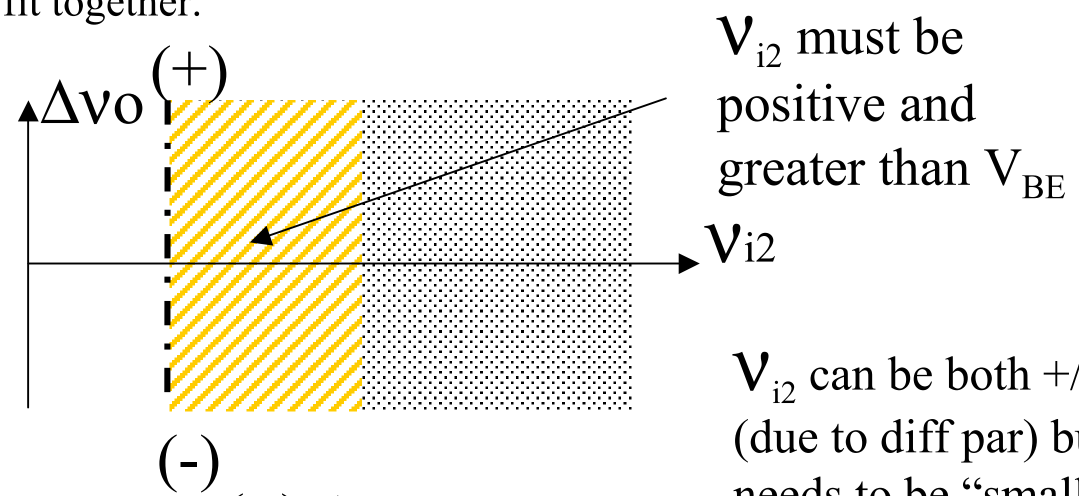
2X2 Quad==4Quad Multiplier

The following are qualitative and conceptual points about 2Q & 4Q Multipliers. Basically Prelab 2 and Lab 2 moves fairly quickly into useful chips that do multiplying. At one level you won't have to struggle with biasing etc. Nonetheless, it is really nice to have a grasp of how things fit together.

2Quad:
$+\nu_{i2}$ only



4Quad:
+&- ν_{i2}

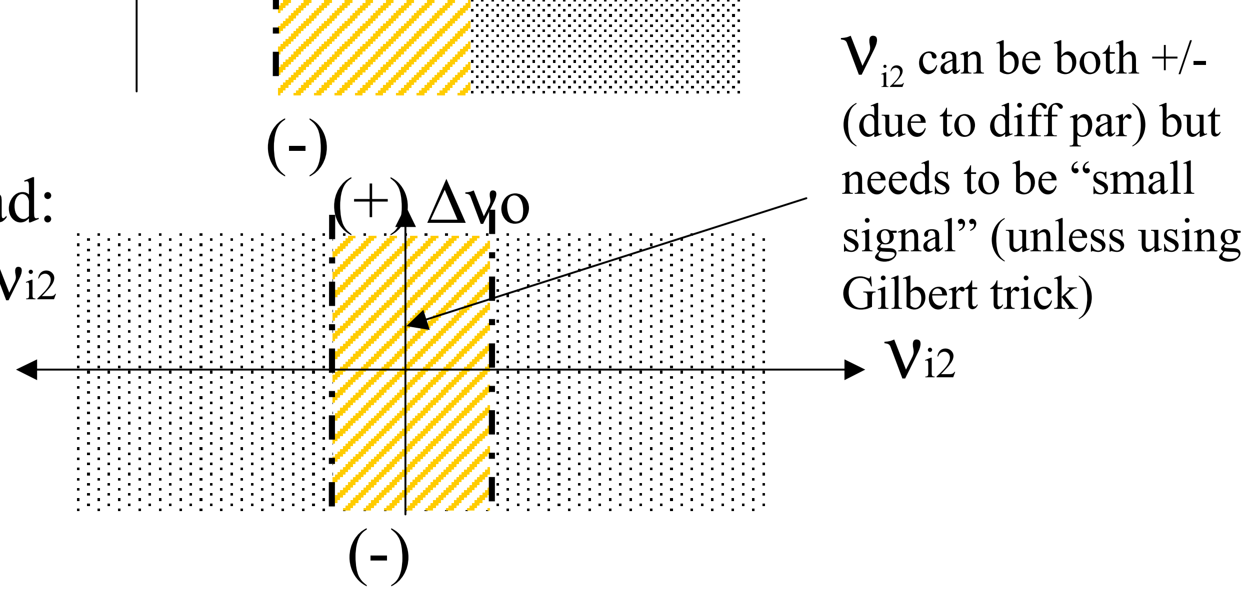
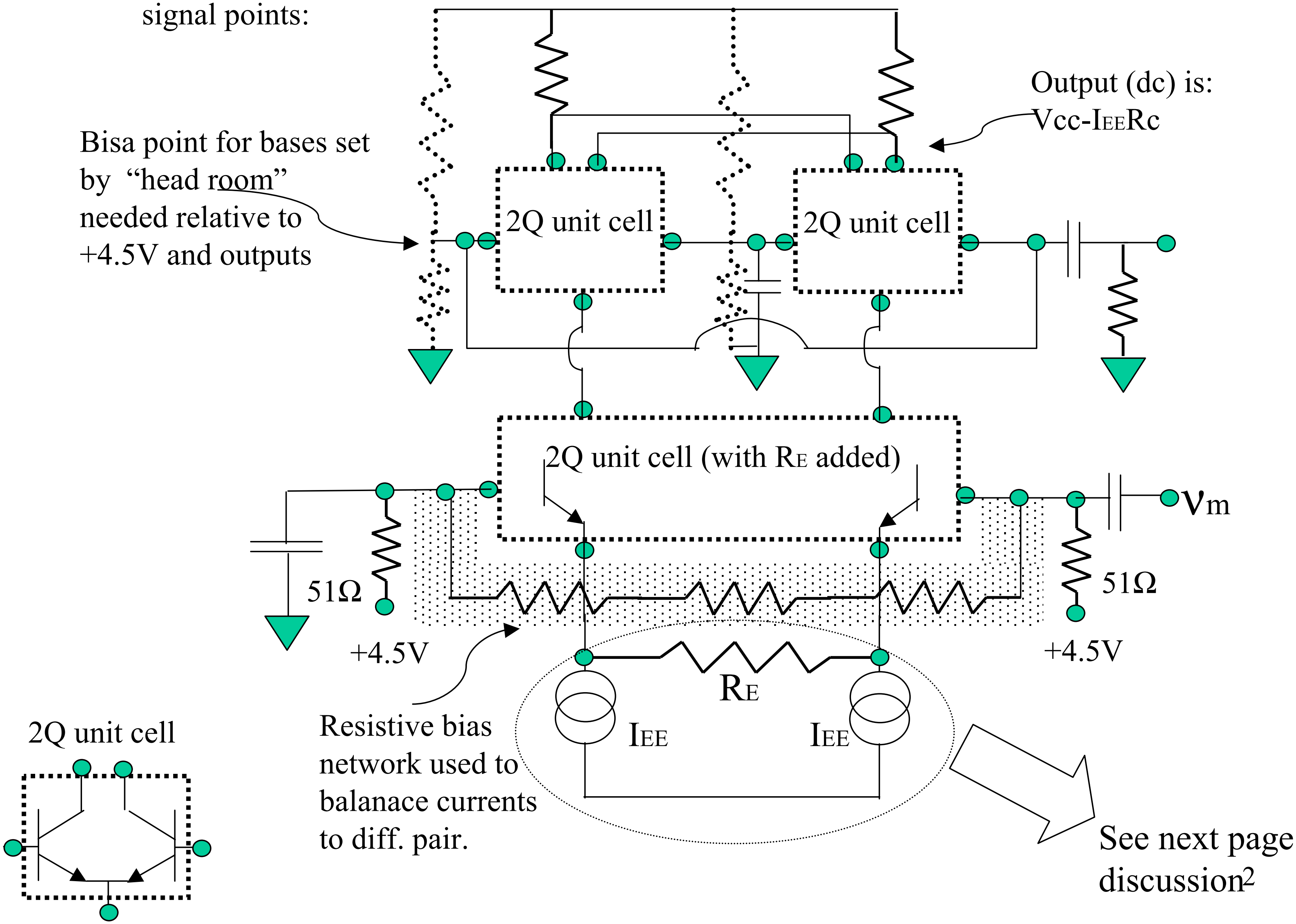
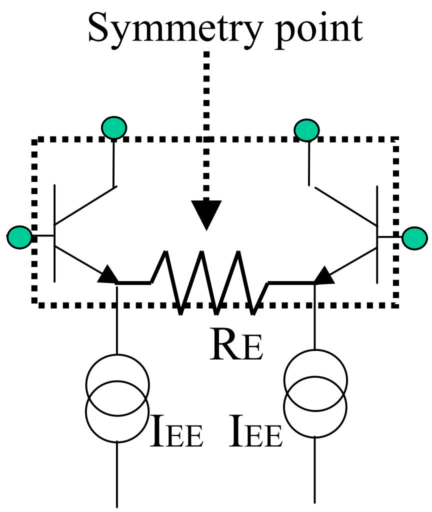
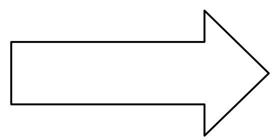


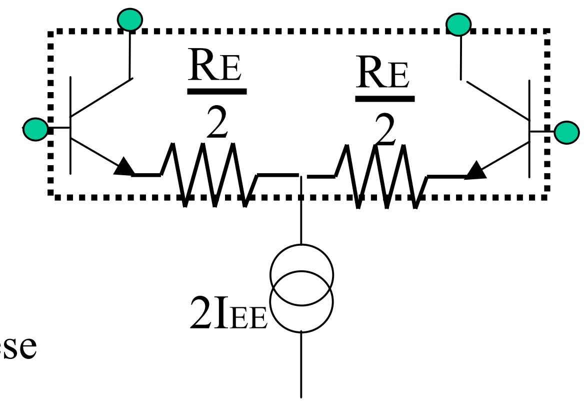
Fig 3 (prelab 2) can be redrawn as follows, primarily to emphasize the symmetry and bias vs signal points:



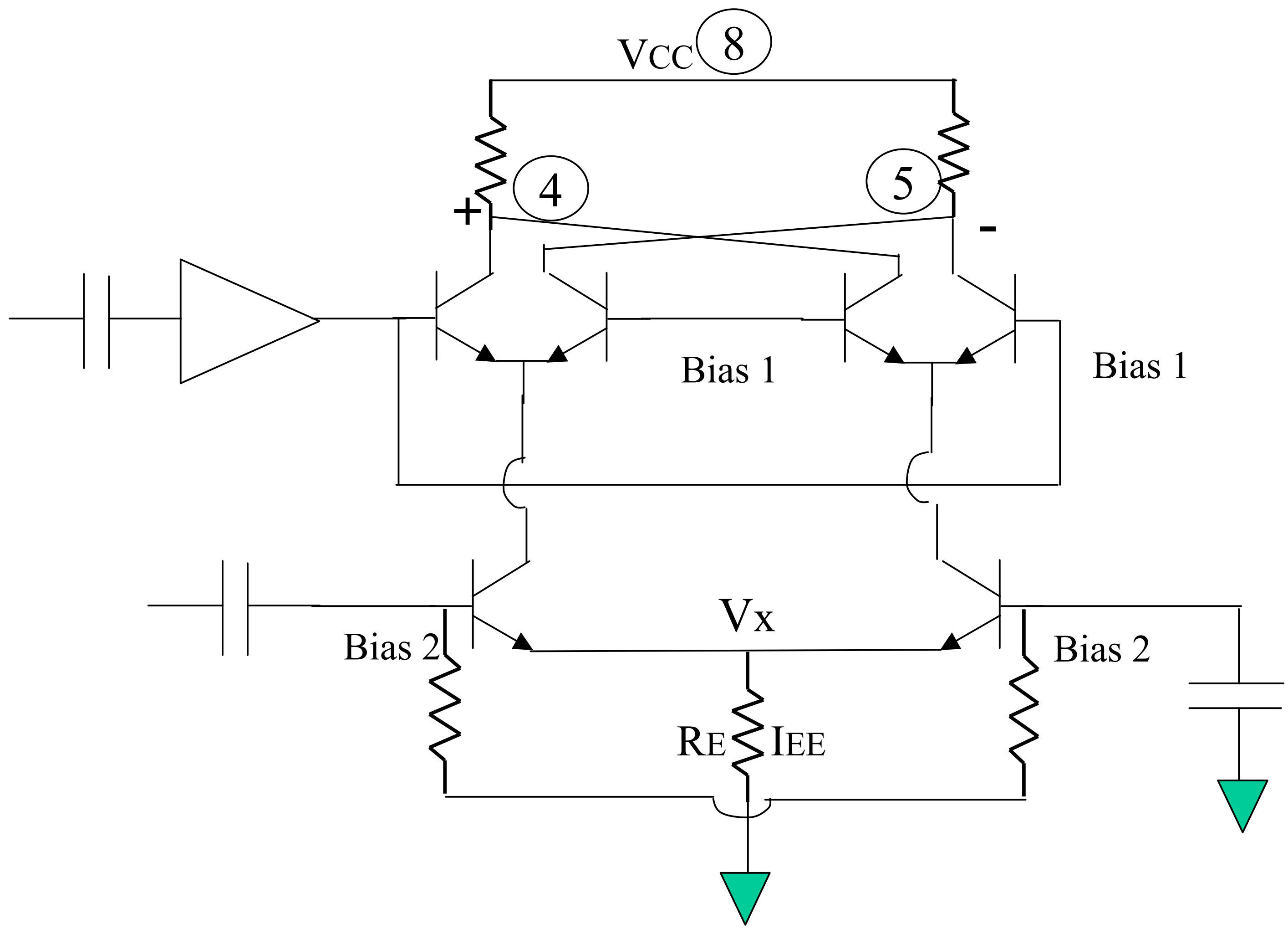




Equivalence of these two circuits…both give same input-output per base & collectors



Now, assuming all that made sense, look at Lab circuit ala 602:



Where are bias points (i.e. What voltages?) Start from nodes 4 & 5

$$\frac{V_{cc} - V4}{R_c} = I_1 \quad ; \quad \frac{V_{cc} - V5}{R_c} = I_2$$

$$I_{EE} = I_1 + I_2$$

If we know R_E => $V_x = I_{EE} R_E$

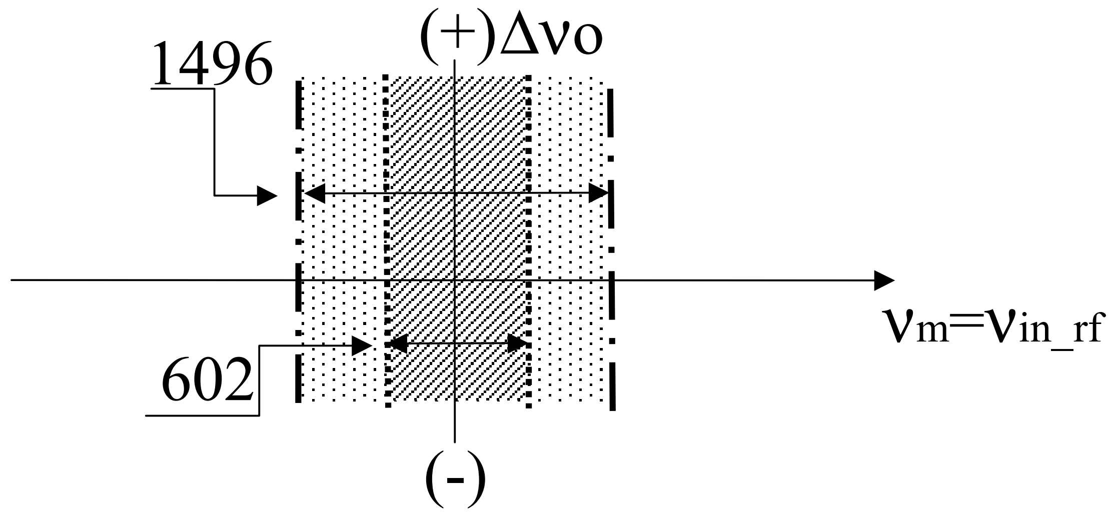
$\therefore V_{BIAS2} = V_x + V_{BE}$

$$V_{BIAS1}\Big|_{\min} = V_{BIAS2} + V_{BE} + V_{CE(sat)}$$

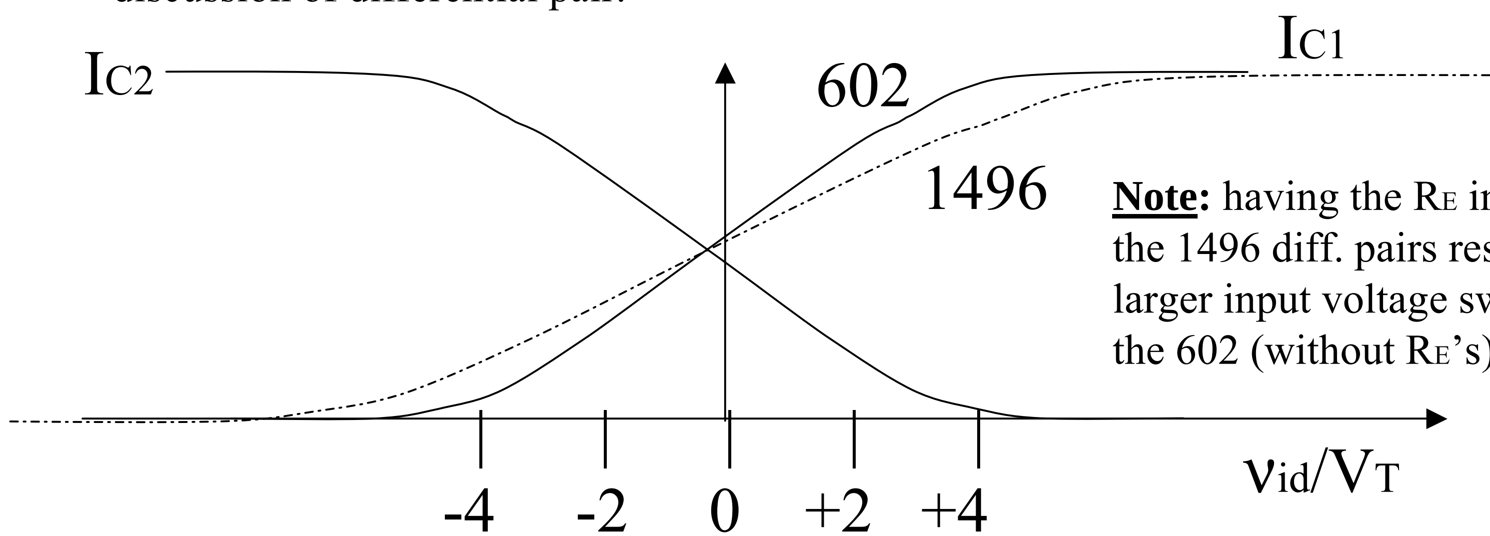
$$V_{BIAS1}\Big|_{\max} = V_4 = V_5$$

Basically, V_{BIAS1} needs to be somewhere between these limits.

Final comments on differences between Fig 3 (Prelab 2; "1496" circuit) and 602:



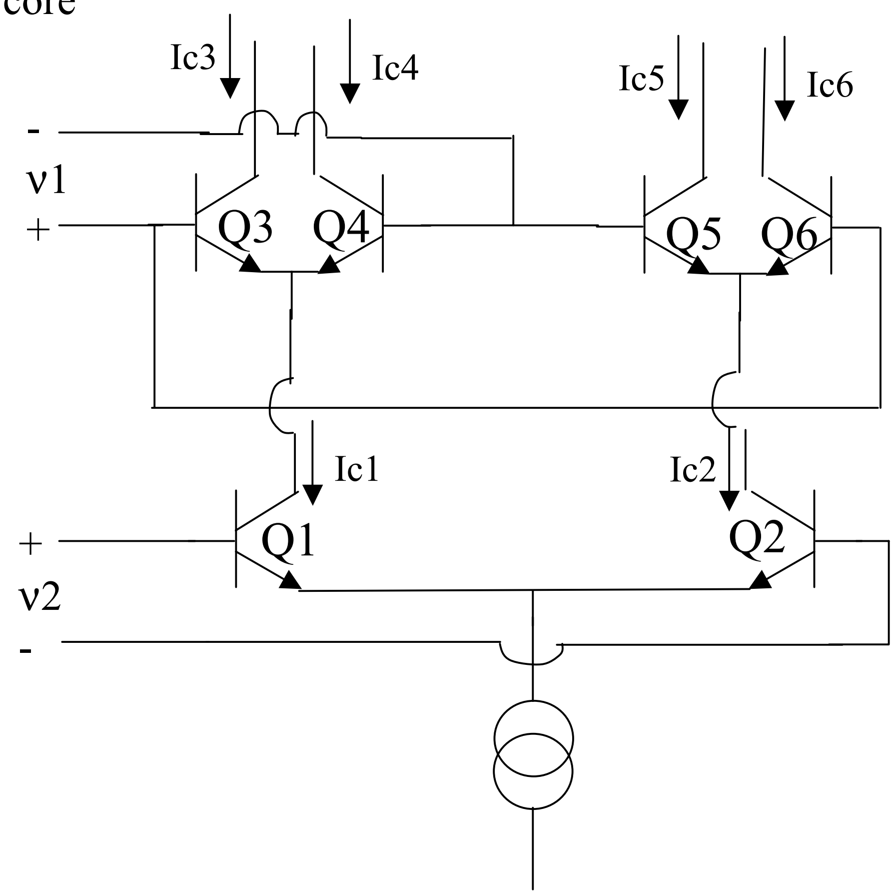
Comment: the bottom diff pair limits how much swing at v_m. To understand, look at earlier discussion of differential pair.



Note: having the R_E included in the 1496 diff. pairs results in a larger input voltage swing vs. the 602 (without R_E's).

Now, time for the down-and-detailed…Four Quad Multiplier:

Even with the beauty and simplicity of the multiplier with diff. Pair and current source, there are limits (2 quad, linearity, dynamic range…) The following is a fully symmetric 4 Quad "core"



By exact analogy to previous derivation for single diff pair

$$I_{c3} - I_{c4} = \frac{I_{c1}}{1 + e^{-v_1/V_T}} - \frac{I_{c1}}{1 + e^{+v_1/V_T}}$$

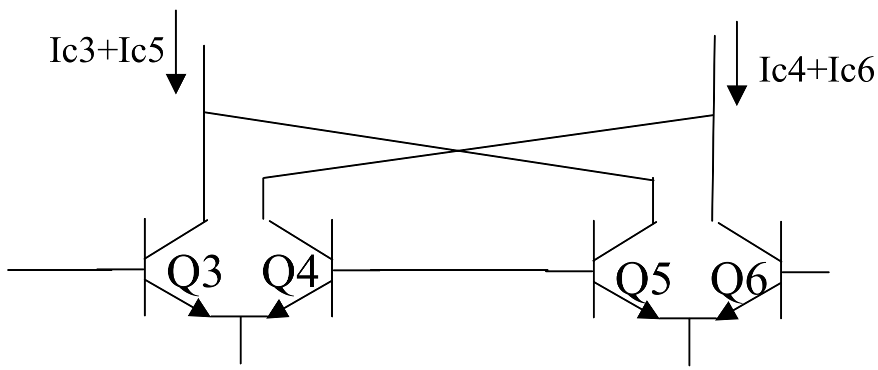
$$= \tanh\left({v_1}/{2V_T} \right) \cdot I_{c1} \xleftarrow{where} \frac{I_{EE}}{1 + e^{-v_2/V_T}}$$

and *for* $Q5 - Q6$

$$I_{c5} - I_{c6} = \tanh\left({v_1}/{2V_T} \right) \cdot I_{c2} \leftarrow \frac{I_{EE}}{1 + e^{+v_2/V_T}}$$

Now taking $(I_{c3}-I_{c4})+(I_{c5}-I_{c6})$, the exponential terms involving $\exp\left(\pm v_2/V_T\right)$ Also reduce to tanh:

$$\left(I_{c3}-I_{c4}\right)+\left(I_{c5}-I_{c6}\right)=I_{EE}\tanh\left(\frac{v_1}{2V_T}\right)\cdot\tanh\left(\frac{v_2}{2V_T}\right)$$



The circuit connectivity to actually realize this tanh(x) · tanh(y) function is cross-coupled as shown and voltage output $R(I_{c3}+I_{c5})-R(I_{c4}+I_{c6})$

As noted in previous diff pair is equal to (2 Quad) discussions, there are both : limitations due to tanh functions and ways to "fix" and get around them:

1) Use small v_1 and v_2 (not always useful)
2) "linearize" lower diff pair by adding R_E to each emitter (quite useful)
3) "prescale" both x and y arguments (i.e. $x=\tanh^{-1}v_1$ and $y=\tanh^{-1}v_2$)

This last approach is the very elegant (and useful) Gilbert multiplier "with all the trimmings").

There will be a handout on the fully prescaled Gilbert multiplier (but it will not be discussed further)

Distortion

We will consider two main kinds: harmonic (HD) and intermodulation (IM), lets start with HD and then come back to IM.

$$a\cos(\omega_1 t) \rightarrow A \rightarrow b_1 \cos(\omega_1 t + \phi_1) + b_2 \cos(2\omega_1 t + \phi_2) + b_3 \ldots$$

$$HD_2 \equiv \frac{|b_2|}{|b_1|} \quad ; \quad HD_3 \equiv \frac{|b_3|}{|b_1|}$$

$$THD \equiv \frac{\left[b_2^2 + b_3^2 + \ldots\right]^{1/2}}{|b_1|}$$

Using power series expansion on collector current, expressions for the diff pair:

$$\frac{I_{c2}}{I_{EE}} = \frac{1}{1+\exp(d)} = \frac{1}{2} - \frac{d}{4} + \frac{1}{48}d^3 - \frac{1}{480}d^5 \ldots$$

↑

note: d^2 term is not here!

This is different for the expression (and harmonic terms) we noted in Ch. 4 Krauss for a single CE amplifier)

Using d = a cosω_1t and doing a bit of math/algebra
(see Mayaram & Pederson, Kluwer '91 p.17)

$$\frac{I_{c2}}{I_{EE}} = b_o^1 + b_1 \cos\omega_1 t + b_2 \cos 2\omega_1 t + b_3 \cos 3\omega_1 t \ldots$$

$$b_o^1 = {}^1\!/_2 \qquad b_3 = \frac{1}{192}\left(\frac{a}{V_T}\right)^3$$

$$b_1 = {}^1\!/_4\left({}^a\!/_{V_T} - \frac{1}{16}\left(\frac{a}{V_T}\right)^3 \right) \qquad \vdots$$

$$b_2 = 0$$

Figure 1.16 (Mayaram & Pederson) shows how these harmonics vary vs v_1/V_T (input signal in normalized units of V_T):

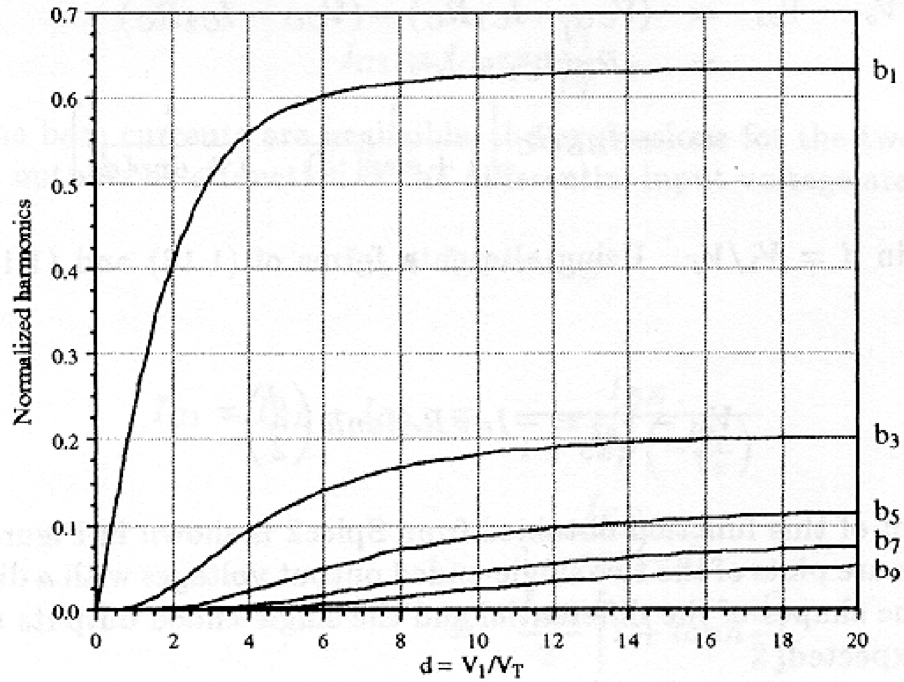


Figure 1.16: Normalized harmonics of the collector current in ECP.

From this figure we can easily estimate HD_3… and THD. As noted above, $b_2 = 0 \quad \therefore \quad HD_2 = 0$ (good news for using diff. Pair). Analytically, HD_3 can be expressed:

$$HD_3 \simeq \frac{1}{48}\left(\frac{a}{V_T}\right)^2$$

$$\uparrow \frac{kT}{q}$$

Following a similar procedure for MOS device using the simple current expression:

$$I_D = k' W/L \left[(V_{Gs} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

$$HD_3(MOS) = \frac{1}{32}\left(\frac{a}{V_{GG} - V_{TH}(MOS)}\right)^2$$

If we use a single transistor (common emitter) amplifier, things: 1) get more complex and 2) performance is worse.

$$I_c = I_{CA} e^{v1/VT} \quad where \quad I_{CA} = I_s e^{\mathrm{VBE}/\mathrm{VT}}$$

$$\simeq I_{CA}\left[1 + \frac{v_1}{V_T} + \frac{1}{2}\left(\frac{v_1}{V_T}\right)^2 + \frac{1}{6}\left(\frac{v_1}{V_T}\right)^3 \ldots\right]$$

Considering the case where $\frac{v_1}{VT} \equiv d$

$$e^{(d\cos\omega_1 t)} = I_o(d) + 2I_1(d)\cos\omega_1 t + 2I_2(d)\cos 2\omega_1 t + \ldots 2I_n(d)\cos n\omega_1 t..$$

Where the I_n's are modified Besset function. (order n)

$$I_c = \underbrace{I_{CA} I_o(d)}_{I_{dc}} \left[1 + \frac{2I_1(d)}{I_o(d)} \cos \omega_1 t + \frac{2I_n(d)}{I_o(d)} \cos n\omega_1 t \ldots \right]$$

$$b_1 = I_{dc} \left[\frac{2I_1(d)}{I_o(d)} \right] \quad HD_2 = \frac{I_2(d)}{I_1(d)} \quad \left(\simeq \frac{1}{4} d, \begin{matrix} \text{for} \\ d \ll 1 \end{matrix} \right)$$

$$b_2 = I_{dc} \left[\frac{2I_2(d)}{I_o} \right] \quad HD_3 = \frac{I_3(d)}{I_1(d)}$$

$$\vdots \qquad\qquad \vdots$$

Plotting the respective ratios $I_n(d)/I_o(d)$ we see something qualitatively like Fig 1.16 (Mayaram & Pederson) but much more severe quantitatively in terms of HD

Figure 2.6 (Mayaram & Pederson)

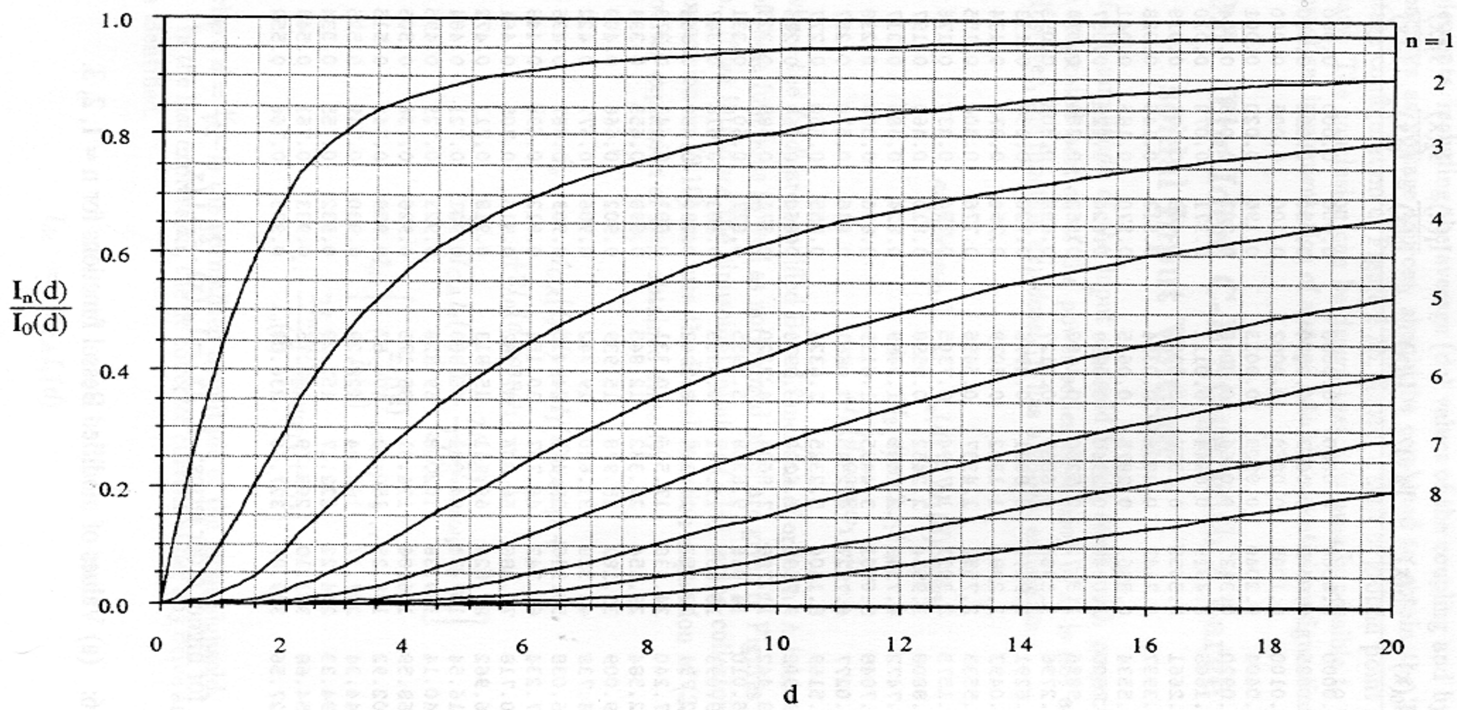


Figure 2.6: (b) Plot of normalized Bessel functions for n = 1 to 8.

Intermodulation (IM) distortion arises when multiple frequencies are present

$$v_i = v_{1a}\cos\omega_1 t + v_{2a}\cos\omega_2 t$$

$$v_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots$$

Using υ_i in υ_o expression gives <u>lots</u> of terms. The primary ones of concern come from 3rd order as follows:.

$$a_3 v_i^3 = \ldots a_{32}\cos(\omega_1 t \pm 2\omega_2 t) + a_{33}\cos(2\omega_1 t \pm \omega_2 t)$$

$$a_{32} = \frac{3a_3}{4} V_{1a} V_{2a}^2 \quad a_{33} = \frac{3a_3}{4} V_{1a}^2 V_{2a}$$

Assuming $V_{1a}=V_{2a}$, then approximately

$$IM_3 = \frac{V_o\left[IM_3\right]}{a_1 V_{1a}} = \frac{\frac{3}{4}a_3 V_{1a}^3}{a_1 V_{1a}} = \frac{3}{4}\frac{a_3}{a_1}V_{1a}^2$$

There are several basic points to note as well as practical considerations:

- the IM3 grows super linearly
- this means it overtakes the linear gain term (a big problem)
- we need to control output power to avoid such IM distortion problems (see figure and physical interpretation)