# Analysis and Design of Analog Integrated Circuits

Third Edition

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## 11.3 Noise Models of Integrated-Circuit Components

In the above sections, the various physical sources of noise in electronic circuits were described. In this section, these sources of noise are brought together to form the small-signal equivalent circuits including noise for diodes and for bipolar and field-effect transistors.

### 11.3.1 Junction Diode

The equivalent circuit for a junction diode was considered briefly in the consideration of shot noise. The basic equivalent circuit of Fig. 11.4 can be made complete by adding series resistance $r_s$ as shown in Fig. 11.12. Since $r_s$ is a physical resistor due to the resistivity of the silicon, it exhibits thermal noise. Experimentally it has been found that any flicker noise present can be represented by a current generator in shunt with $r_d$, and this is conveniently combined with the shot-noise

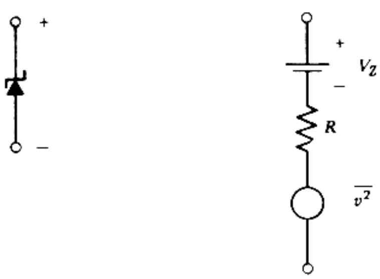



**Figure 11.11** Equivalent circuit of a Zener diode including noise.

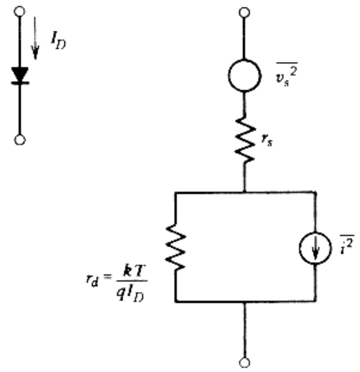



**Figure 11.12** Complete diode small-signal equivalent circuit with noise sources.

generator as indicated by (11.10) to give

$$\overline{v_s^2} = 4kTr_s\Delta f \tag{11.9}$$

$$\overline{i^2} = 2qI_D\Delta f + K\frac{I_D^a}{f}\Delta f \tag{11.10}$$

### 11.3.2 Bipolar Transistor[11]

In a bipolar transistor in the forward-active region, minority carriers diffuse and drift across the base region to be collected at the collector–base junction. Minority carriers entering the collector-base depletion region are accelerated by the field existing there and swept across this region to the collector. The time of arrival at the collector–base junction of the diffusing (or drifting) carriers is a purely random process, and thus the transistor collector current consists of a series of random current pulses. Consequently, collector current $I_C$ shows *full shot noise* as given by (11.2), and this is represented by a shot noise current generator $\overline{i_c^2}$ from collector to emitter as shown in the equivalent circuit of Fig. 11.13.

Base current $I_B$ in a transistor is due to recombination in the base and base-emitter depletion regions and also to carrier injection from the base into the

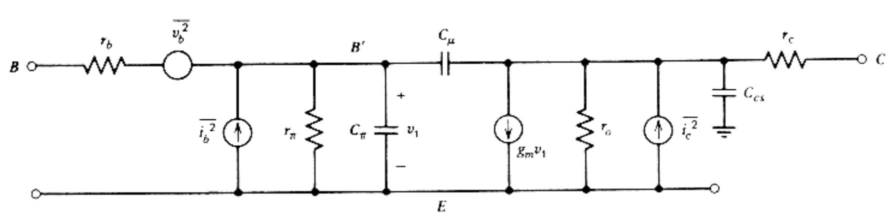



**Figure 11.13** Complete bipolar transistor small-signal equivalent circuit with noise sources.

emitter. All of these are independent random processes, and thus $I_B$ also shows *full shot noise*. This is represented by shot noise current generator $\overline{i_b^2}$ in Fig. 11.13.

Transistor base resistor $r_b$ is a physical resistor and thus has thermal noise. Collector series resistor $r_c$ also shows thermal noise, but since this is in series with the high-impedance collector node, this noise is negligible and is usually not included in the model. Note that resistors $r_\pi$ and $r_o$ in the model are *fictitious* resistors that are used for modeling purposes only, and they do *not* exhibit thermal noise.

Flicker noise and burst noise in a bipolar transistor have been found experimentally to be represented by current generators across the internal base–emitter junction. These are conveniently combined with the shot noise generator in $\overline{i_b^2}$. Avalanche noise in bipolar transistors is found to be negligible if $V_{CE}$ is kept at least 5 V below the breakdown voltage $BV_{CEO}$, and this source of noise will be neglected in subsequent calculations.

The full small-signal equivalent circuit including noise for the bipolar transistor is shown in Fig. 11.13. Since they arise from separate, independent physical mechanisms, all the noise sources are *independent* of each other and have mean-square values:

$$\overline{v_b^2} = 4kTr_b\Delta f \tag{11.11}$$

$$\overline{i_c^2} = 2qI_C\Delta f \tag{11.12}$$

$$\overline{i_b^2} = \underbrace{2qI_B\Delta f}_{\text{Shot noise}} + \underbrace{K_1\frac{I_B^a}{f}\Delta f}_{\text{Flicker noise}} + K_2\frac{I_B^c}{\underbrace{1+\left(\frac{f}{f_c}\right)^2}_{\text{Burst noise}}}\Delta f \tag{11.13}$$

This equivalent circuit is valid for both *npn* and *pnp* transistors. For *pnp* devices, the magnitudes of $I_B$ and $I_C$ are used in the above equations.

The base-current noise spectrum can be plotted using (11.13), and this has been done in Fig. 11.14, where burst noise has been neglected for simplicity. The shot noise and flicker noise asymptotes meet at a frequency $f_a$, which is called the

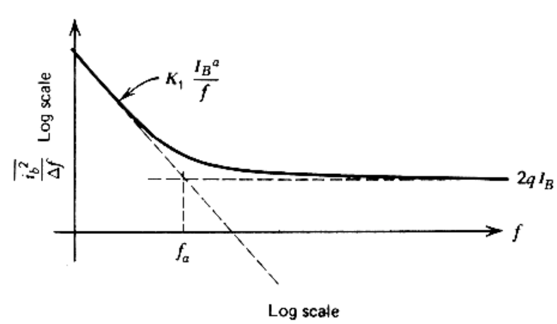



**Figure 11.14** Spectral density of the base-current noise generator in a bipolar transistor.

flicker noise "corner" frequency. In some transistors using careful processing, $f_a$ can be as low as 100 Hz. In other transistors $f_a$ can be as high as 10 MHz.

### 11.3.3 Field-Effect Transistor[12,13]

The structure of field-effect transistors (FET) was described in Chapter 1. There it was shown that the resistive channel joining source and drain is modulated by the gate-source voltage so that the drain current is controlled by the gate-source voltage. Since the channel material is *resistive*, it exhibits *thermal noise*, and this is the major source of noise in FETs (both JFET and MOSFET). It can be shown that this noise source can be represented by a noise-current generator $\overline{i_d^2}$ from drain to source in the FET small-signal equivalent circuit of Fig. 11.15. Flicker noise in the FET is also found experimentally to be represented by a drain-source current generator, and these two can be lumped into one noise generator $\overline{i_d^2}$. The other source of noise in FETs is shot noise generated by the gate leakage current. This is represented by $\overline{i_g^2}$ in Fig. 11.15 and is usually very small. It becomes significant only when the driving-source impedance connected to the FET gate is very large. The generators in Fig. 11.15 are *all independent* of each other and have values

$$\overline{i_g^2} = 2qI_G\Delta f \tag{11.14}$$

$$\overline{i_d^2} = \underbrace{4kT\left(\frac{2}{3}g_m\right)\Delta f}_{\text{Thermal noise}} + \underbrace{K\frac{I_D^a}{f}\Delta f}_{\text{Flicker noise}} \tag{11.15}$$

where

$I_G$ is the gate leakage current
$I_D$ is the drain bias current
$K$ is a constant for a given device
$a$ is a constant between 0.5 and 2
$g_m$ is the device transconductance at the operating point

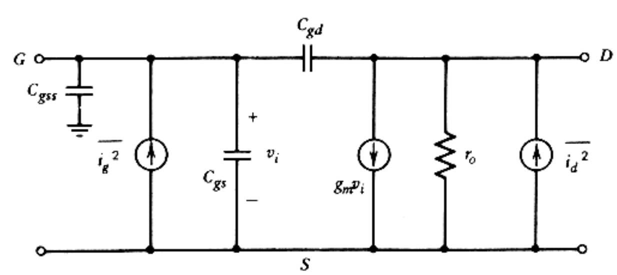



**Figure 11.15** FET small-signal equivalent circuit with noise sources.

### 11.3.4 Resistors

Monolithic and thin-film resistors display thermal noise as given by (11.4) and (11.5), and the circuit representation of this is shown in Fig. 11.7. As mentioned in Section 11.2.3, discrete carbon resistors also display flicker noise, and this should be considered if such resistors are used as external components to the integrated circuit.

### 11.3.5 Capacitors and Inductors

Capacitors are common elements in integrated circuits, either as unwanted parasitics or as elements introduced for a specific purpose. Inductors in general cannot be realized on the silicon die but are sometimes used as external elements, particularly in integrated communication circuits. There are *no sources of noise* in *ideal* capacitors or inductors. In practice, real components have parasitic resistance that *does* display noise as given by the thermal noise formulas of (11.4) and (11.5). In the case of integrated-circuit capacitors, the parasitic resistance usually consists of a small value in series with the capacitor. Parasitic resistance in inductors can be modeled by either series or shunt elements.

## 11.4 Circuit Noise Calculations [14,15]

The device equivalent circuits including noise that were derived above can be used for the calculation of circuit noise performance. First, however, methods of circuit calculation with noise generators as sources must be established, and attention is now given to this problem.

Consider a noise current source with mean-square value

$$\overline{i^2} = S(f)\,\Delta f \tag{11.16}$$

where $S(f)$ is the *noise spectral density*. The value of $S(f)$ is plotted versus frequency in Fig. 11.16*a* for an arbitrary noise generator. In a small bandwidth $\Delta f$, the mean-square value of the noise current is given by (11.16), and the rms value can be written as

$$i = \sqrt{S(f)\,\Delta f} \tag{11.17}$$

The noise current in bandwidth $\Delta f$ can be represented approximately[14] by a sinusoidal current generator with rms value $i$ as shown in Fig. 11.16*b*. If the noise current in bandwidth $\Delta f$ is now applied as an input signal to a circuit, its effect can be calculated by substituting the sinusoidal generator and performing circuit analysis in the usual fashion. When the circuit response to the sinusoid is calculated, the mean-square value of the output sinusoid gives the mean-square value of the output noise in bandwidth $\Delta f$. *Thus network noise calculations reduce to familiar sinusoidal circuit-analysis calculations.* The only difference occurs when multiple noise sources are applied, as is usually the case in practical circuits. Each

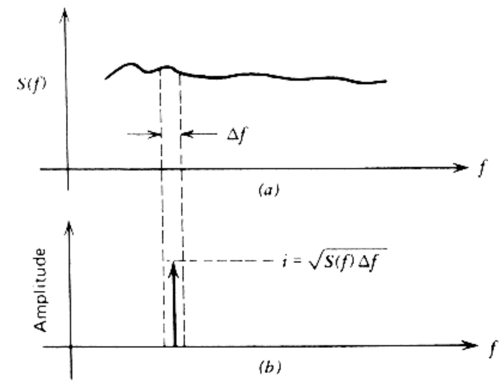



**Figure 11.16** Representation of noise in a bandwidth $\Delta f$ by an equivalent sinusoid with the same rms value.

noise source is then represented by a separate sinusoidal generator, and the output contribution of each one is separately calculated. The total output noise in bandwidth $\Delta f$ is calculated as a *mean-square* value by *adding* the individual *mean-square* contributions from each output sinusoid. This depends, however, on the original noise sources being *independent*, as will be shown below. This requirement is always satisfied if the equivalent noise circuits derived in previous sections are used, as all the noise sources arise from separate mechanisms and are thus independent.

For example, consider two resistors $R_1$ and $R_2$ connected in series as shown in Fig. 11.17. Resistors $R_1$ and $R_2$ have respective noise generators

$$\overline{v_1^2} = 4kTR_1\Delta f \tag{11.18}$$

$$\overline{v_2^2} = 4kTR_2\Delta f \tag{11.19}$$

In order to calculate the mean-square noise voltage $\overline{v_T^2}$ produced by the two resistors in series, let $v_T(t)$ be the instantaneous value of the total noise voltage and

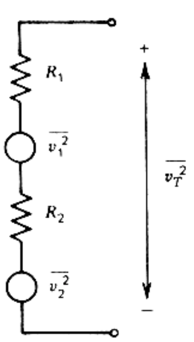



**Figure 11.17** Circuit for the calculation of the total noise $\overline{v_T^2}$ produced by two resistors in series.

$v_1(t)$ and $v_2(t)$ the instantaneous values of the individual generators. Then

$$v_T(t) = v_1(t) + v_2(t) \tag{11.20}$$

and thus

$$\begin{aligned}\overline{v_T(t)^2} &= \overline{[v_1(t) + v_2(t)]^2} \\ &= \overline{v_1(t)^2} + \overline{v_2(t)^2} + \overline{2v_1(t)v_2(t)}\end{aligned} \tag{11.21}$$

Now, since noise generators $v_1(t)$ and $v_2(t)$ arise from separate resistors, they must be *independent*. Thus the *average* value of their product $v_1(t)v_2(t)$ will be zero and (11.21) becomes

$$\overline{v_T^2} = \overline{v_1^2} + \overline{v_2^2} \tag{11.22}$$

Thus the mean-square value of the sum of a number of independent noise generators is the sum of the individual mean-square values. Substituting (11.18) and (11.19) in (11.22) gives

$$\overline{v_T^2} = 4kT(R_1 + R_2)\Delta f \tag{11.23}$$

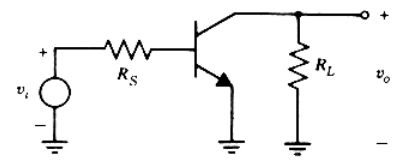
Equation 11.23 is just the value that would be predicted for thermal noise in a resistor $(R_1 + R_2)$ using (11.4), and thus the results are consistent. These results are also consistent with the representation of the noise generators by *independent* sinusoids as described earlier. It is easily shown that when two or more such generators are connected in series, the mean-square value of the total voltage is equal to the sum of the individual mean-square values.

In the above calculation, two noise voltage sources were considered connected in series. It can be similarly shown that an analogous result is true for independent noise *current* sources connected in *parallel*. The mean-square value of the combination is the sum of the individual mean-square values. This result was assumed in the modeling of Section 11.3 where, for example, three independent noise-current generators (shot, flicker, and burst) were combined into a single base-emitter noise source for a bipolar transistor.

### 11.4.1 Bipolar Transistor Noise Performance

As an example of the manipulation of noise generators in circuit calculations, consider the noise performance of the simple transistor stage with the ac schematic shown in Fig. 11.18*a*. The small-signal equivalent circuit including noise is shown in Fig. 11.18*b*. (It should be pointed out that, for noise calculations, the equivalent circuit analyzed must be the actual circuit configuration used. That is, Fig. 11.18*a* cannot be used as a half-circuit representation of a differential pair for the purposes of noise calculation because noise sources in each half of a differential pair affect the total output noise.)

In the equivalent circuit of Fig. 11.18*b*, the external input signal $v_i$ has been



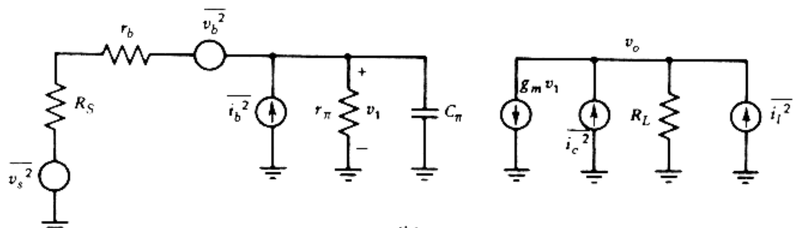




**Figure 11.18** (*a*) Simple transistor amplifier ac schematic. (*b*) Small-signal equivalent circuit with noise sources.

ignored so that output signal $v_o$ is due to noise generators only. $C_\mu$ is assumed small and is neglected. Output resistance $r_o$ is also neglected. The transistor noise generators are as described previously and in addition

$$\overline{v_s^2} = 4kTR_S\Delta f \tag{11.24}$$

$$\overline{i_l^2} = 4kT\frac{1}{R_L}\Delta f \tag{11.25}$$

The total output noise can be calculated by considering each noise source in turn and performing the calculation *as if* each noise source were a sinusoid with rms value equal to that of the noise source being considered. Consider first the noise generator $v_s$ due to $R_S$. Then

$$v_1 = \frac{Z}{Z + r_b + R_S} v_s \tag{11.26}$$

where

$$Z = r_\pi \left\| \frac{1}{j\omega C_\pi} \right. \tag{11.27}$$

The output noise voltage due to $v_s$ is

$$v_{o1} = -g_m R_L v_1 \tag{11.28}$$

Use of (11.26) in (11.28) gives

$$v_{o1} = -g_m R_L \frac{Z}{Z + r_b + R_S} v_s \tag{11.29}$$

The phase information contained in (11.29) is irrelevant because the noise signal has random phase and the only quantity of interest is the mean-square value of the output voltage produced by $v_s$. From (11.29) this is

$$\overline{v_{o1}^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2} \overline{v_s^2} \tag{11.30}$$

By similar calculations it is readily shown that the noise voltage produced at the output by $\overline{v_b^2}$ and $\overline{i_b^2}$ is

$$\overline{v_{o2}^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2} \overline{v_b^2} \tag{11.31}$$

$$\overline{v_{o3}^2} = g_m^2 R_L^2 \frac{(R_S + r_b)^2 |Z|^2}{|Z + r_b + R_S|^2} \overline{i_b^2} \tag{11.32}$$

Noise at the output due to $\overline{i_l^2}$ and $\overline{i_c^2}$ is

$$\overline{v_{o4}^2} = \overline{i_l^2} R_L^2 \tag{11.33}$$

$$\overline{v_{o5}^2} = \overline{i_c^2} R_L^2 \tag{11.34}$$

Since all five noise generators are *independent*, the total output noise is

$$\overline{v_o^2} = \sum_{n=1}^{5} \overline{v_{on}^2} \tag{11.35}$$

$$= g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2} (\overline{v_s^2} + \overline{v_b^2} + (R_S + r_b)^2 \overline{i_b^2}) + R_L^2 (\overline{i_l^2} + \overline{i_c^2}) \tag{11.36}$$

Substituting expressions for the noise generators we obtain

$$\frac{\overline{v_o^2}}{\Delta f} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2} [4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B] + R_L^2 \left(4kT \frac{1}{R_L} + 2qI_C\right) \tag{11.37}$$

where flicker noise has been assumed small and neglected. Substituting for $Z$ from

(11.27) in (11.37) we find

$$\frac{\overline{v_o^2}}{\Delta f} = g_m^2 R_L^2 \frac{r_\pi^2}{(r_\pi + R_S + r_b)^2} \frac{1}{1 + \left(\frac{f}{f_1}\right)^2} [4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B]$$
$$+ R_L^2\left(4kT\frac{1}{R_L} + 2qI_C\right) \qquad \mathbf{(11.38)}$$

where

$$f_1 = \frac{1}{2\pi[r_\pi \| (R_S + r_b)]C_\pi} \qquad \mathbf{(11.39)}$$

The output noise-voltage spectral density represented by (11.38) has a frequency-dependent part and a constant part. The frequency dependence arises because the gain of the stage begins to fall above frequency $f_1$, and noise due to generators $\overline{v_s^2}$, $\overline{v_b^2}$, and $\overline{i_b^2}$, which appears amplified in the output, also begins to fall. The constant term in ((1.38) is due to noise generators $\overline{i_l^2}$ and $\overline{i_c^2}$. Note that this noise contribution would also be frequency dependent if the effect of $C_\mu$ had not been neglected. The noise-voltage spectral density represented by (11.38) has the form shown in Fig. 11.19.

## EXAMPLE

In order to give an appreciation of the numbers involved, specific values will now be assigned to the parameters of (11.38), and the various terms in the equation will be evaluated. Assume that

$$I_C = 100\,\mu\text{A} \qquad \beta = 100 \qquad r_b = 200\,\Omega$$
$$R_S = 500\,\Omega \qquad C_\pi = 10\,\text{pF}$$
$$R_L = 5\,\text{k}\Omega$$

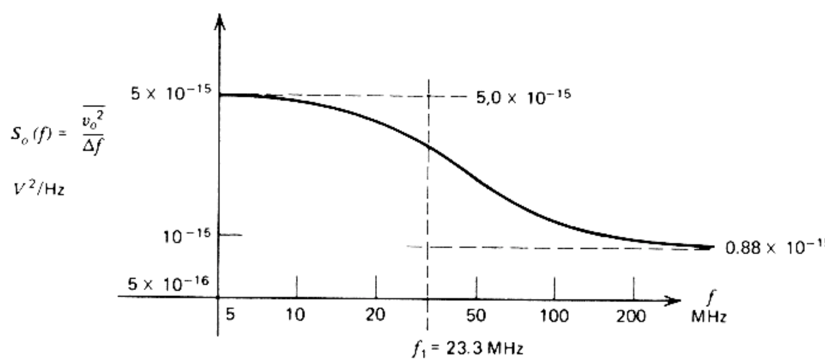



**Figure 11.19** Noise voltage spectrum at the output of the circuit of Fig. 11.18.

Substituting these values in (11.38) and using $4kT = 1.66 \times 10^{-20}$ V-C gives

$$\frac{\overline{v_o^2}}{\Delta f} = \left[ 5.82 \times 10^{-18} \frac{1}{1 + \left(\frac{f}{f_1}\right)^2} (700 + 9.4) + 1.66 \times 10^{-20} (5000 + 48{,}080) \right] \text{V}^2/\text{Hz}$$

$$= \frac{4.13 \times 10^{-15}}{1 + \left(\frac{f}{f_1}\right)^2} + 0.88 \times 10^{-15} \text{ V}^2/\text{Hz} \qquad \textbf{(11.40)}$$

Equation 11.39 gives

$$f_1 = 23.3 \text{ MHz} \qquad \textbf{(11.41)}$$

Equation 11.40 shows that the output noise-voltage spectral density is $5.0 \times 10^{-15}$ V²/Hz at low frequencies, and it approaches $0.88 \times 10^{-15}$ V²/Hz at high frequencies. The major contributor to the output noise in this case is the source resistance $R_S$, followed by the base resistance of the transistor. The noise spectrum given by (11.40) is plotted in Fig. 11.19.

## EXAMPLE

Suppose the amplifier in the above example is followed by later stages that limit the bandwidth to a sharp cutoff at 1 MHz. Since the noise spectrum as shown in Fig. 11.19 does not begin to fall significantly until $f_1 = 23.3$ MHz, the noise spectrum may be assumed constant at $5.0 \times 10^{-15}$ V²/Hz over the bandwidth 0 to 1 MHz. Thus the *total* noise voltage at the output of the circuit of Fig. 11.18*a* in a 1-MHz bandwidth is

$$\overline{v_{oT}^2} = 5.0 \times 10^{-15} \times 10^6 \text{ V}^2 = 5.0 \times 10^{-9} \text{ V}^2$$

and thus

$$v_{oT} = 71 \, \mu\text{V rms} \qquad \textbf{(11.42)}$$

Now suppose that the amplifier of Fig. 11.18*a* is *not* followed by later stages that limit the bandwidth but is fed directly to a wideband detector (this could be an oscilloscope or a voltmeter). In order to find the total output noise voltage in this case, the contribution from each frequency increment $\Delta f$ must be summed at the output. This reduces to *integration* across the bandwidth of the detector of the noise-voltage spectral-density curve of Fig. 11.19. For example, if the detector had a 0 to 50-MHz bandwidth with a sharp cutoff, then the total output noise would be

$$\overline{v_{oT}^2} = \sum_{f=0}^{50\times 10^6} S_o(f)\,\Delta f = \int_0^{50\times 10^6} S_o(f)\,df \tag{11.43}$$

where

$$S_o(f) = \frac{\overline{v_o^2}}{\Delta f} \tag{11.44}$$

is the noise spectral density defined by (11.40). In practice, the exact evaluation of such integrals is often difficult and approximate methods are often used. Note that if the integration of (11.43) is done graphically, the noise spectral density versus frequency must be plotted on *linear scales.*

### 11.4.2 Equivalent Input Noise and the Minimum Detectable Signal

In the previous section, the output noise produced by the circuit of Fig. 11.18 was calculated. The significance of the noise performance of a circuit is, however, the limitation it places on the smallest input signals the circuit can handle before the noise degrades the quality of the output signal. For this reason, the noise performance is usually expressed in terms of an *equivalent input noise signal*, which gives the same output noise as the circuit under consideration. In this way, the equivalent input noise can be compared directly with incoming signals and the effect of the noise on those signals is easily determined. For this purpose, the circuit of Fig. 11.18 can be represented as shown in Fig. 11.20, where $\overline{v_{iN}^2}$ is an input noise-voltage generator that produces the same output noise as all of the original noise generators. All other sources of noise in Fig. 11.20 are considered removed. Using the same equivalent circuit as in Fig. 11.18*b*, we obtain, for the output noise from Fig. 11.20,

$$\overline{v_o^2} = g_m^2 R_L^2 \frac{|Z|^2}{|Z + r_b + R_S|^2}\,\overline{v_{iN}^2} \tag{11.45}$$

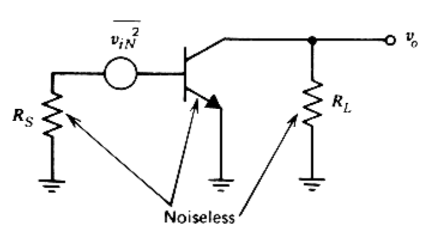



**Figure 11.20** Representation of circuit noise performance by an equivalent input noise voltage.

If this noise expression is equated to $\overline{v_o^2}$ from (11.37), the equivalent input noise voltage for the circuit can be calculated as

$$\frac{\overline{v_{iN}^2}}{\Delta f} = 4kT(R_S + r_b) + (R_S + r_b)^2 2qI_B + \frac{1}{g_m^2 R_L^2}\frac{|Z + r_b + R_S|^2}{|Z|^2} R_L^2\left(4kT\frac{1}{R_L} + 2qI_C\right) \qquad \textbf{(11.46)}$$

Note that the noise-voltage spectral density given by (11.46) *rises* at high frequencies because of the variation of $|Z|$ with frequency. This is due to the fact that as the *gain* of the device falls with frequency, output noise generators $\overline{i_c^2}$ and $\overline{i_f^2}$ have a larger effect when referred back to the input.

## EXAMPLE

Calculate the *total* input noise voltage $\overline{v_{iNT}^2}$ for the circuit of Fig. 11.18 in a bandwidth of 0 to 1 MHz.

This could be calculated using (11.46), derived above. Alternatively, since the total output noise voltage $\overline{v_{oT}^2}$ has already been calculated, this can be used to calculate $\overline{v_{iNT}^2}$ (in a 1-MHz bandwidth) by dividing by the circuit voltage gain squared. If $A_v$ is the low-frequency, small-signal voltage gain of Fig. 11.18, then

$$A_v = \frac{r_\pi}{r_b + r_\pi + R_S} g_m R_L$$

Use of the previously specified data for this circuit gives

$$A_v = \frac{26{,}000}{200 + 26{,}000 + 500}\,\frac{5000}{260} = 18.7$$

Since the noise spectrum is flat up to 1 MHz, the low-frequency gain can be used to calculate $\overline{v_{iNT}^2}$ as

$$\overline{v_{iNT}^2} = \frac{\overline{v_{oT}^2}}{A_v^2} = \frac{5 \times 10^{-9}}{18.7^2}\,\text{V}^2 = 14.3 \times 10^{-12}\,\text{V}^2$$

Thus we have

$$v_{iNT} = 3.78\,\mu\text{V rms}$$

The above example shows that in a bandwidth of 0 to 1 MHz, the noise in the circuit *appears to come* from a 3.78-$\mu$4 rms noise-voltage source in series with the input. This noise voltage can be used to estimate the smallest signal that the circuit can effectively amplify, sometimes called the *minimum detectable signal* (MDS). This depends strongly on the nature of the signal and the application. If no special

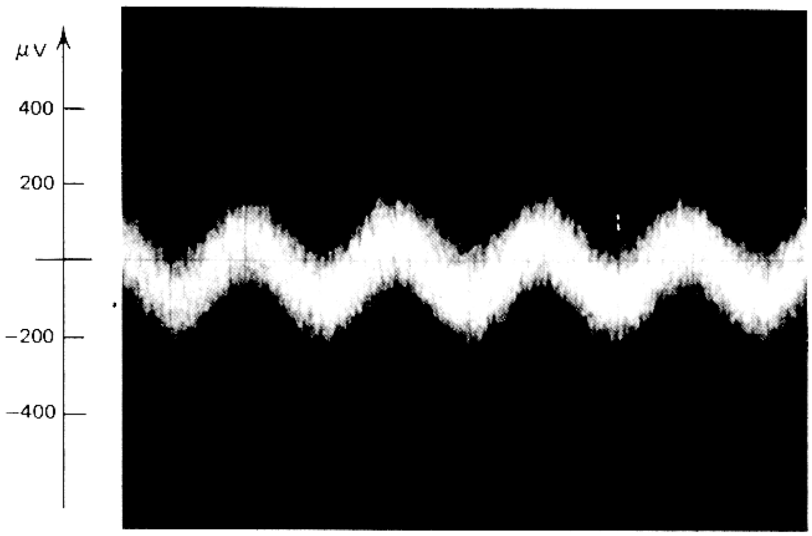



**Figure 11.21** Output voltage waveform of the circuit of Fig. 11.18 with a 3.78-$\mu$V rms sinewave applied at the input. The circuit bandwidth is limited to 1 MHz, which gives an equivalent input noise voltage of 3.78 $\mu$V rms.

filtering or coding techniques are used, the MDS can be taken as equal to the equivalent input noise voltage in the passband of the amplifier. Thus, in this case

$$\text{MDS} = 3.78\ \mu\text{V rms}$$

If a sinewave of magnitude 3.78 $\mu$V rms were applied to this circuit, and the output in a 1-MHz bandwidth examined on an oscilloscope, the sine wave would be barely detectable in the noise, as shown in Fig. 11.21. The noise waveform in this figure is typical of that produced by shot and thermal noise.