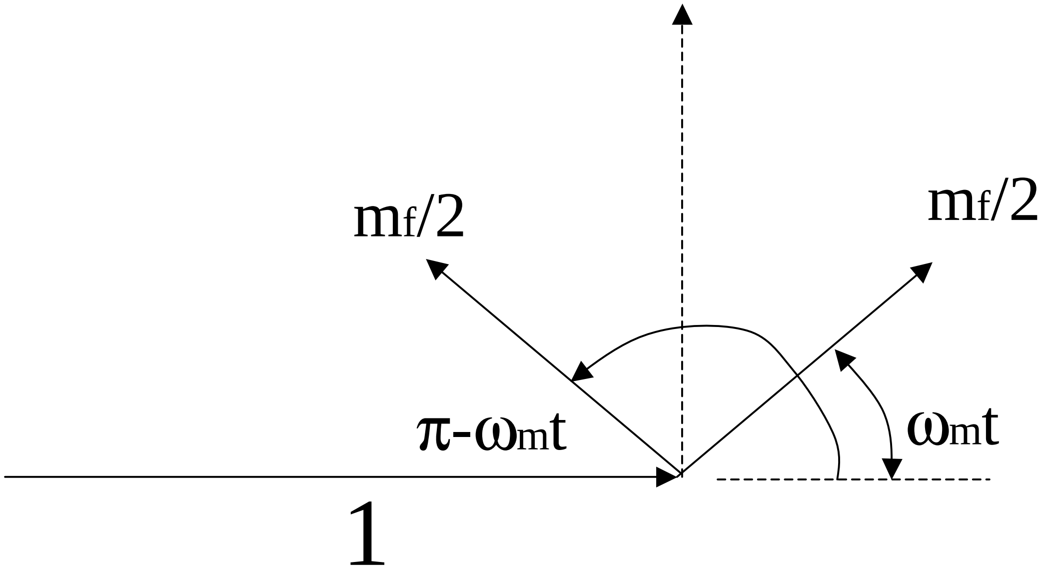


Actually, getting quantitative about these two figures:

$$v_o\Big|_{FM}(t) = \cos\omega_c t - \frac{m_f}{2}\left[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t\right]$$

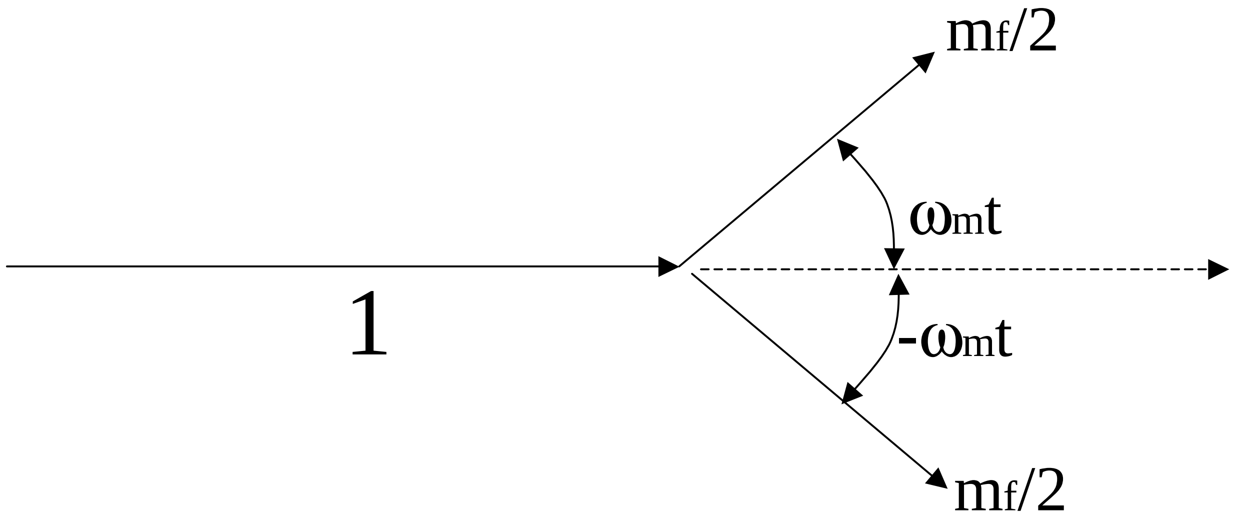
$$= R_e\left[e^{j\omega_c t}\left(1 - \frac{m_f}{2}e^{-j\omega_m t} + \frac{m_f}{2}e^{+j\omega_m t}\right)\right]$$

$$\left((-) \equiv shift\ by + \pi\right)$$



$$v_o\Big|_{AM}(t) = \cos\omega_c t + \frac{m_f}{2}\left[\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right]$$

$$= R_e\left[e^{j\omega_c t}\left(1 + \frac{m_f}{2}e^{j\omega_m t} + \frac{m_f}{2}e^{-j\omega_m t}\right)\right]$$



WBFM (or just FM with m_f>0.25)

$$v_o(t) = \sin(\omega_c t + m_f \sin \omega_m t)$$

$$= \sin \omega_c t \cos(m_f \sin \omega_m t) + \cos \omega_c(t) \sin(m_f \sin \omega_m t)$$

Without using small angles…Bessel Functions!

$$v_o\Big|_{WBFM}(t) = J_o(m_f)\sin \omega_c t +$$

$$2J_1(m_f)\cos \omega_c t \sin \omega_m t +$$

$$2J_2(m_f)\sin \omega_c t \cos 2\omega_m t +$$

$$2J_3(m_f)\cos \omega_c t \sin 3\omega_m t + ...$$

The attached pages give:

- Table* of Bessel Function values as a function of m_f
- A graphical version of the data*
- An example of what the resultant "vector diagram*" looks like for m_f=1

Comments:

- The J's are higher order terms (harmonics beyond $\omega_c \underline{+} \omega_m$
- This is wide band (WBFM) and hence < gets larger (m_f> 0.25)

*Figures from Chapter 9, book by P.H. Young, Electronic Communication Techniques, 1990, Merrill/Macmillan Publishing

Bessel functions are solutions of the Bessel equation:

$$x^2 y'' + xy' + \left(x^2 - p^2\right)y = 0$$

$$J_0(x) = \sum_{o}^{\infty} (-1)^k \frac{x^{2k}}{2^{2k}(k!)^2}$$

$$J_1(x) = \sum_{0}^{\infty} (-1)^k \frac{x^{2k+1}}{2^{2k+1} k!(k+1)!}$$

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$$

Related formulations for trigonometric functions

$$\cos(m\cos x) = J_o(m) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(m)\cos(2kx)$$

$$\cos(m\sin x) = J_o(m) + \sum_{k=1}^{\infty} J_{2k}(m)\cos(2kx)$$

$$\sin(m\cos x) = 2\sum_{k=1}^{\infty}(-1)^k J_{2k+1}(m)\cos(2k+1)x$$

$$\sin(m\sin x) = 2\sum_{k=0}^{\infty} J_{2k+1}(m)\sin(2k+1)x$$

TABLE 9–1 Bessel Functions of the First Kind, $J_n(m_f)$

m_f	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—
2.4	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—
10.0	−0.25	0.05	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01

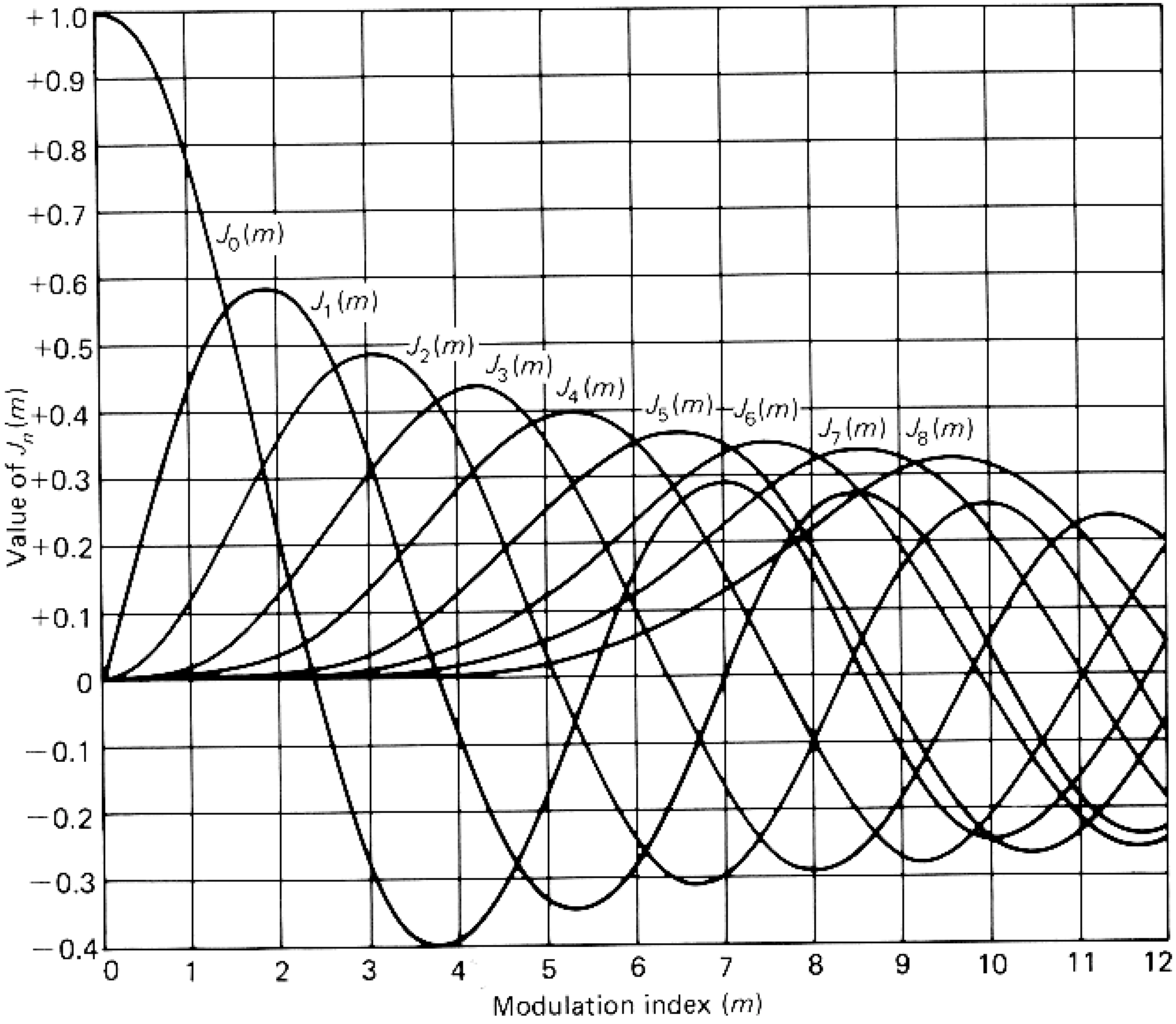
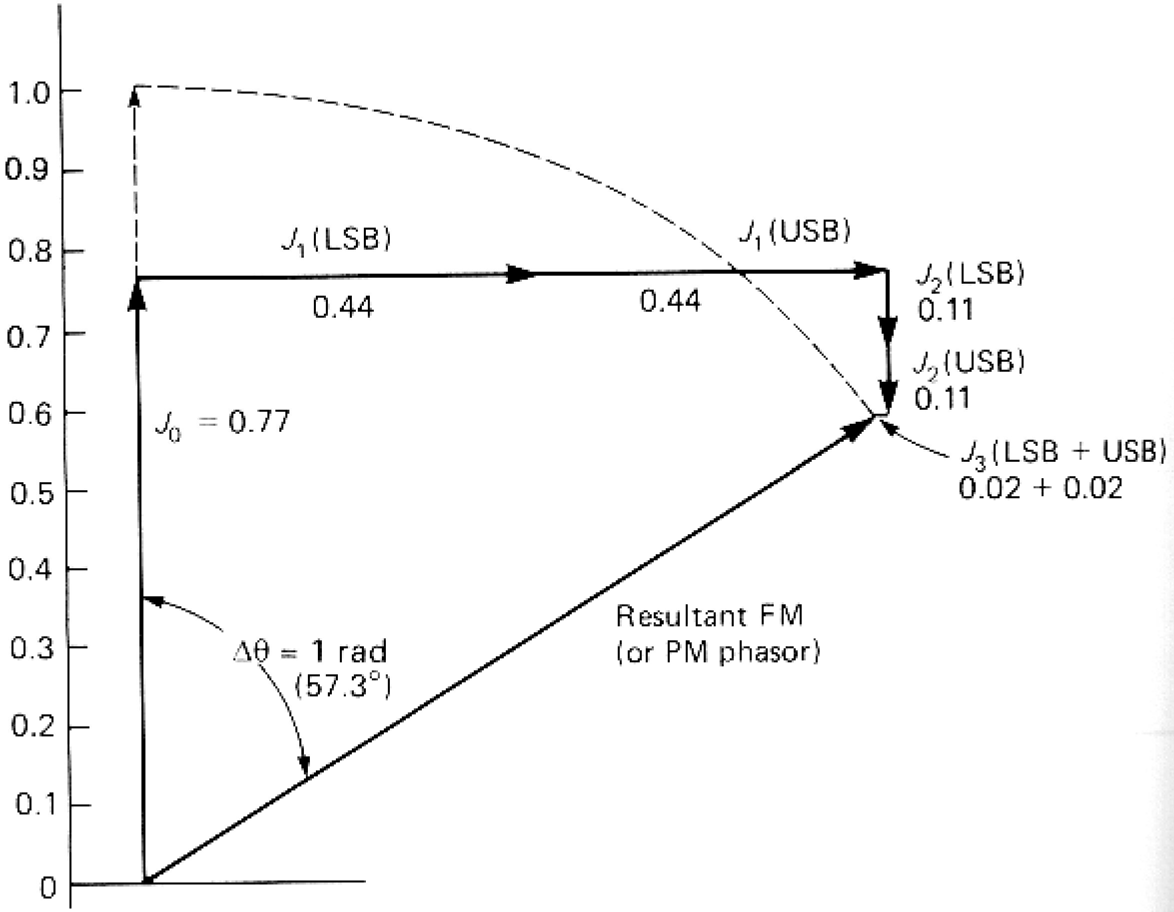


FIGURE 9–5 Amplitudes of carrier and sidebands for FM (and PM) relative to the unmodulated carrier.

FIGURE 9–7 Carrier and sidebands add up to transmitted FM (or PM) phasor.



Phase Modulation

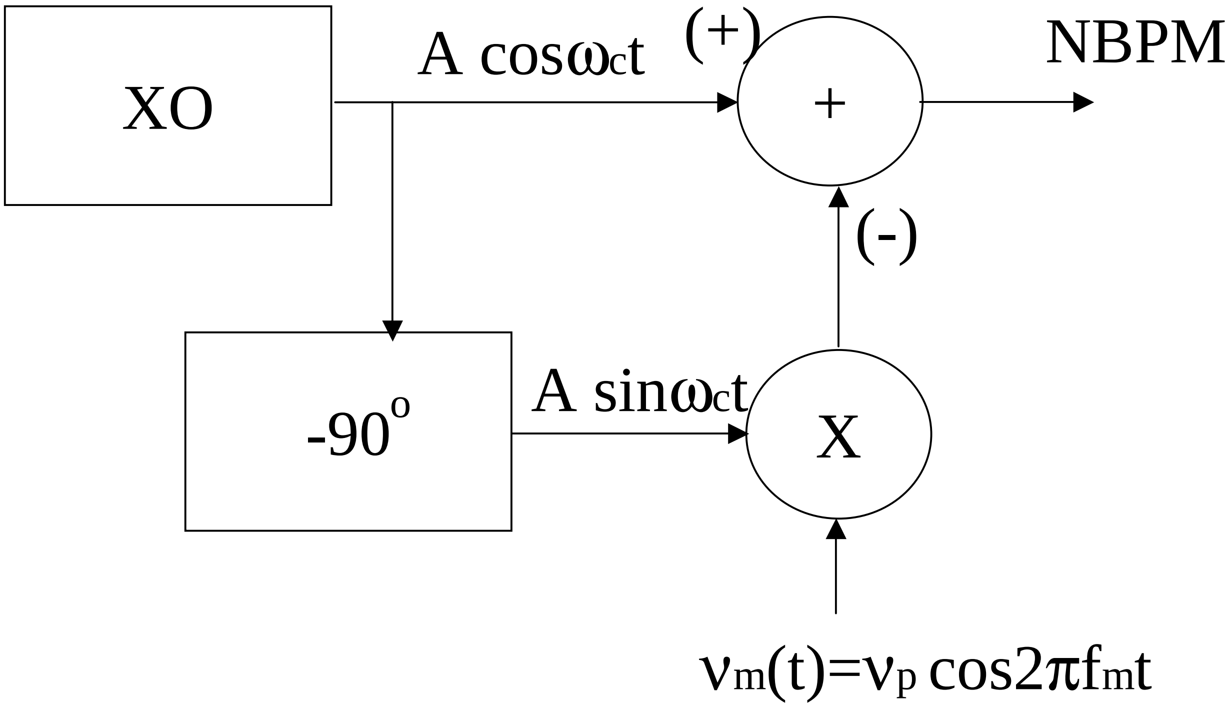
Recall…

$$v_o(t)=A\cos\theta(t)$$

$$\overbrace{\left(\underset{\text{FM}}{\underset{\uparrow}{\omega t}}+\underset{\text{PM}}{\underset{\uparrow}{\phi}}\right)}$$

Now, let’s go for PM

$$\theta(t)=\omega_c t+K_p v_m(t)$$



XO

$A \cos\omega_c t$

(+)

+

NBPM

(-)

-90^o

$A \sin\omega_c t$

X

$\nu_m(t)=\nu_p \cos 2\pi f_m t$

$$v_o(t) = A\cos\omega_c t - A(m_p\cos\omega_m t)\sin\omega_c t$$

$$\nearrow$$

$$m_p = \Delta\phi = k_p v_p$$

$$\left[\text{Note: for FM, } m_f = \frac{K_o v_p}{f_m}\right.$$

$$\left.\therefore \text{ for PM constant } \frac{\omega}{f_m} \text{ vs. not so with FM}\right]$$

Also, XO (crystal osc.) can be made super stable with temp. etc. Now the burden of signal processing is to get -90°, mixer etc.