
ANALOG INTEGRATED CIRCUITS FOR COMMUNICATION
Principles, Simulation and Design

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14.2 A Simple Circuit Model of a PLL

The basic properties and operation of a PLL can be illustrated using a simple circuit model for a PLL, such as can be constructed using the elements available in the Spice simulators. Circuit simulation then provides waveforms and numerical data to document the operation and properties of the PLL. One circuit possibility is shown in Figure 14.2a. The Spice input file is given in Figure 14.2b. The subcircuits of the PLL model (often referred to as a macromodel) include the VCO implemented with a Wien-type oscillator and a lowpass, RC filter. The amplifier of the PLL is achieved by setting the desired value of the gain as the constant of a voltage-controlled voltage source, which also models the PC. In the following, each of these subcircuits is described.

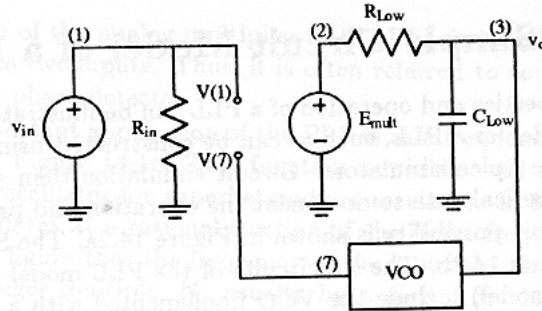
VCO: The VCO of the PLL macromodel is an idealized Wien-type oscillator as shown in Figure 14.2c. A prototype of this configuration is given in Figure 9.11 and operational results are given in Section 9.6. The two voltage-controlled conductors, described below, provide a linear change of oscillation frequency with the output voltage of the filter (PLL). The gain element of the oscillator is produced with a simple voltage-controlled voltage source providing a gain of 3.05. Two diode, voltage-source clamp combinations are used to limit the output voltage of the idealized amplifier to $\pm 10V$ and provide the necessary nonlinearity for the oscillator. A pulse voltage source is included in one conductor combination of the VCO to initialize the circuit for transient operation and to reduce the time needed to let the startup transients die off.

In the Wien oscillator, it is convenient to let the resistors and capacitors of the feedback network circuit be equal. $R_1 = R_2 = R$ and $C_1 = C_2 = C$. The steady-state frequency of the oscillator is then approximately

$$\omega_{osc} = \frac{1}{RC} = \frac{G}{C} \quad (14.4)$$

where $G = \frac{1}{R}$. It is now convenient to use conductance notation rather than resistance notation for the oscillator. The two conductances, $G_1 = \frac{1}{R_1}$ and $G_2 = \frac{1}{R_2}$, which have identical values, are each implemented, as shown in Figure 14.2c, as the parallel combination of a fixed resistance and variable conductance,

$$\begin{aligned} G_1 &= \frac{1}{R_a} + G_{var1} \\ G_2 &= \frac{1}{R_b} + G_{var2} \end{aligned} \quad (14.5)$$



(a)

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PLL MACROMODEL, FIG 14.2B
.TRAN 0.5U 20U 0 0.1U
.PLOT TRAN V(3)
* INPUT *
VIN 1 0 SIN(0 1 1.0MEG)
*VIN 1 0 SIN(0 1 1.1MEG)
*VIN 1 0 SIN(0 1 0.9MEG)
*VIN 1 0 SFFM(0 1 10MEG 7.5 10K)
RIN 1 0 1K
* PHASE DETECTOR MULTIPLIER *
EMULT 2 0 POLY(2) 1 0 7 0 0 0 0 0 1
* LOW PASS FILTER BW=20KHZ *
RLOW 2 3 1K
CLOW 3 0 7.958NF
* WIEN-TYPE VOLTAGE CONTROLLED OSCILLATOR *
* KO=100KHZ/VOLT FO=1MHZ *
* WEIN FEEDBACK CIRCUIT *
RB 4 10 1K
GVAR2 4 10 POLY(2) 4 10 3 0 0 0 0 0 1E-4
VTRIG 10 0 PULSE(0 5 0 0 0 1NS)
C2 4 0 159.15PF
RA 4 6 1K
GVAR1 4 6 POLY(2) 4 6 3 0 0 0 0 0 1E-4
C1 6 7 159.15PF
* POSITIVE-GAIN BLOCK *
EGAIN 5 0 4 0 3.05
RO 5 7 10
D1 7 8 MD
V1 8 0 10V
D2 0 9 MD
V2 9 7 10V
.MODEL MD D IS=1E-16
* CONTROL CARDS *
.OPTIONS NOPAGE ITL5=0 LIMPTS=2000
.OPTIONS RELTOL=1E-4
.END

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(b)

Figure 14.2: (a) Circuit model for a PLL. (b) SPICE input file.

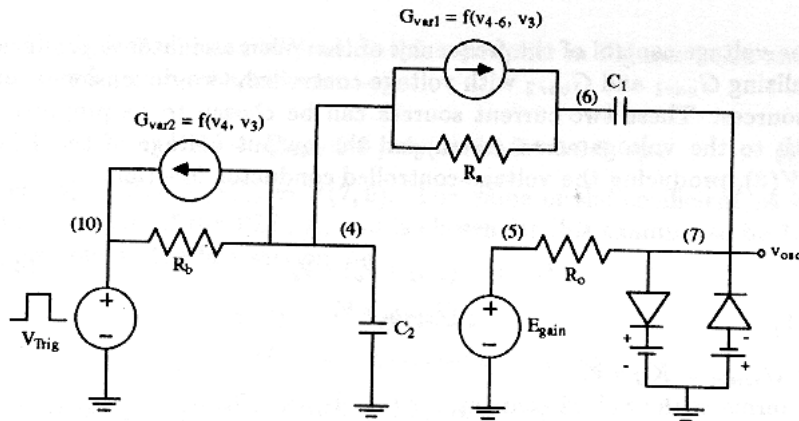


Figure 14.2: (c) The VCO of the PLL macromodel.

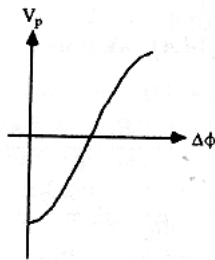


Figure 14.3: Plot of the output voltage with the phase difference.

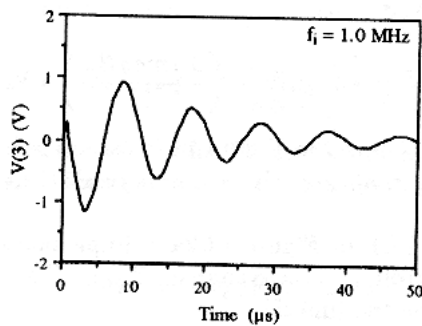


Figure 14.4: (a) The transient response of a PLL.

The voltage control of the frequency of the Wien oscillator is produced by realizing G_{var1} and G_{var2} with voltage-controlled, two-dimensional current sources. These two current sources can be chosen to be proportional both to the voltage across them and the output voltage of the PLL, $V_o = V(3)$, producing the voltage-controlled conductor function.

$$\begin{aligned} I_x &= (K_G * V_o) * V_x \\ &= G_{equiv} * V_x \end{aligned} \quad (14.6)$$

where $G_{equiv} = K_G * V_o$.

In terms of the Spice2 element, for the element G_{var1} :

$$I_x = G_{var1} \ 4 \ 10 \ poly(2) \ 4 \ 10 \ 3 \ 0 \ 0 \ 0 \ 0 \ 0 \ K_G \quad (14.7)$$

The nodes of the element are 4 and 10. The control voltage is $V(3,0)$. The value of the constant K_G in (14.6) and (14.7), for the example listed in Figure 14.2b, is $1 \cdot 10^{-4} \text{ mho/V}$.

Using the above results in (14.4), we obtain

$$\begin{aligned} \omega_{osc} &= \frac{(1/R_a + K_G * V_o)}{C} \\ &= \frac{1}{R_a C} + \frac{K_G * V_o}{C} \\ &= \omega_{free} + K_o V_o \end{aligned} \quad (14.8)$$

where $\omega_{free} = \frac{1}{R_a C}$ and $K_o = \frac{K_G}{C}$.

For the values of the Spice example of Figure 14.2, $f_{free} = \frac{\omega_{free}}{2\pi} = 1 \text{ megHz}$ and $k_o = 100 \text{ kHz/V}$, where the lower-case constant denotes the cyclic value: $k_o = (\frac{1}{2\pi}) K_o$.

$$f_{osc} = \frac{\omega_{osc}}{2\pi} = 1 \text{ megHz} + \left(\frac{0.1 \text{ megHz}}{V} \right) * V_o \quad (14.9)$$

FILTER: In the PLL shown in Figure 14.2a, a one-pole, lowpass filter is included, but a more complicated filter is easily substituted.

PC: The PC of the PLL of Figure 14.2a is implemented with a two-dimensional voltage-controlled voltage source, E_{mult} . The output voltage for this element should be the function:

$$V(2) = K V_i * V_{osc} \quad (14.10)$$

The Spice2 element to provide this multiplier in Figures 14.2a and b is E_{mult} and has the description:

$$V(2) = E_{mult} \ 2 \ 0 \ poly(2) \ 1 \ 0 \ 7 \ 0 \ 0 \ 0 \ 0 \ p4 \quad (14.11)$$

where $V_i = V(1, 0)$, $V_{osc} = V(7, 0)$. The value of the coefficient $p4$ is the 'gain constant' of the PLL, A , and is chosen for this example to be 1.

As done in the last section, let

$$\begin{aligned} v_i(t) &= V_{iA} \cos \omega_i t \\ v_{osc}(t) &= V_{oscA} \cos(\omega_{osc} t + \Delta\phi) \end{aligned} \quad (14.12)$$

Now set $\omega_i = \omega_{osc}$. The output from E_{mult} is

$$V_p = V(2) = - \left(\frac{V_{iA} V_{oscA}}{2} \right) \cos(\Delta\phi) \quad (14.13)$$

$$+ \left(\frac{V_{iA} V_{oscA}}{2} \right) \cos[(\omega_i + \omega_{osc})t + \Delta\phi] \quad (14.14)$$

As mentioned in the last section, these sum and difference frequency terms are the same as those in the output of a mixer. After filtering out the high-frequency component, the output signal (voltage) is proportional to the cosine of the phase difference between the two sinusoids. A plot of the output voltage with the phase difference, V_p vs $\Delta\phi$, for this simple case is shown in Figure 14.3 and is the negative of a segment of a cosine function.

$$V_p = -K_p \cos(\Delta\phi), \quad 0 < \Delta\phi < \pi \quad (14.15)$$

The sign of the function above is necessary to produce a positive output voltage with a positive change of the frequency difference. The constant K_p is the slope of this function for $V_p = 0$ ($\phi = \frac{\pi}{2}$).

$$K_p = -K_p \left(\frac{d \cos \Delta\phi}{d \Delta\phi} \right) \Big|_{\Delta\phi = \frac{\pi}{2}} \quad (14.16)$$

(Note that K_p is used both as the PC coefficient and the multiplier of the $\cos \Delta\phi$ expression.) For the values of the Spice example, $V_{iA} = 1V$ and $V_{oscA} = 10V$.

$$K_p = \frac{1}{2} V_{iA} V_{oscA} = 5V \quad (14.17)$$

PLL OPERATION: As noted, for equal input and VCO free-running frequencies, the phase difference of the two signals must be 90° to provide

a zero dc output voltage for the PC. This can be readily verified from Spice simulation. In the Spice input file of Figure 14.2b, several input signals can be used. For the operative input for the present analysis, without the leading *, the input frequency is 1megHz . The transient response of the PLL is shown in Figure 14.4a. After a transient startup condition, the dynamics of which are studied in the next section, the output voltage settles to zero volts indicating that the PLL is locked to the input frequency. The input frequency is equal to the free running-frequency of the VCO.

A more detailed plot of the input signal and the output of the VCO is given in Figure 14.4b. In lock, the expected 90° phase difference between the two sinusoids can be seen with v_{osc} leading v_i by 90° . This aspect is also established analytically in the next section.

The output waveform of Figure 14.4c is produced when the input frequency to the PLL is changed to 1.1megHz . After the transients have decayed, the output voltage settles toward $+1V$ and controls the VCO to provide an output frequency of $1\text{megHz} + k_o(1V) = 1.1\text{megHz}$. It is to be noted that the 'startup transients' in this case are quite different than that of the first example where $f_i = f_{free}$. This is due to the capture phenomena which is analyzed in a later section.

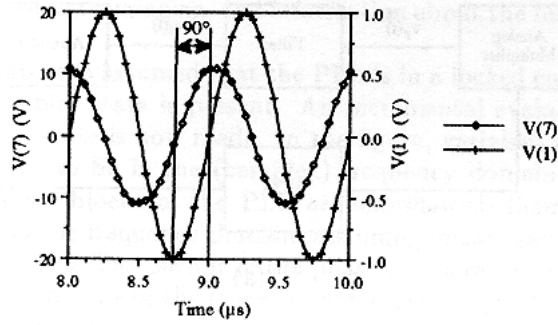
Similarly, for an input frequency of 0.9megHz , the output voltage settles to $-1V$ to produce a VCO frequency of 0.9megHz . The output waveform of the PLL is shown in Figure 14.4d.

The rejection of the filter is not perfect and a portion of the sum frequency from the analog multiplier (PC) output appears at the PLL output. This is shown, for an input frequency of 1megHz , in the more detailed output of Figure 14.4e.

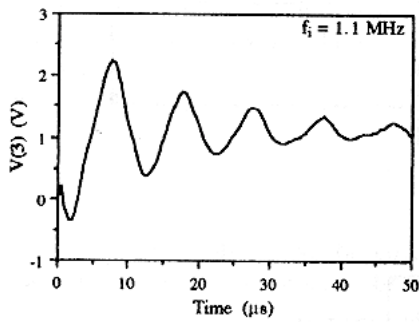
14.3 The Small-Signal Analysis of the PLL

The block diagram of a PLL is repeated in Figure 14.5a. Note that in this diagram, all input and output variables are denoted as voltage variables in the time domain. As mentioned in the last sections, the analog multiplier serves as the phase comparator. The low-frequency output of the PC is equal to the voltage V_p which is proportional to the phase difference of its two input signals, v_i and v_{osc} . As brought out earlier, although the two input signals to the phase comparator may be voltages and currents, the desired 'error' signal is the phase difference between the signals.

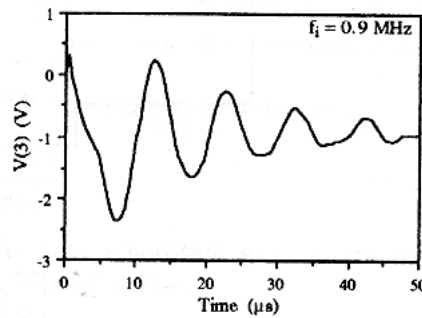
Instantaneous frequencies are of prime concern, but the error signal of the PC must be the differences of the phases of the two inputs, not the frequency difference. Comparable to the situation for frequency modulation, as mentioned in Chapter 13, frequency differences cannot be used directly



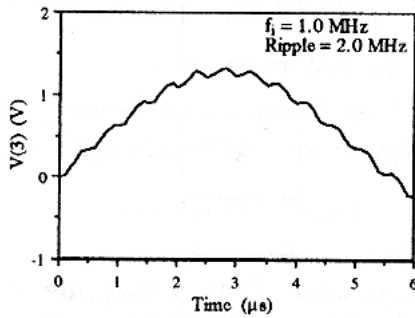
(b)



(c)

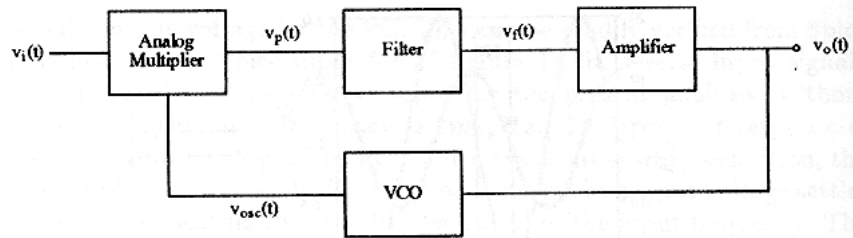


(d)

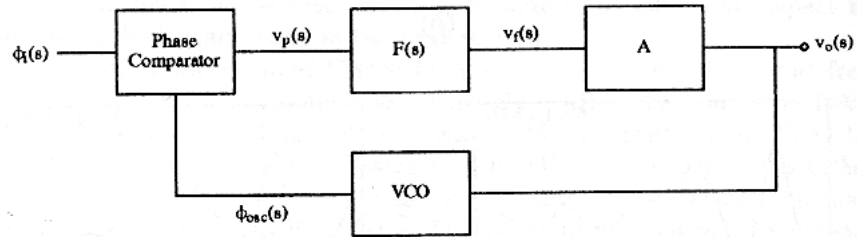


(e)

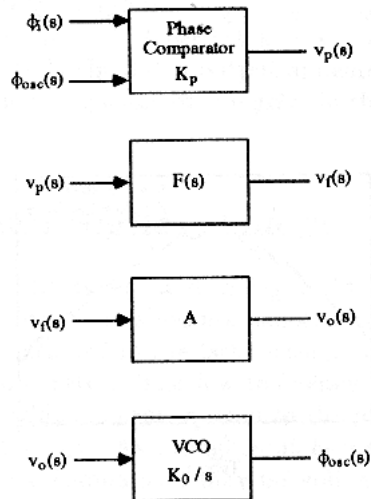
Figure 14.4: (b) Detailed plot of the input signal and the VCO output. Output voltage waveform for an input frequency of (c) 1.1 MHz, and (d) 0.9 MHz. (e) Output of the PLL showing the ripple content.



(a)



(b)



(c)

Figure 14.5: Block diagram of the PLL in the (a) time domain, and (b) frequency domain. (c) The individual blocks of the PLL.

since this introduces an averaging, and information about the instantaneous frequency is lost.

In Figure 14.5b, it is assumed that the PLL is in a locked condition and that a definite dc bias state is present. An incremental evaluation of the PLL about this dc state is now made. In the figure, variables of the block diagram are chosen to be in the (complex) frequency domain. In Figure 14.5c, the individual blocks of the PLL are shown with their input and output variables in the frequency domain, assuming linear operation about the dc operating point. The output of the phase comparator for this small-signal situation is the slope of the transfer characteristic, (14.16), about the operating point, $\Delta\phi = \frac{\pi}{2}$.

$$v_p(s) = K_p(\phi_i - \phi_{osc}) \quad (14.18)$$

where $\Delta\phi = \phi_i - \phi_{osc}$ and where $\cos(\frac{-\pi}{2} + x) = \sin x \simeq x$ for small x is used. K_p is often called the conversion ('gain' or transfer) constant of the comparator.

The filter is usually lowpass and its transfer function is denoted

$$F(s) = \frac{v_f(s)}{v_p(s)} \quad (14.19)$$

The loop amplifier can be chosen to have a constant gain A ; any frequency effects are assumed to be included in the filter.

$$v_o(s) = Av_f(s) \quad (14.20)$$

The voltage-controlled oscillator (VCO) provides a sinusoidal output voltage with a controlled frequency. In the time domain,

$$\omega_{osc}(t) = \omega_{free} + K_o v_o(t) \quad (14.21)$$

where ω_{free} is the free-running frequency of the VCO and K_o is the control constant. In the frequency domain, the incremental frequency can be written:

$$\Delta\omega_{osc}(s) = K_o v_o(s) \quad (14.22)$$

Again, since we are concentrating on the small-signal behavior about the operating state, the constant free-running term can be neglected. The phase function of the oscillator output is of primary interest. This is obtained in the time domain by using the relation that the phase function is the integral of the frequency function.

$$\phi_{osc}(t) = \int \Delta\omega_{osc}(t) dt \quad (14.23)$$

In the frequency domain,

$$\phi_{osc}(s) = \left(\frac{1}{s}\right) \Delta\omega_{osc}(s) \quad (14.24)$$

Therefore, the phase output from the VCO can be expressed

$$\phi_{osc}(s) = \left(\frac{1}{s}\right) K_o v_o(s) \quad (14.25)$$

Choosing the phase as the output variable of the VCO introduces an inherent integration, $1/s$.

To repeat, the PLL is assumed to be in a locked state, i.e., the VCO is locked to the input frequency. The closed-loop transfer function of the PLL is established as follows: The amplifier output is

$$v_o(s) = AF(s)K_p(\phi_i - \phi_{osc}) \quad (14.26)$$

The oscillator output in terms of its phase response is given in (14.25). The closed-loop transfer function in terms of the phase, ϕ_i , of the input signal is

$$\frac{v_o(s)}{\phi_i(s)} = \frac{[K_p FA]}{1 + \frac{K_p FAK_o}{s}} \quad (14.27)$$

The frequency of the input signal can be introduced using a relation for $\Delta\omega_i(s)$ comparable to (14.24). Note that $\Delta\omega_i$ must be used, i.e., the change of input frequency about the reference operating state.

$$\begin{aligned} \frac{v_o(s)}{\Delta\omega_i(s)} &= \frac{v_o(s)}{[s\phi_i(s)]} \\ &= \frac{[K_p FA]}{[s + K_p FAK_o]} \end{aligned} \quad (14.28)$$

In the next section, this transfer function is used to explore for the locked condition the dynamics of the PLL and its response characteristics.

Note that (14.27) has the form of a closed-loop feedback function,

$$\begin{aligned} A_f &= \frac{a_f}{(1 - a_f f)} \\ &= \frac{a_f}{(1 + a_L)} \end{aligned} \quad (14.29)$$

where the 'open-loop gain', a_f , and 'loopgain' function, a_L , are defined as:

$$a_f = K_p F A \quad (14.30)$$

$$a_L = -a_f f = \frac{K_L}{s} \quad (14.31)$$

$$K_L = K_p F A K_o \quad (14.32)$$

For the situation where no filter is included in the PLL, $F = 1$, and a_L has the form of a hyperbolic function and can be considered as a degenerate or limiting form of a lowpass function. The product $K_L = K_p F A K_o$ can be identified as the magnitude of transfer function around the loop, i.e., the magnitude of the loopgain constant. The loopgain function is dimensionless. However, note that the loopgain constant, K_L , has the dimension of radial frequency.

An alternate output of the closed-loop PLL is $\Delta\phi$, the phase difference between the input sinusoid and that of the VCO output. Using (14.26) and (14.27), we obtain

$$\begin{aligned} \Delta\phi &= \phi_i - \phi_{osc} \\ &= \frac{\phi_i s}{(s + K_L)} \end{aligned} \quad (14.33)$$

For K_L very large,

$$\Delta\phi = \frac{\phi_i s}{K_L} \quad (14.34)$$

Therefore, for a large magnitude of loopgain constant, the magnitude of the phase error reduces to zero with a $+90^\circ$ phase shift with respect to the input sinusoid. This result is consistent with the physical reasoning and observations of the last sections, cf., Figures 14.3 and 14.4b.

14.4 Dynamics of the PLL in the Locked Condition

In this section, the dynamical response characteristics of the PLL are examined for typical filter components for the PLL. In the simplest case, no lowpass filter is included, and $F = 1$. Of course, there is no filtering of high-order intermodulation terms produced by the PC. All of these terms appear unattenuated in the output voltage. Nonetheless, for reference purposes, the results from the closed-loop response for this case are included.

For $F = 1$, the inherent integration in the VCO variables introduces the only frequency effect. The closed-loop transfer function of the PLL from (14.28) is

$$\begin{aligned} \frac{v_o}{\Delta\omega_i} &= \frac{\left(\frac{1}{K_o}\right)K_L}{[s + K_L]} \\ &= \frac{H}{(s + \omega_a)} \end{aligned} \quad (14.35)$$

where H and ω_a are auxiliary constants. K_L is the loopgain constant introduced in the last section.

$$K_L = K_p A K_o \quad (14.36)$$

For $F = 1$, the closed-loop transfer function of the PLL, $\frac{v_o}{\Delta\omega_i}$, has a single real pole at $-\omega_a = -K_L$, as shown in Figure 14.6a. Note that for $K_L = 0$, which is the open-loop case, the pole lies at the origin which moves out into the LHP as K_L increases. The negative-real axis is the locus of the closed-loop pole with K_L . (Remember for negative feedback that the locii on the real axis lie to the left of an odd number of poles and zeros.) The closed-loop PLL transfer function, (14.35), for any value of K_L , has a first-order, lowpass transfer characteristic. The -3dB frequency of the magnitude of the transfer function for $s = j\omega$ is K_L , as illustrated in Figure 14.6b. (With this identification, K_L is often referred to as the closed-loop bandwidth for $F = 1$, or more simply as the 'loop bandwidth'.)

For the above example, K_L is the effective output bandwidth for frequency deviations of the input signal, $\Delta\omega_i = 2\pi\Delta f_i$. For example, if the input is a FM signal, the PLL in lock follows the frequency variation of the input at a cyclic rate equivalent to the modulation frequency f_m . The low-frequency component of the control voltage to the VCO is a lowpass signal with a frequency variation equal to the modulation frequency of the FM signal. Therefore, FM demodulation is achieved if this voltage is taken as the output of the PLL. To the extent that the PC and VCO have linear transfer characteristics, the demodulation is linear. If the passband width of the modulation is less than K_L , the -3db bandwidth of the closed-loop PLL, little (lowpass) frequency distortion is introduced although the output contains unrejected components of the higher-order intermodulation terms.

For a numerical example, we choose the parameters of the PLL macro-model of Figures 14.2a and b. $f_{free} = 1\text{megHz}$ and $k_o = \frac{0.1\text{megHz}}{\sqrt{V}}$. In radial measure,

$$\omega_{osc} = 2\pi(1 \cdot 10^6 + 0.1 \cdot 10^6 V_o) \quad (14.37)$$

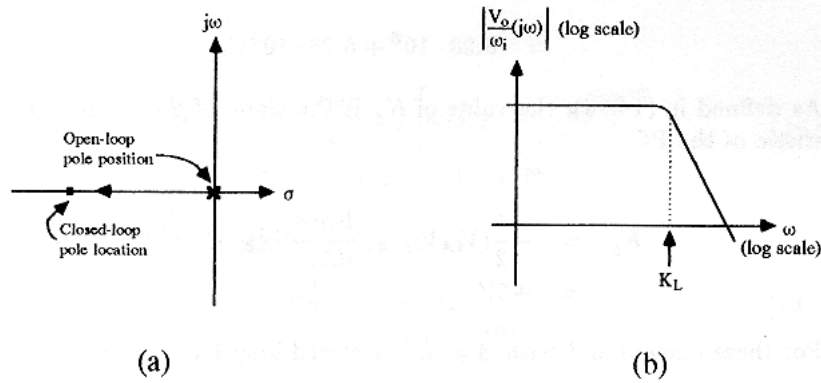
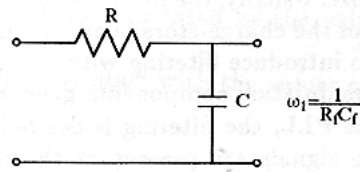
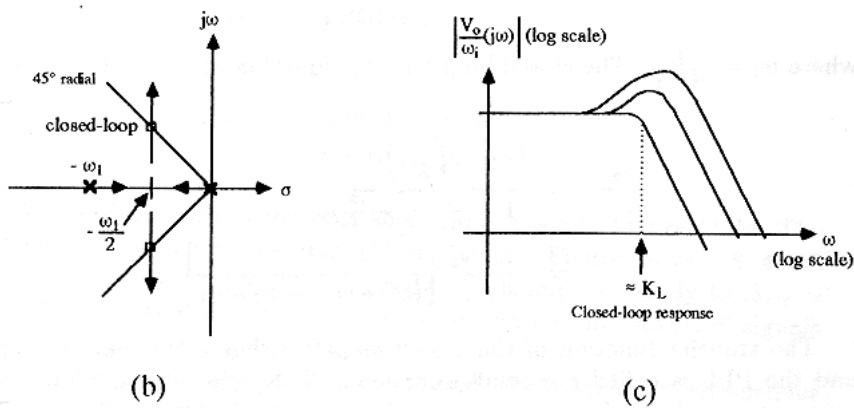


Figure 14.6: (a) Pole locations with no filter. (b) Magnitude response of the closed-loop transfer function.



(a)



(b)

(c)

Figure 14.7: (a) Circuit for a one-pole filter. (b) Pole locations and locii. (c) Magnitude response of the closed-loop transfer function.

$$= 6.28 \cdot 10^6 + 6.28 \cdot 10^5 V_o$$

As defined in (14.16), the value of K_p is the slope of the transfer characteristic of the PC.

$$\begin{aligned} K_p &= -\frac{1}{2}(V_{iA} V_{oscA}) \frac{d \cos \phi}{d \Delta \phi} \Big|_{\Delta \phi = \frac{\pi}{2}} \\ &= +5V \end{aligned} \quad (14.38)$$

For these choices and with $A = 1$, the closed-loop bandwidth is

$$\begin{aligned} K_L &= K_p A K_o = 3.14 \cdot 10^6 \frac{r}{s} \\ k_L &= \frac{K_L}{2\pi} = 500 \text{ kHz} \end{aligned} \quad (14.39)$$

ONE-POLE FILTER: Usually, the presence of a lowpass filter in the loop is inevitable because of the charge-storage effects in the amplifier. Further, it is often desirable to introduce filtering with other RC elements to reject high-frequency intermodulation components generated in the PC. For the overall response of the PLL, the filtering is desired for applications where additional, interfering signals are present at the input to the PLL. In a common situation for an IC PLL, the filter has a single-pole response and takes the form shown in Figure 14.7a. For this situation,

$$F(s) = \frac{1}{\left[1 + \frac{s}{w_1}\right]} \quad (14.40)$$

where $w_1 = \frac{1}{R_1 C_1}$. The closed-loop transfer function of the PLL becomes

$$\begin{aligned} \frac{v_o}{\Delta \omega_i} &= \frac{\left(\frac{1}{K_o}\right)}{1 + \frac{s}{K_L} + \frac{s^2}{(\omega_1 K_L)}} \\ &= \frac{K_L \omega_1}{K_o} \left[\frac{1}{(s^2 + \omega_1 s + \omega_1 K_L)} \right] \end{aligned} \quad (14.41)$$

The transfer function of the closed-loop PLL has a two-pole response and the PLL is called a second-order loop. The poles of the closed-loop response are shown in Figure 14.7b where the locii of the poles are also shown as the parameter K_L is varied. The closed-loop poles, if complex, are

$$\begin{aligned}
 s_1, s_2 &= \frac{-\omega_1}{2} \pm j \left[\omega_1 K_L - \left(\frac{\omega_1}{2} \right)^2 \right]^{0.5} \\
 &= -\epsilon \omega_n \pm j \omega_n [1 - \epsilon^2]^{0.5}
 \end{aligned}
 \tag{14.42}$$

where the auxiliary constants are

$$\begin{aligned}
 \omega_n^2 &= \omega_1 K_L \\
 \epsilon &= \frac{1}{2} \left[\frac{\omega_1}{K_L} \right]^{0.5}
 \end{aligned}
 \tag{14.43}$$

(Note that this epsilon is not related to the van der Pol parameter.)

A set of steady-state magnitude responses are plotted for a sequence of the closed-loop pole locations in Figure 14.7c. Peaking occurs for $\epsilon < 0.707$. For $\epsilon = 0.707$, $\omega_1 = 2K_L$ and a maximally flat magnitude response is obtained with a -3dB bandwidth of $\omega_{-3dB} = \omega_n = 1.414K_L$. Here again, the -3dB bandwidth of the loop is dictated by the value of the loop gain, K_L .

For a numerical example, we start with the values of the Spice input of Figure 14.2b.

$$\begin{aligned}
 \omega_1 &= \frac{1}{R_f C_f} = \frac{1}{(1k\Omega * 8nF)} \\
 &= 2\pi 20kHz \\
 K_L &= 2\pi 500kHz \\
 \omega_n &= 2\pi 100kHz \\
 \epsilon &= \frac{1}{10} \\
 s_1, s_2 &= 2\pi [0.1 \pm j1] 10^5 \\
 &= 2\pi (10 \pm j100) 10^3
 \end{aligned}
 \tag{14.44}$$

A sketch of the closed-loop pole locations is given in Figure 14.7d. The magnitude response for this example is given in Figure 14.7e. A 400% peaking is observed, and the -3dB bandwidth is approximately to $\Delta\omega_i = 2\pi 155kHz$. This type of magnitude response can permit 'outband' signals to enter the PLL and appear at the output.

Clearly, the closed-loop poles of the example above are not positioned for a maximally flat, closed-loop magnitude function. For this type of response, the pole of the filter should be located at $-\omega_1 = -2K_L = -2\pi 10^6$. The -3dB frequency for this condition would be $1.414megHz$ and the filtering

of the higher-order intermodulation components of the PC output would not be large.

We can check on the validity of the above evaluation by determining the closed-loop pole positions from the data of Figure 14.4a. First recall the form of the closed loop response for the small-signal situation.

$$v_o(s) = A(s)\Delta\omega_1(s) \quad (14.45)$$

$\Delta\omega_1$ must be considered to be a constant and for the small-signal case is the deviation from w_{free} .

$$\Delta\omega_1(s) = \frac{\Delta\omega_1}{s} \quad (14.46)$$

For the case at hand, where a one-pole filter is used,

$$A(s) = \frac{H_1}{[s^2 + as + b]} = \frac{H_1}{[(s - s_1)(s - s_2)]} \quad (14.47)$$

where H_1 is an auxiliary constant. For the example in question, the closed-loop poles, s_1, s_2 , are a complex pair. The output voltage in the time domain has the form:

$$v_o(t) = H_2 e^{-\frac{at}{2}} \cos(Bt) + H_3 \quad (14.48)$$

where $B = [b - (\frac{a}{2})^2]^{0.5}$. There is an oscillatory decay to the steady-state, the constant H_3 , since the system is underdamped. From the period of the response, we obtain an estimate for the value of the imaginary part of the poles. The period is approximately $10\mu s$, leading to $B = \text{Imag}(s_1) = 2\pi \cdot 10^5$, which is equal to the estimated value of (14.44). The decay of the envelope of the waveform is assumed to have a simple exponential form of (14.48).

$$v(t) = A e^{-\frac{at}{2}} \quad (14.49)$$

From the ratio of the successive positive peaks or negative peaks, $\frac{v_1}{v_2}$, we obtain

$$t_2 - t_1 = \text{period} = \left(\frac{2}{a}\right) \ln\left(\frac{v_1}{v_2}\right) \quad (14.50)$$

$$\frac{a}{2} = \left[\frac{\ln\left(\frac{v_1}{v_2}\right)}{(t_2 - t_1)} \right]$$

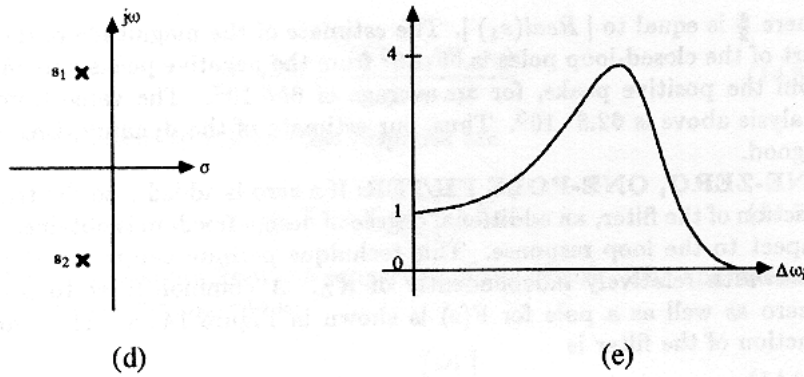


Figure 14.7: (d) A sketch of the closed-loop pole locations. (e) The magnitude response.

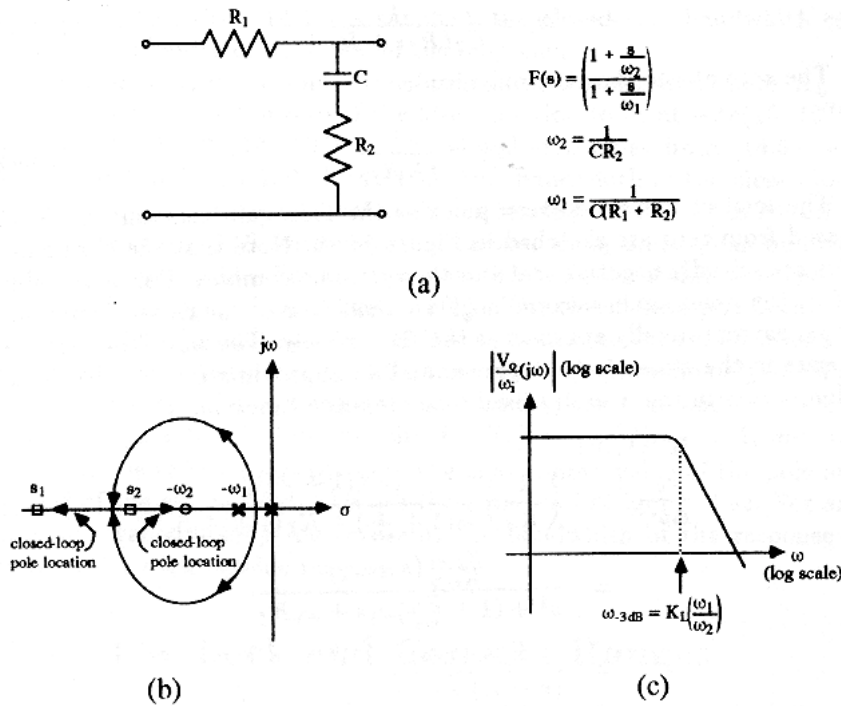


Figure 14.8: (a) A one-pole, one-zero filter. (b) Pole locations and locii. (c) Magnitude response of the closed-loop transfer function.

where $\frac{\sigma}{\omega}$ is equal to $| \text{Real}(s_1) |$. The estimate of the magnitude of the real part of the closed-loop poles is $65 \cdot 10^3$ from the negative peaks and $56 \cdot 10^3$ from the positive peaks, for an average of $61 \cdot 10^3$. The value from the analysis above is $62.8 \cdot 10^3$. Thus, our estimate of the dynamical response is good.

ONE-ZERO, ONE-POLE FILTER: If a zero is added into the transfer function of the filter, an additional degree of design freedom is obtained with respect to the loop response. This technique permits one to set the loop bandwidth relatively independently of K_L . A common filter to provide a zero as well as a pole for $F(s)$ is shown in Figure 14.8a. The transfer function of the filter is

$$F(s) = \frac{v_o}{v_p} = \frac{\left[1 + \frac{s}{\omega_2}\right]}{\left[1 + \frac{s}{\omega_1}\right]} \quad (14.51)$$

The pole at $-\omega_1$ has the magnitude

$$\omega_1 = \frac{1}{C_f(R_{f1} + R_{f2})} \quad (14.52)$$

The zero at $-\omega_2$ has the magnitude

$$\omega_2 = \frac{1}{C_f R_{f2}} \quad (14.53)$$

The locii of the closed-loop poles as the loop gain constant K_L is increased from zero are sketched in Figure 14.8b. Note that the closed-loop poles are initially negative real and then become complex. For large values of K_L , the poles again become negative real. One of the closed-loop poles, say s_2 , asymptotically approaches the zero at $-\omega_2$. The zero, however, also appears in the closed-loop response, and an approximate pole-zero cancellation occurs in the overall closed-loop response function.

$$\begin{aligned} \frac{v_o}{\Delta\omega_i} &= \left(\frac{1}{K_o}\right) \frac{[K_L(1 + \frac{s}{\omega_2})]}{[s(1 + \frac{s}{\omega_1}) + K_L(1 + \frac{s}{\omega_2})]} \quad (14.54) \\ &= \frac{\frac{K_L\omega_1}{K_o\omega_2}(s + \omega_2)}{s^2 + (1 + \frac{K_L}{\omega_2})\omega_1 s + \omega_1 K_L} \\ &= \frac{H(s + \omega_2)}{(s - s_1)(s - s_2)} \end{aligned}$$

When the pole, s_2 , and the zero, $-\omega_2$, approximately cancel for large values of K_L , the closed-loop response is approximately a one-pole response.

$$\frac{v_o}{\Delta\omega_i} = \frac{(K_L\omega_1/K_o\omega_2)}{(s - s_1)} \quad (14.55)$$

The conditions to achieve this response are

$$\omega_1 < \omega_2 < \left(\frac{\omega_1}{\omega_2}\right) K_L \quad (14.56)$$

where $\omega_1 < \omega_2$ follows from the properties of the circuit. The approximate value of the remaining pole is:

$$s_1 \approx -K_L \left[\frac{\omega_1}{\omega_2}\right] \quad (14.57)$$

The magnitude response is sketched in Figure 14.8c. The closed-loop bandwidth is the magnitude of the remaining pole, $|s_1|$.

$$BW_{-3dB} = |s_1| \quad (14.58)$$

To the extent that (14.56) is satisfied, the closed-loop bandwidth can be set independently of the value of the loop gain.

To continue with the numerical example of this section where $K_L = 2\pi \cdot 5 \cdot 10^5$, we choose the zero of the filter function to lie at $-2\pi(20 \cdot 10^3)\frac{r}{s}$ and the pole at $-2\pi(10 \cdot 10^3)\frac{r}{s}$. The closed-loop poles from (14.54) are $-2\pi(21 \cdot 10^3)$ and $-2\pi(239 \cdot 10^3)$. The $-3dB$ bandwidth of the closed-loop response is approximately $239kHz$. The estimate from (14.58) is $250kHz$.

To reduce the $-3dB$ frequency, new choices for ω_1 and ω_2 can be made using (14.58) as a guide. For an approximate $50kHz$ bandwidth, we choose $\omega_1 = 2\pi(0.5 \cdot 10^3)\frac{r}{s}$ and $\omega_2 = 2\pi(5 \cdot 10^3)\frac{r}{s}$. The closed-loop pole locations are $-2\pi(5.56 \cdot 10^3)\frac{r}{s}$ and $-2\pi(44.9 \cdot 10^3)\frac{r}{s}$. s_2 is close to the zero at $-\omega_2$ and the magnitude response has a $-3dB$ bandwidth of approximately $45kHz$.

Another simple filter configuration providing one pole and one zero is shown in Figure 14.8d. For this circuit $F(0) = \frac{R_{f2}}{(R_{f1} + R_{f2})} < 1$, and the value of K_L is reduced accordingly. The zero is now inside of the pole and the locii of the closed-loop poles are those shown in Figure 14.8e. For any reasonable value of loopgain constant, the bandwidth of the response is determined by the outside pole, s_2 .