EE 155/255: Homework 1 solutions

1. (a) The current in the inductor ramps up in position *a* and ramps down in position *b*. The key is to find how much the inductor ramps up or down (ΔI) and what the average inductor current is in order to find $I_{L_{Max}}$ and $I_{L_{Min}}$.

   To find the amount the current ramps up, we need the inductor charging or discharging time. It happens in this example that charging time and discharging time are the same with a duty of 0.5, but do not get them confused in other problems and interchange them. In this case the charging time is 5 microseconds.

$$\frac{di}{dt} = \frac{V_l}{L}$$

$$\Delta i = \frac{(\text{-}Vc)Tb}{L} = \frac{\text{-}5V * 5\mu s}{100\mu H} = \text{-}0.25A = \text{-}(I_{L_{Max}} - I_{L_{Min}})$$

   and with the fact that

$$I_{avg} = \frac{I_{L_{Max}} + I_{L_{Min}}}{2}$$

   we have a system of equations that solves:

$$I_{L_{Max}} = 10.125A, I_{L_{Min}} = 9.875A$$

   (b) We use volt-seconds balance from our steady state condition and solve for duty factor. We must ensure that the rise current is equal to our fall current during T_b and T_a respectively - this is steady state; no net change in current every cycle.

$$15V * T_a + -5V * T_b = 0$$

$$\frac{T_a}{T_b} = \frac{1}{3}$$

   With an arbitrary time such as:

$$T_a + T_b = 1$$

   We can solve the system for:

$$\frac{T_a}{T_a + T_b} = 0.25 = d_a$$

   The reason volt-seconds balance works is that the inductor differential equations from above show $\frac{di}{dt} = \frac{V(t)}{L}$ and so since we are analyzing the same inductor, we drop the L and instead ensure our integral $\int V(t)dt$ remains equal to 0 per cycle, giving us 0 net change in current per cycle (steady state).

(c) We reuse ideas from (a) with a new V_s and T_a.

$$\Delta i = \frac{(Vs\text{-}Vc)Ta}{L} = \frac{15V * 2.5\mu s}{100\mu H} = 0.375A = I_{L_{Max}} - I_{L_{Min}}$$

$$I_{avg} = 50W/5V = 10A$$

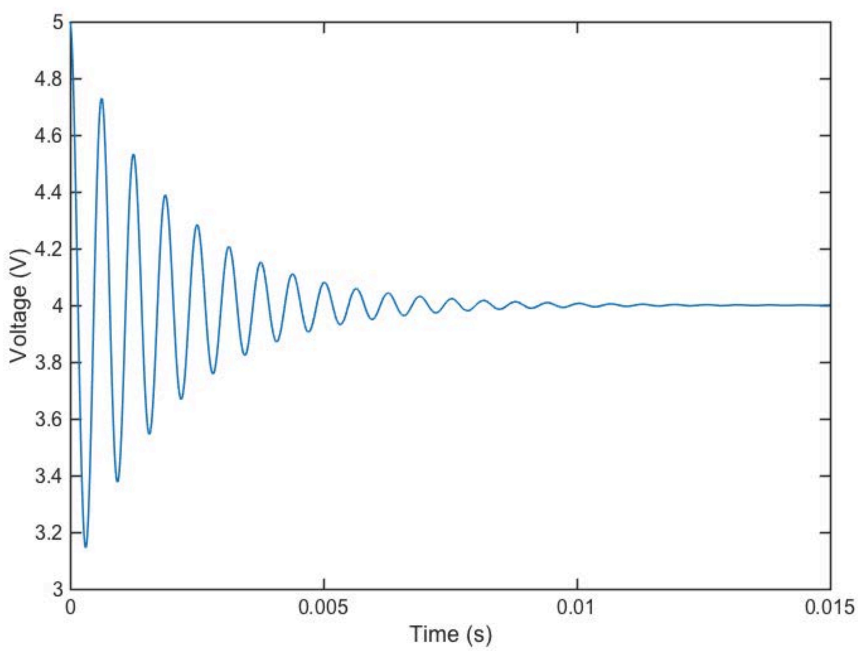
$$I_{L_{Max}} = 10.1875A, I_{L_{Min}} = 9.8125A$$

2. The transient response when the duty factor immediately drops to 40% shows a long term decrease in voltage to a 4V steady state.

   We want to see the low frequency analysis of the buck converter getting into steady state from a change in duty cycle.

We know the voltage is going to go down because our duty cycle has decreased. This means the load current will decrease and so will the average inductor current. We can use a matlab scrip to iteratively calculate the parameters during this transition period. For each loop, we calculate the change in average inductor current, the change in the capacitor voltage, and the change in the load current. **The Q Factor of the parallel RLC system is simply R*sqrt(C/L) = 10.**

We can see how the voltage drops to a steady 4V output after some oscillations:



We know the following equations from nodal analysis.

$$di_L = \frac{(Vs\text{-}Vc)t_a + (\text{-}Vc)t_b}{L} \qquad \frac{V_c}{dt} = \frac{(i_L\text{-}i_R)}{C}$$

The Matlab Code shows the difference equations used to step the simulation:

Matlab Code

```
tcy = 1/(100*1000); % Cycle time
Vs = 10; % Source Voltage
Vc = 5;  % Starting Output Voltage
duty = 0.4; % New duty Cycle
L = 100e-6; %
C = 100e-6;
i = 0.5; % Starting average inductor current
R = 10;
sim_time = 0.015;
time_steps = sim_time/tcy;
p = linspace(0,sim_time,time_steps);
x = p;

for t = 1: time_steps
    iload = Vc/R;
    di = (duty * tcy * (Vs-Vc)/L) + (-Vc/L * (1-duty) * tcy);
    i = i + di;
    dv = tcy * (i-iload)/C;
    p(t) = Vc;
    Vc = Vc + dv;
end

plot(x,p);
xlabel('Time (s)');
ylabel('Voltage (V)');
```