

# EE155/255 Green Electronics

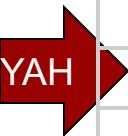
Feedback Control  
10/11/17

Prof. William Dally  
Computer Systems Laboratory  
Stanford University



# Course Logistics

- Solar Day is 10/23
- HW3 due Monday 10/16
- Lab2 signed off this week
- Lab3 out



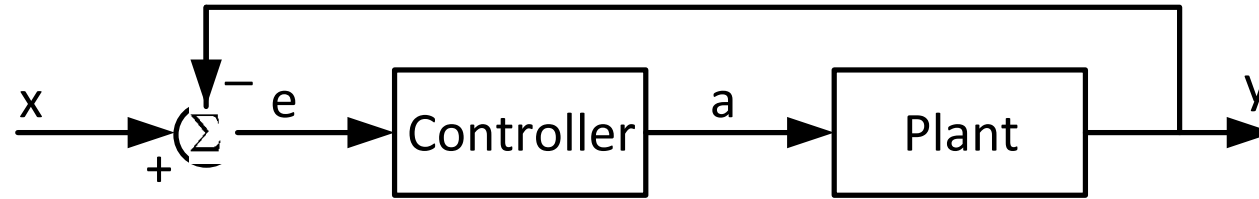
| No | Date     | Topic                              | HW out | HW in | Lab out | Lab ck | Lab                  | HW                      |
|----|----------|------------------------------------|--------|-------|---------|--------|----------------------|-------------------------|
| 1  | 9/25/17  | Intro (basic converters)           | 1      |       | 1       |        | Intro to ST32F3      | Periodic Steady State   |
| 2  | 9/27/17  | Embedded Prog/Power Elect.         |        |       |         |        |                      |                         |
| 3  | 10/2/17  | Power Electronics - 1 (switches)   | 2      | 1     | 2       | 1      | AC Energy Meter      | Power Devices           |
| 4  | 10/4/17  | Power Electronics - 2 (circuits)   |        |       |         |        |                      |                         |
| 5  | 10/9/17  | Photovoltaics                      | 3      | 2     | 3       | 2      | PV MPPT              | Motor control Matlab    |
| 6  | 10/11/17 | Feedback Control                   |        |       |         |        |                      |                         |
| 7  | 10/16/17 | Electric Motors                    | 4      | 3     | 4       | 3      | Motor control - Lab/ | Feedback                |
| 8  | 10/18/17 | Isolated Converters                |        |       |         |        |                      |                         |
| 9  | 10/23/17 | Solar Day                          | 5/PP   | 4     | 5       | 4      | PS                   | Isolated Converters     |
| 10 | 10/25/17 | Magnetics                          |        |       |         |        |                      |                         |
| 11 | 10/30/17 | Soft Switching                     | 6      | 5/PP  | 6       | 5      | Magnetics            | Magnetics and Inverters |
| 12 | 11/1/17  | Project Discussions                |        |       |         |        |                      |                         |
| 13 | 11/6/17  | Inverters, Grid, PF, and Batteries |        | 6     | P       | 6      | Project              |                         |
| 14 | 11/8/17  | Thermal & EMI                      |        |       |         |        |                      |                         |
| 15 | 11/13/17 | Quiz Review                        |        |       |         | C1     |                      |                         |
| 16 | 11/15/17 | Grounding, and Debugging           |        |       |         |        |                      |                         |
| Q  | 11/15/17 | Quiz - in the evening              |        |       |         |        |                      |                         |
|    | 11/20/17 | Thanksgiving Break                 |        |       |         | C2     |                      |                         |
|    | 11/22/17 | Thanksgiving Break                 |        |       |         |        |                      |                         |
| 17 | 11/27/17 | Guest Lecture                      |        |       |         |        |                      |                         |
| 18 | 11/29/17 | Martin Fornage - Enphase           |        |       |         | C3     |                      |                         |
| 19 | 12/4/17  | Colin Campbell - Tesla             |        |       |         |        |                      |                         |
| 20 | 12/6/17  | No Class                           |        |       |         |        |                      |                         |

# Course to Date

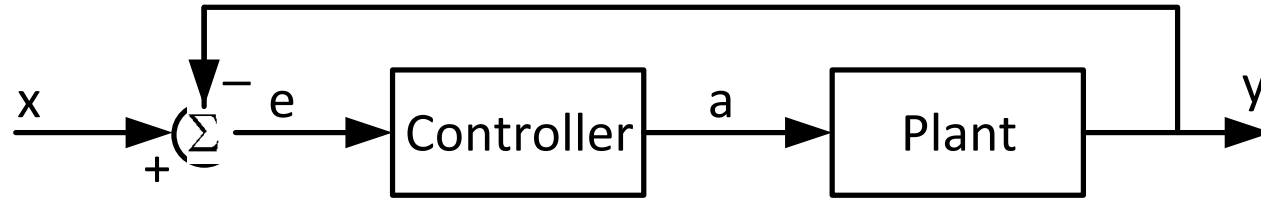
- We need sustainable energy systems
- At the core they are voltage converters
- Periodic steady-state analysis, buck and boost
- Intelligent control (embedded uC) + power path
- Real devices have switching and conduction loss
- Half-bridges, dead-time, and snubbers
- PV cells characterized by a diode-like I-V curve (MPPT)
- PV systems – single-phase store energy

# Feedback Control

# System with Feedback



# System with Feedback



$x$  – input (desired output)

$y$  – output

$e$  – error ( $x-y$ )

$a$  – actuation – controller output

# Example, driving a car

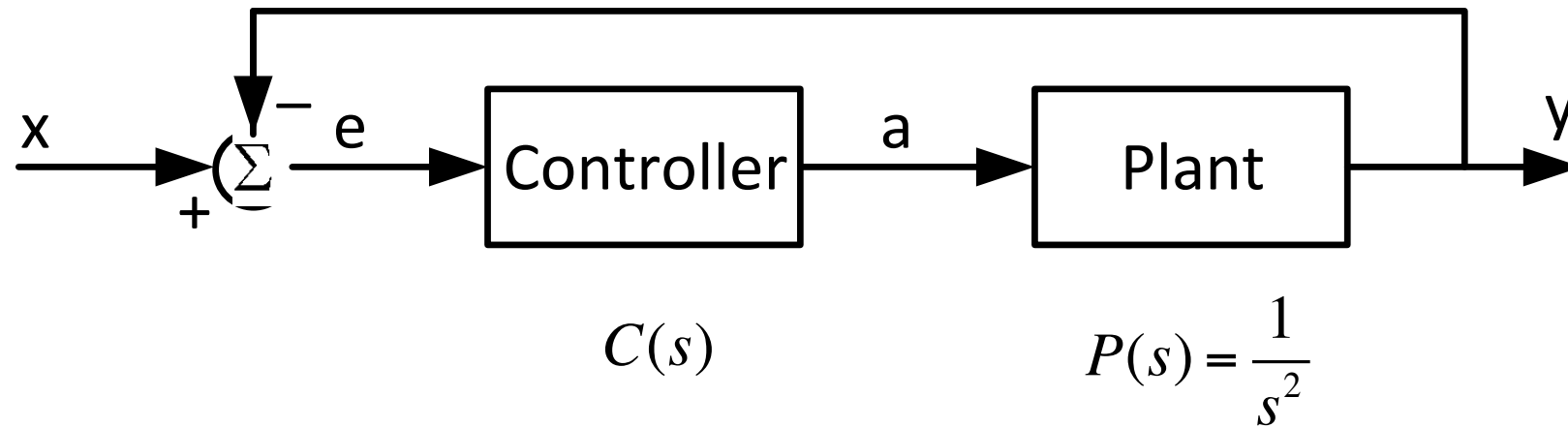
- $x$  – position of car in lane
- $\theta$  – angle of car on road
- $\phi$  – angle of wheels

- $d\theta/dt = \phi$
- $dx/dt = \theta$

$$x(s) = \frac{a\phi(s)}{s^2}$$

- How do you stay on the road?
  - Need a control law

# Feedback Control



$$Y(s) = (X(s) - Y(s))C(s)P(s)$$

$$Y(s) = X(s) \left( \frac{C(s)P(s)}{1 + C(s)P(s)} \right)$$

# Consider Proportional Control

$$C(s) = p$$

$$H(s) = C(s)P(s) = \frac{p}{s^2}$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)} = \frac{p}{s^2 + p}$$

# Three Ways to See this is Unstable

$$C(s) = p$$

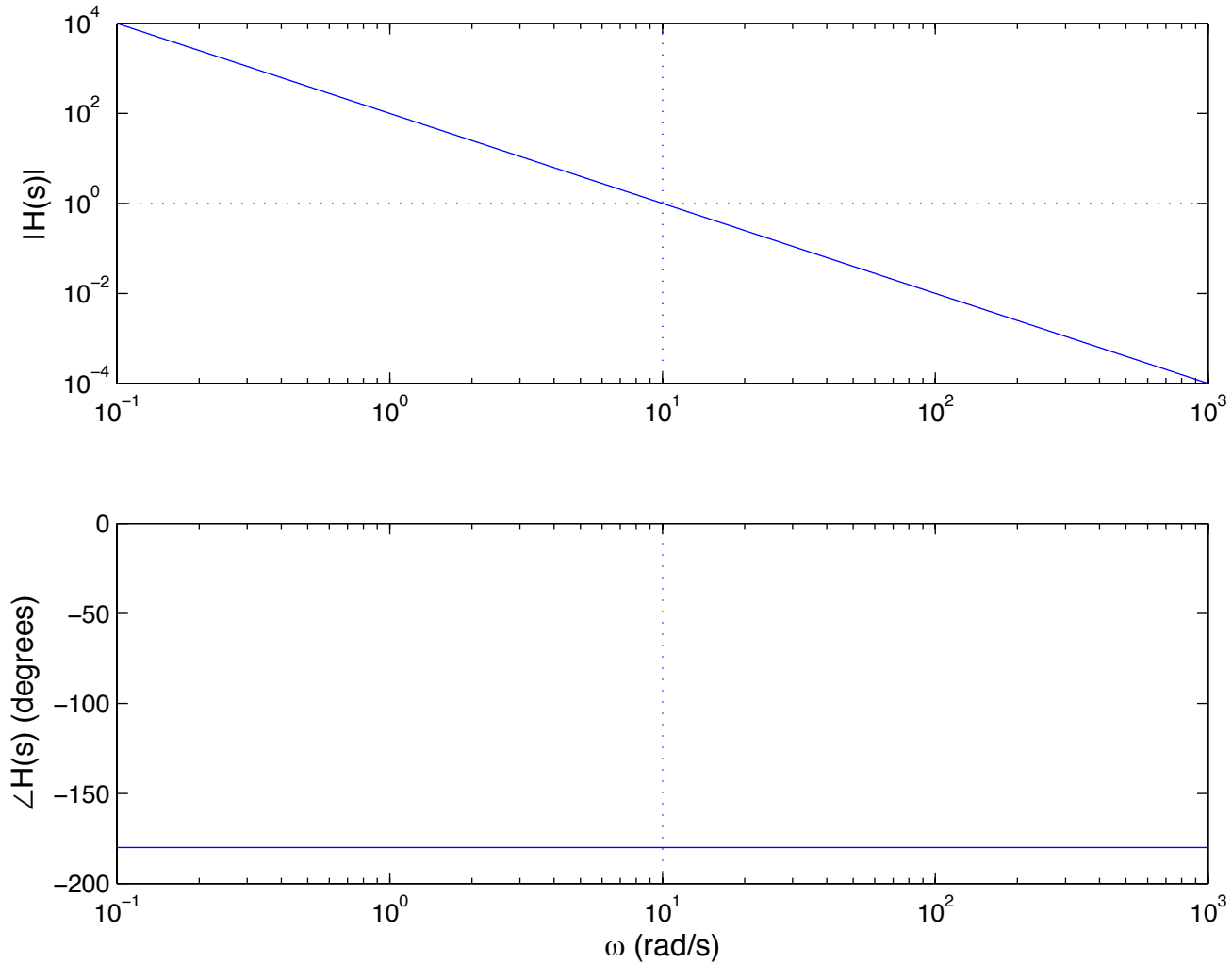
$$H(s) = C(s)P(s) = \frac{p}{s^2}$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)} = \frac{p}{s^2 + p}$$

1. Unity-gain phase of open loop response  $H(s)$
2. Damping factor of closed loop response
3. Simulation

# Open Loop Response

$$H(s) = C(s)P(s) = \frac{p}{s^2}$$



# Damping Factor

$$G(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)} = \frac{p}{s^2 + p} = \frac{k}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\zeta = 0,$$

$$\omega_0 = \sqrt{p}$$

# Damping Factor

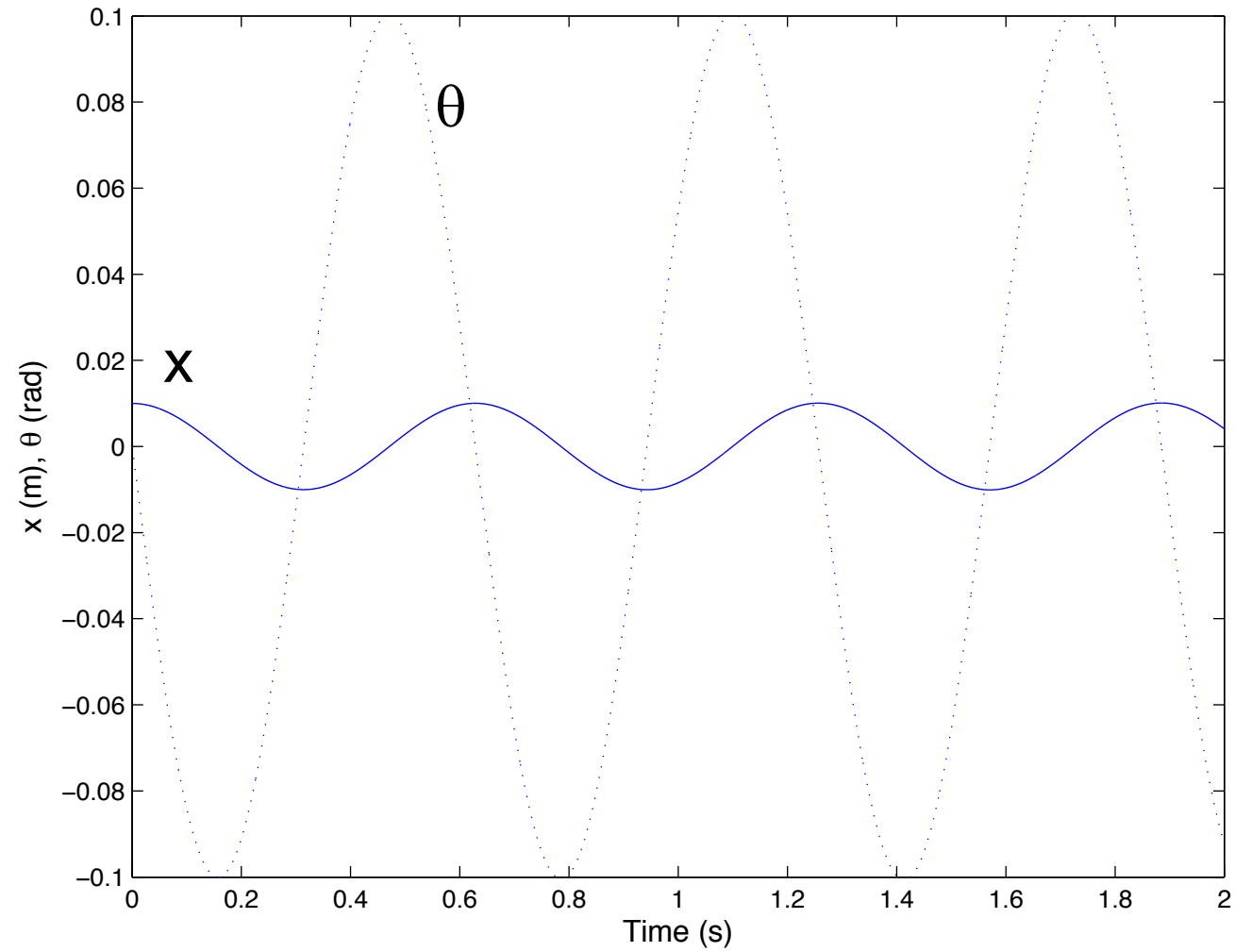
$$G(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)} = \frac{p}{s^2 + p} = \frac{k}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$\zeta = 0,$$

$$\omega_0 = \sqrt{p}$$

Frequency scales with sqrt of proportional gain

# Simulation



# PD Control

Proportional feedback oscillates

Add derivative term to “look ahead”  $T_d$

$$C(s) = p + rs$$

$$H(s) = C(s)P(s) = \frac{p + rs}{s^2}$$

$$G(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)} = \frac{p + rs}{s^2 + rs + p}$$

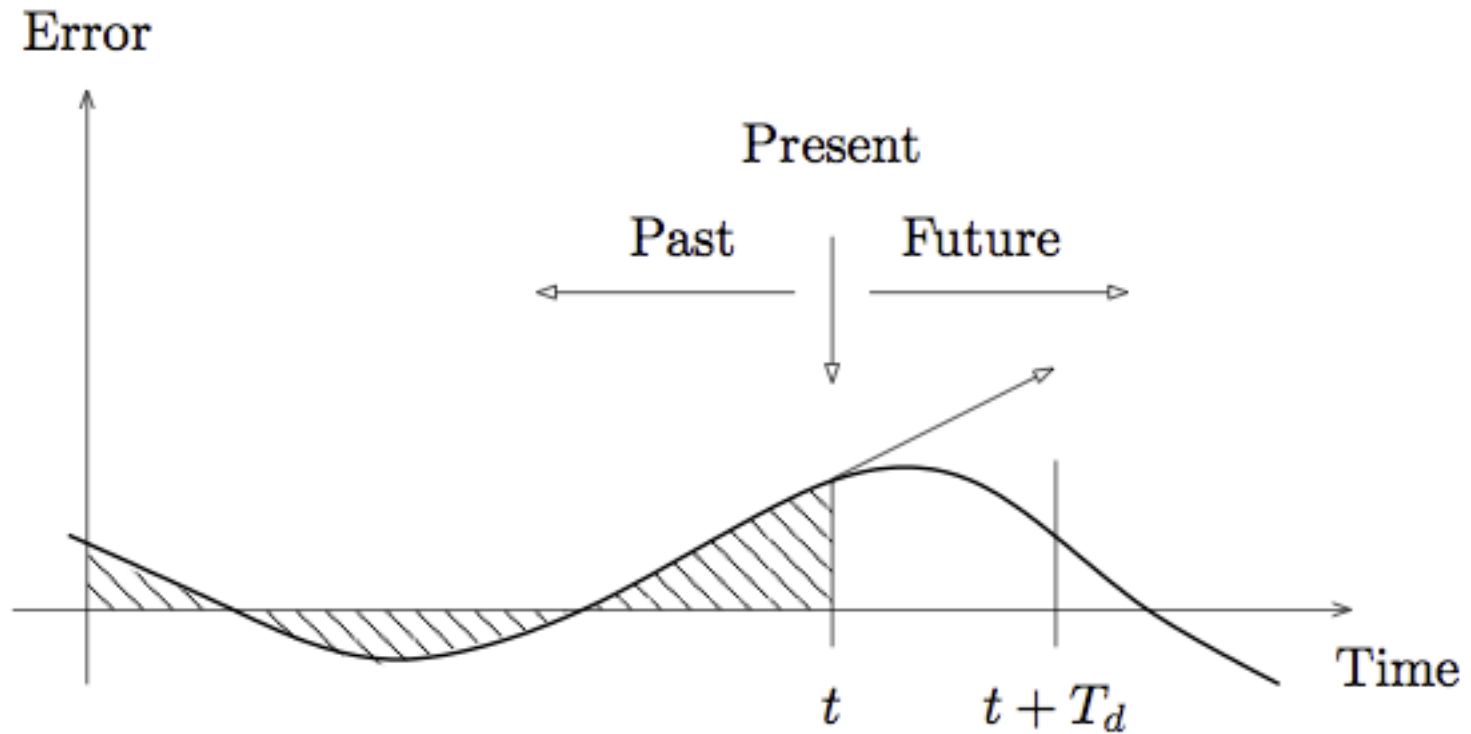
$$\omega_0 = \sqrt{p}$$

$$\xi = \frac{r}{2\omega_0} = \frac{r}{2\sqrt{p}}$$

With  $p=100$ , pick  $r=20$

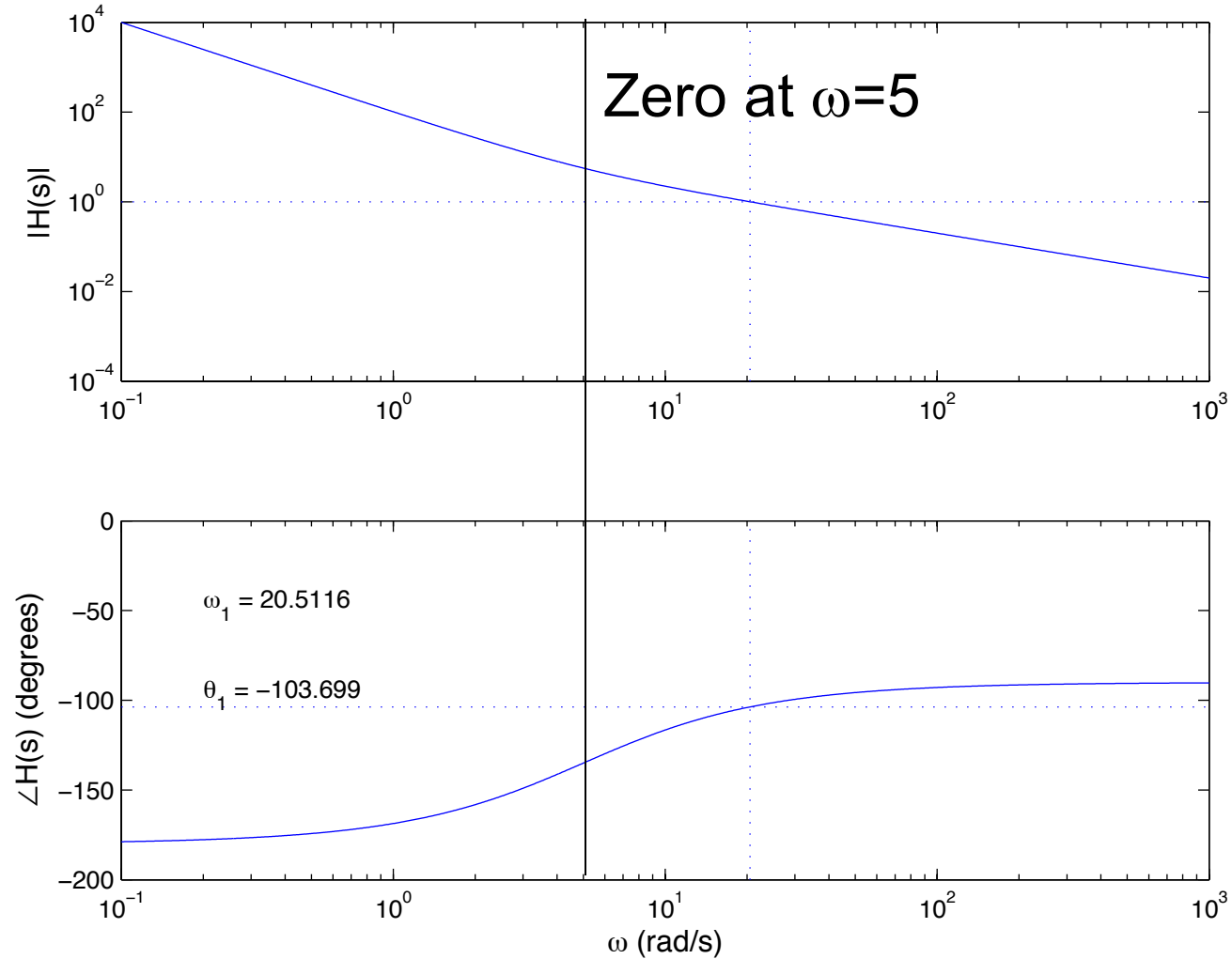
Zero at  $s = p/r$

# PD is Predictive



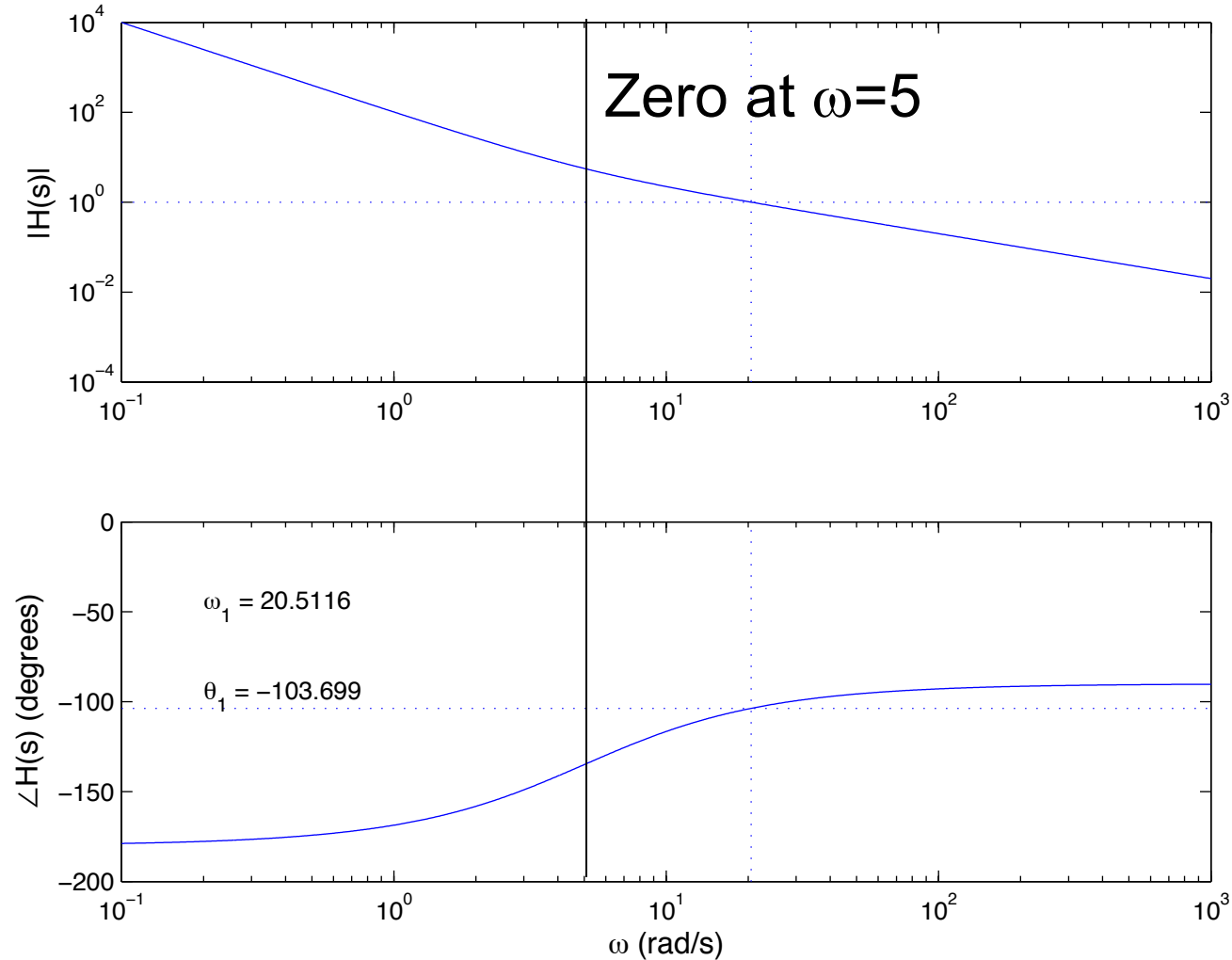
# PD Open Loop

$$H(s) = C(s)P(s) = \frac{p + rs}{s^2}$$

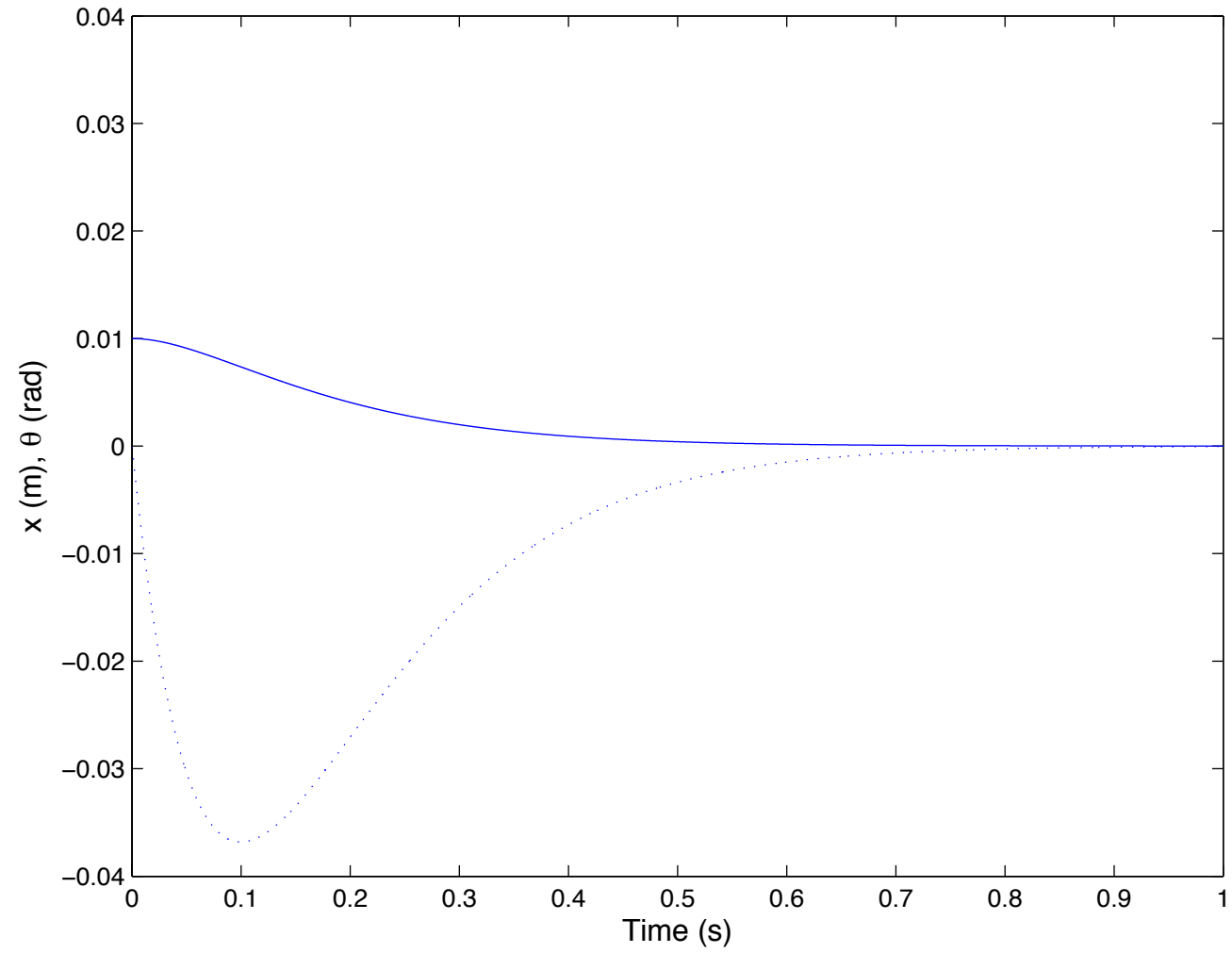


# What happens if zero is moved left or right?

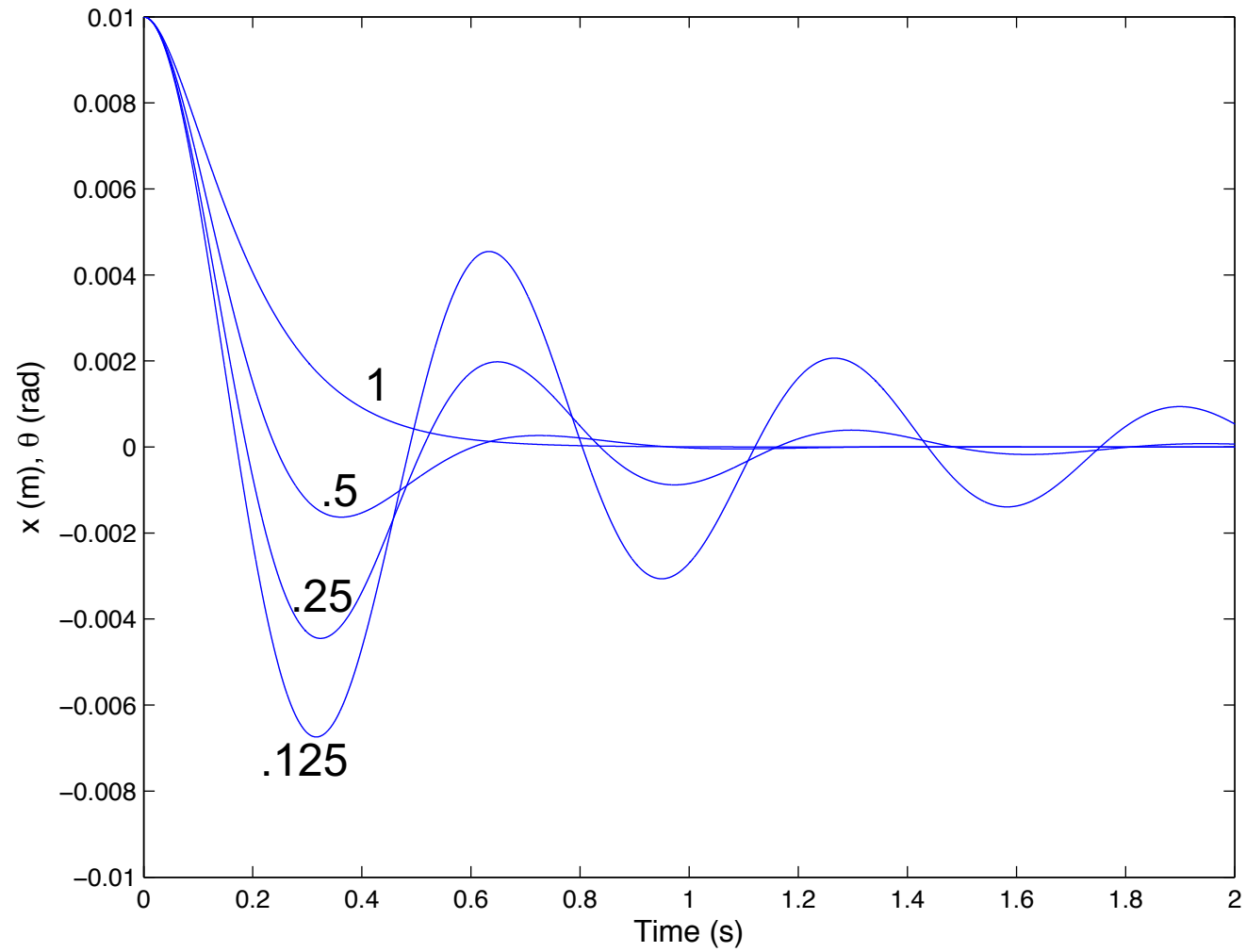
$$H(s) = C(s)P(s) = \frac{p + rs}{s^2}$$



# PD Simulation



# Effect of damping factor



# Bode Plot Intuition

$$\omega_0 = \sqrt{p}$$

$$\xi = \frac{r}{2\omega_0} = \frac{r}{2\sqrt{p}}$$

$$\omega_z = \frac{p}{r} = \frac{\sqrt{p}}{2} \text{ if } \xi = 1$$

Critically damped when zero is at half unity-gain frequency

# What about integral control

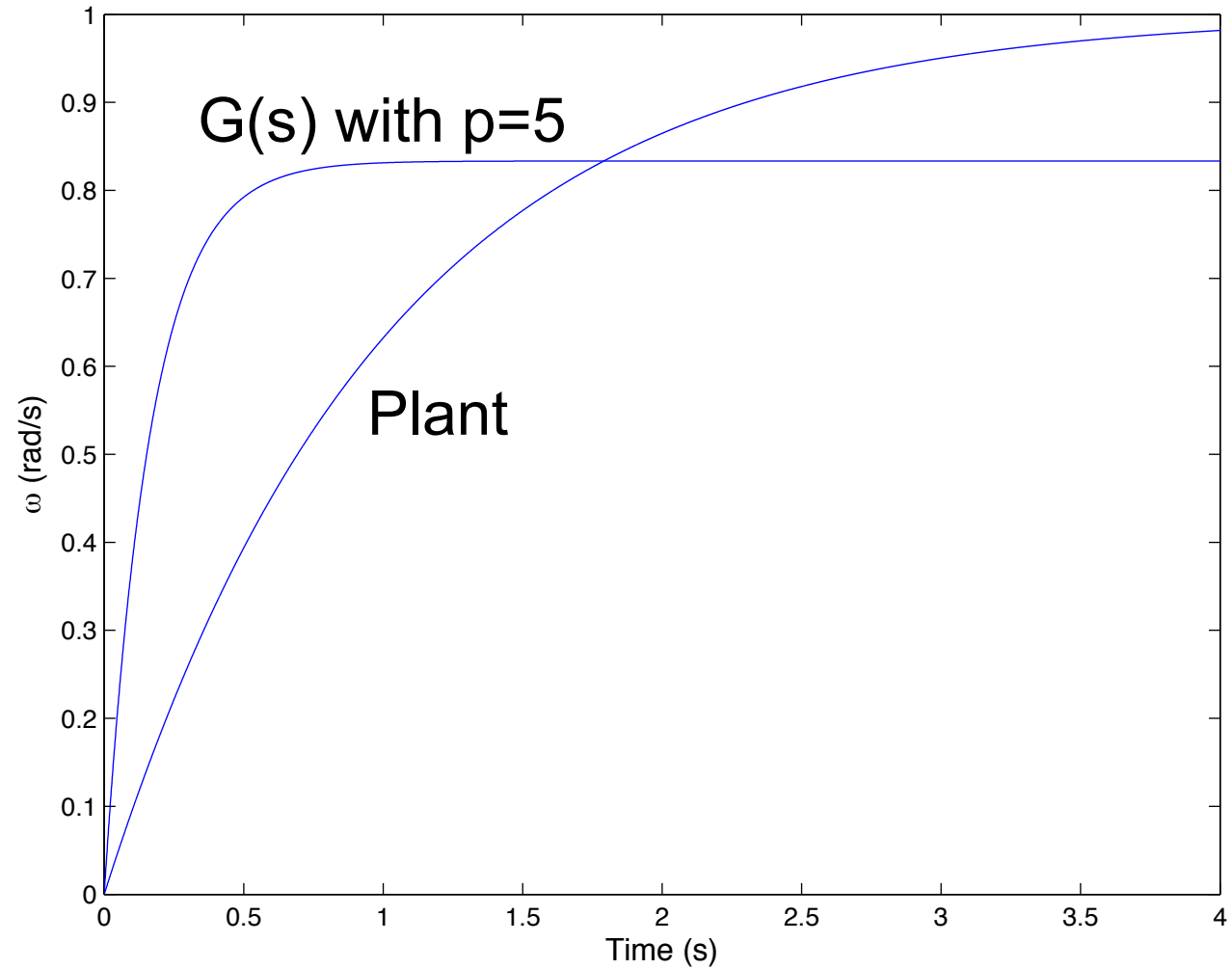
# Consider Motor with $\tau_m = 1\text{s}$

$$P(s) = \frac{1}{s+1}$$

Proportional control leaves residual error  
Derivative feedback doesn't help

$$G(s) = \frac{p}{s + (p + 1)}$$

$$G(s) = \frac{p}{s + (p + 1)}$$



# Integral Term Eliminates Residual Error But Beware “wind up”

$$P(s) = \frac{1}{s + a}$$

$$C(s) = p + \frac{q}{s}$$

$$H(s) = P(s)C(s) = \frac{p + \frac{q}{s}}{s + a} = \frac{ps + q}{s^2 + as}$$

$$G(s) = \frac{ps + q}{s^2 + (a + p)s + q}$$

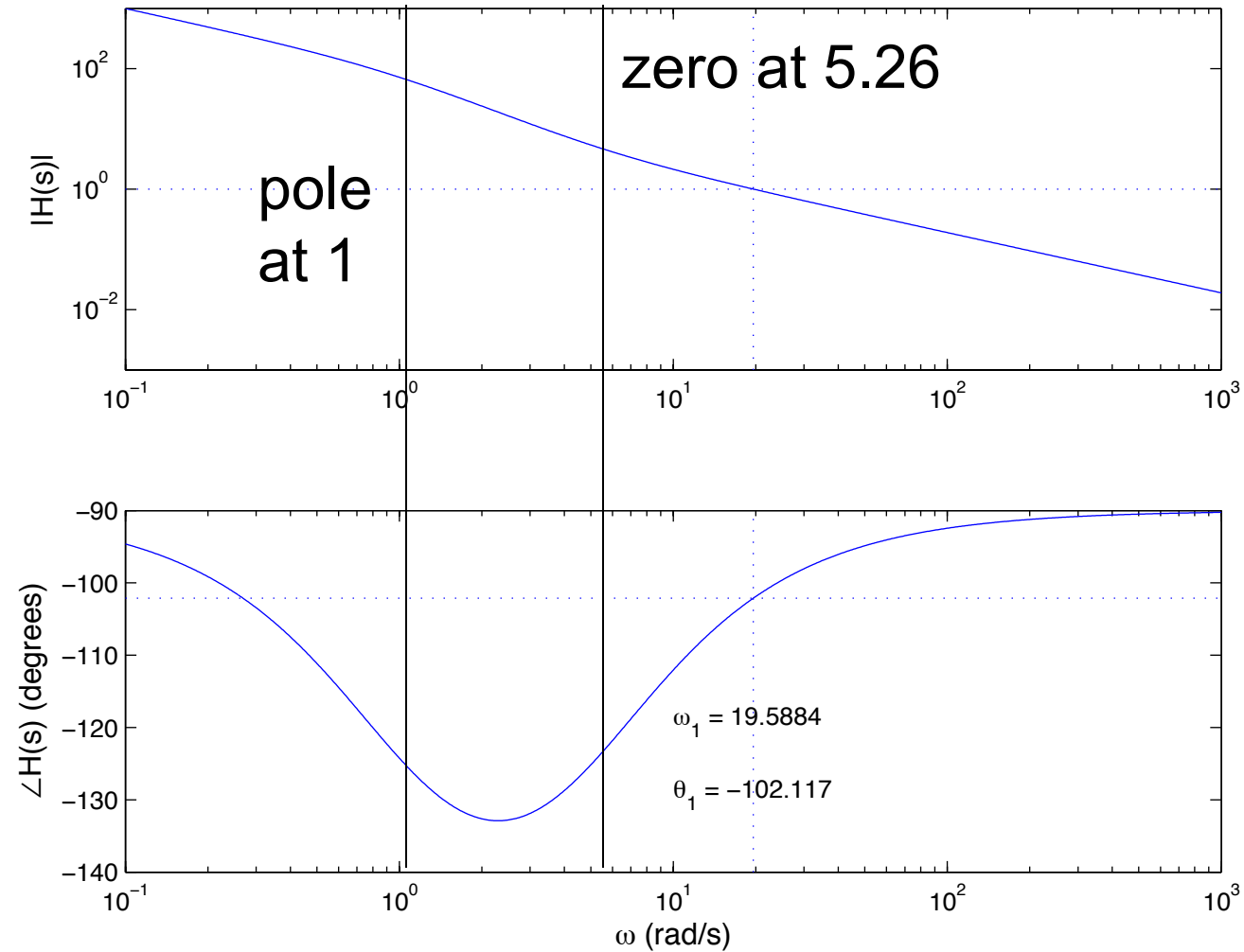
$$\omega_0 = \sqrt{q}$$

$$\xi = \frac{a + p}{2\sqrt{q}}$$

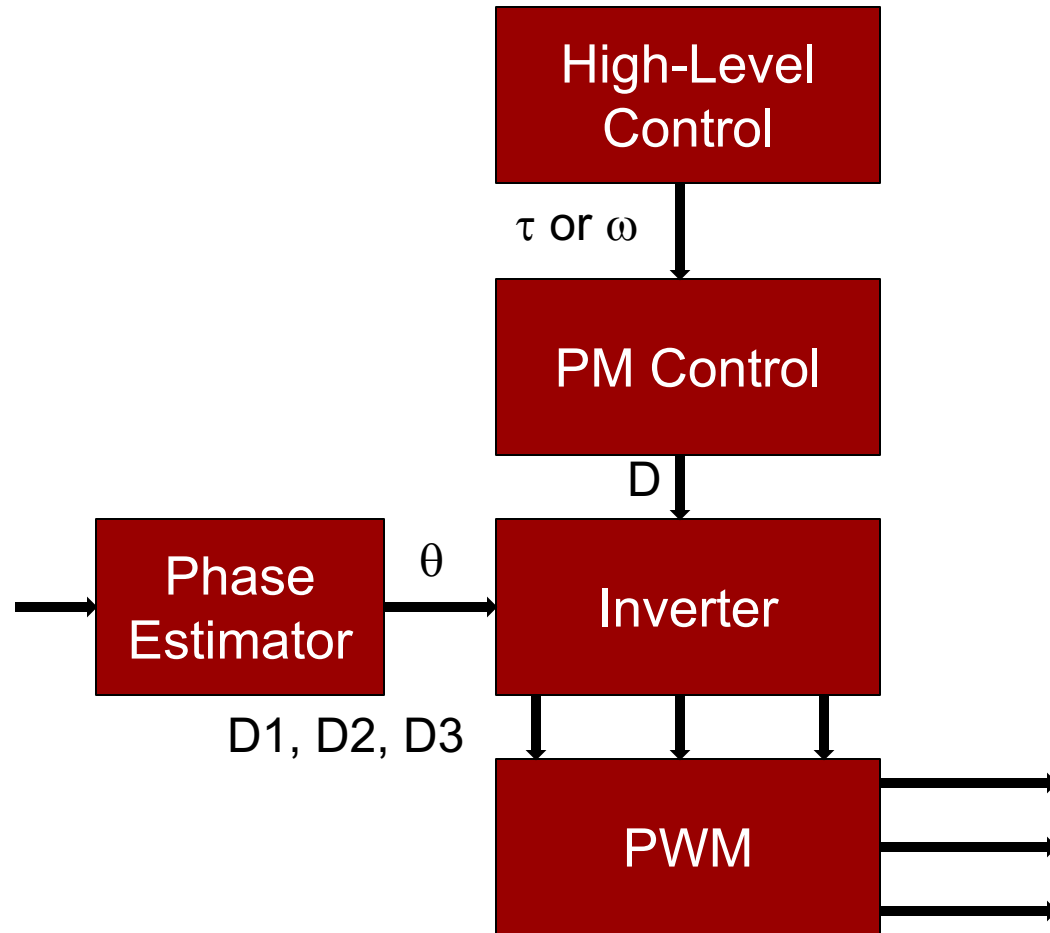
With  $a=1$ , pick  $q=9$   $p=5$

You can also eliminate residual error with  
feed forward control

# Bode Plot of Integral Control $q=100$ $p=19$

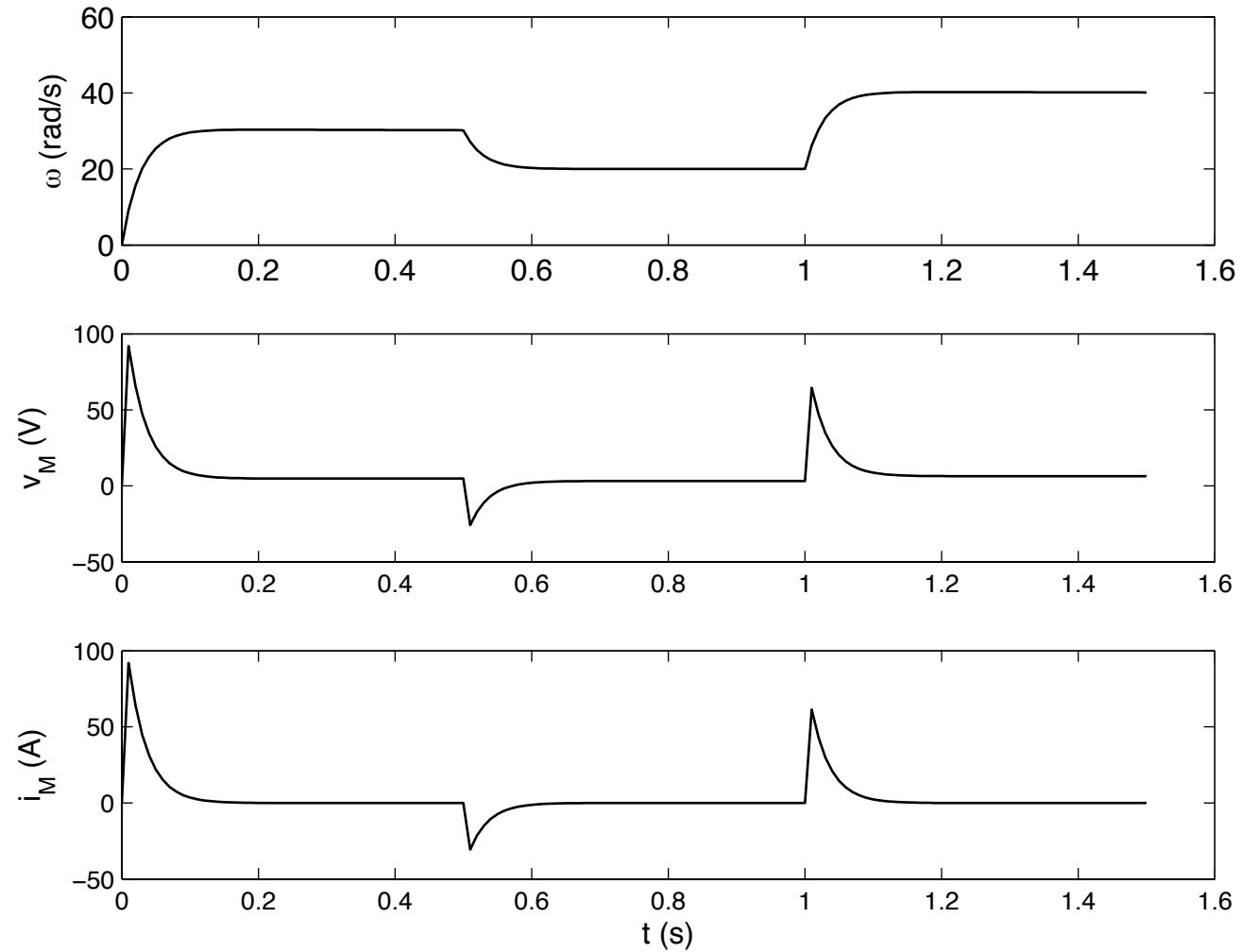


# Layered Control

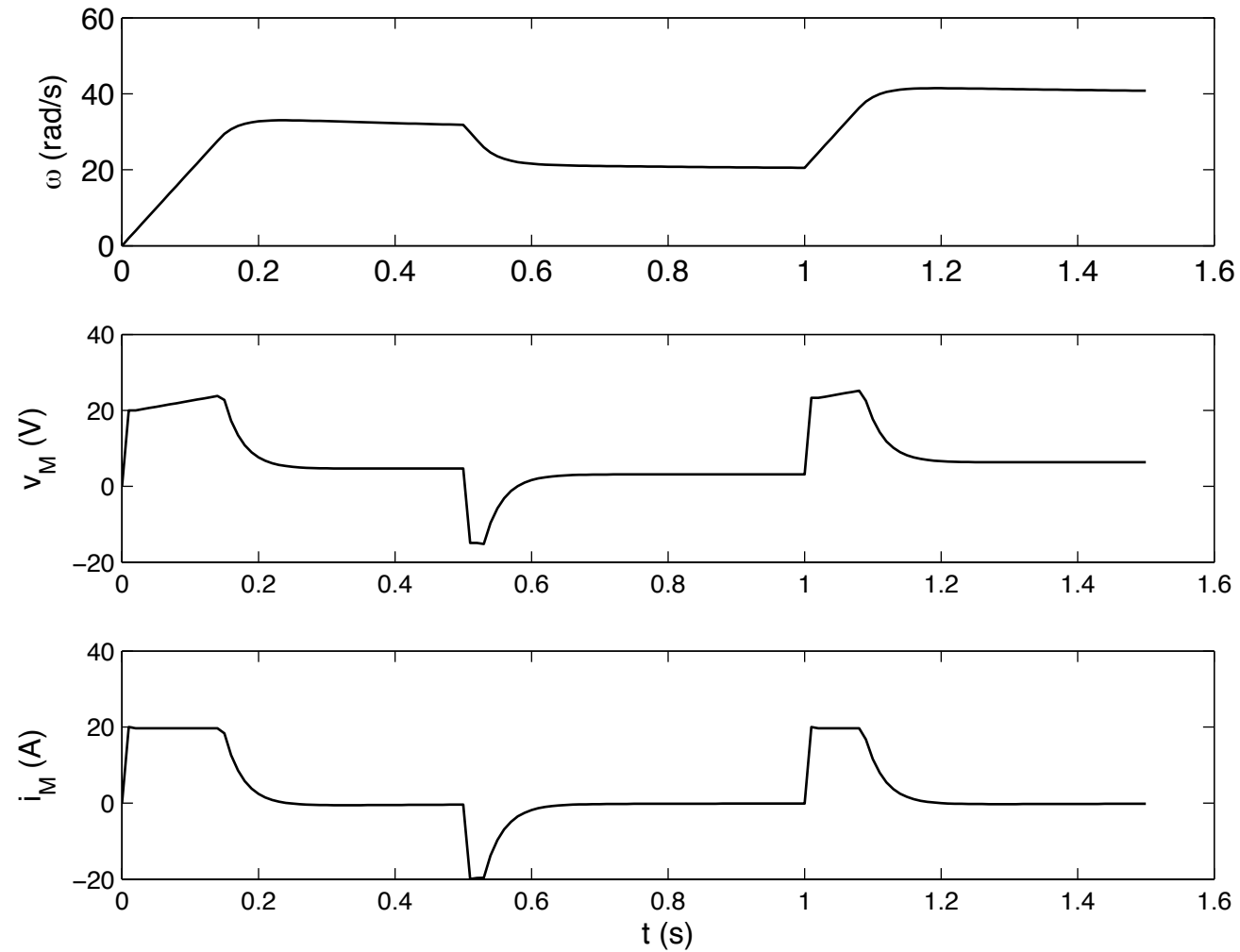


# Adding Current Limit

# Motor Control w/o Limit



# Motor Control with Current Limit at 20A



# Avoiding Integral “Wind Up”

```
err = omega_d - omega ;  
ierr = ierr + err*dt ;  
v_in_1 = err*p + ierr*q ;  
  
v_in_max = i_max*r_m + v_e ;  
v_in_min = -i_max*r_m + v_e ;  
v_in = min(v_in_max, max(v_in_min, v_in_1)) ;  
if(v_in ~= v_in_1)  
    ierr = v_in/q ;  
end
```

# Avoiding Integral “Wind Up”

```
err = omega_d - omega ;  
ierr = ierr + err*dt ;  
v_in_1 = err*p + ierr*q ;  
  
v_in_max = i_max*r_m + v_e ;  
v_in_min = -i_max*r_m + v_e ;  
v_in = min(v_in_max, max(v_in_min, v_in_1)) ;  
if(v_in ~= v_in_1)  
    ierr = v_in/q ;  
end
```

Compute voltage limits to stay within current limit

# Avoiding Integral “Wind Up”

```
err = omega_d - omega ;  
ierr = ierr + err*dt ;  
v_in_1 = err*p + ierr*q ;  
  
v_in_max = i_max*r_m + v_e ;  
v_in_min = -i_max*r_m + v_e ;  
v_in = min(v_in_max, max(v_in_min, v_in_1)) ;  
if(v_in ~= v_in_1)  
    ierr = v_in/q ;  
end
```

Clamp applied voltage (PWM)  
to computed limits

# Avoiding Integral “Wind Up”

```
err = omega_d - omega ;  
ierr = ierr + err*dt ;  
v_in_1 = err*p + ierr*q ;  
  
v_in_max = i_max*r_m + v_e ;  
v_in_min = -i_max*r_m + v_e ;  
v_in = min(v_in_max, max(v_in_min, v_in_1)) ;  
if(v_in ~= v_in_1)  
    ierr = v_in/q ;  
end
```

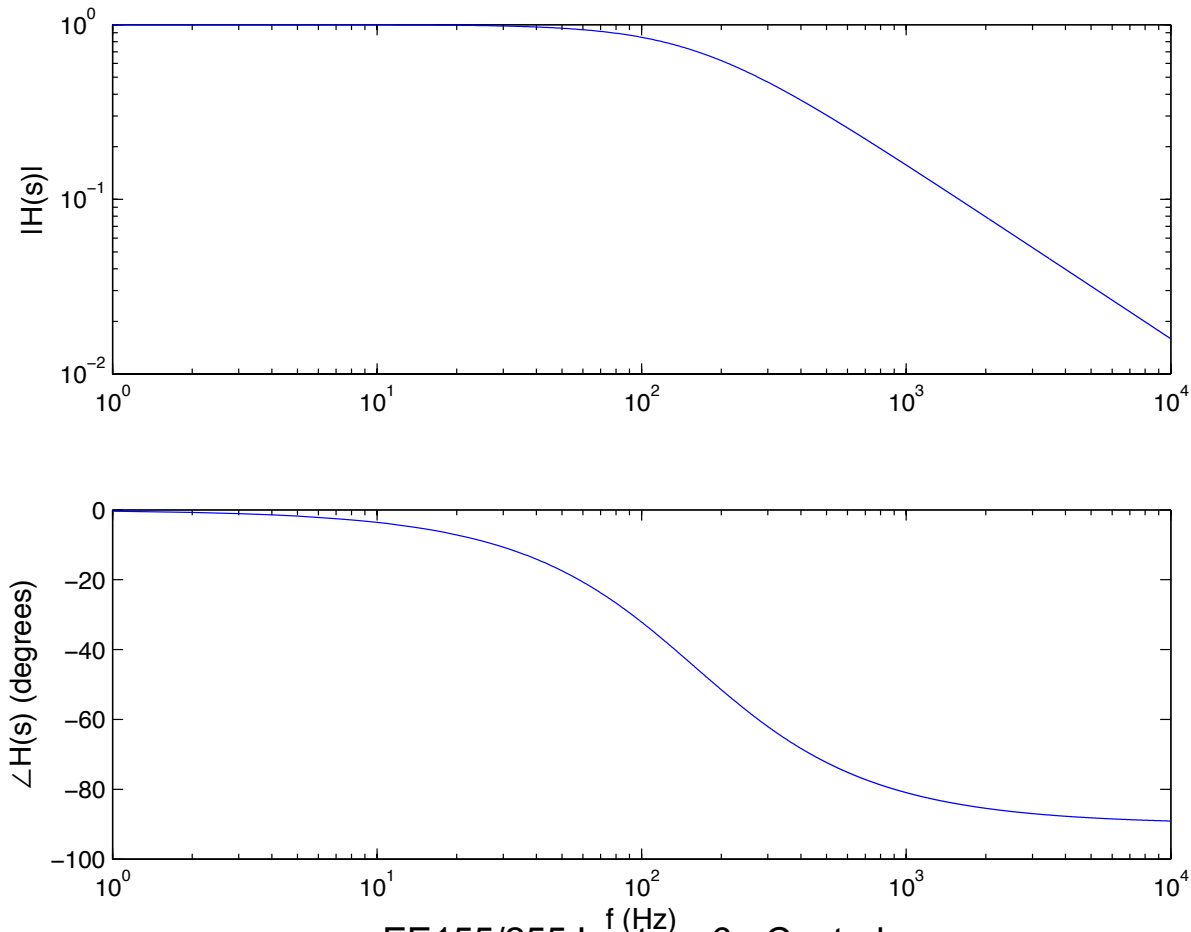
If clamped, force integral error to value that would produce clamped voltage

# A Primer on Bode Plots

# Low-Pass Frequency Response

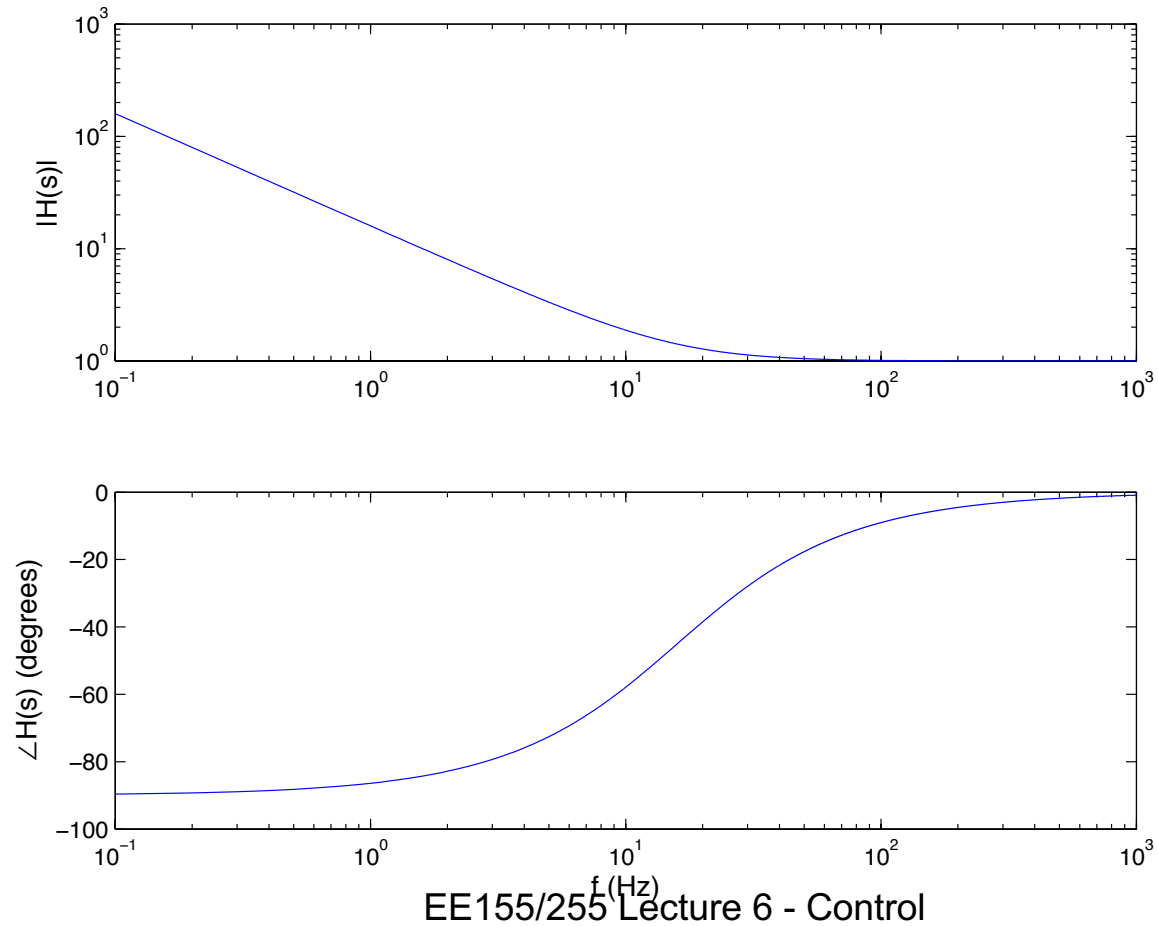
$$R = 1\text{k}\Omega, C = 1\mu\text{F}$$

$$\text{pole at } \omega = 1/RC$$



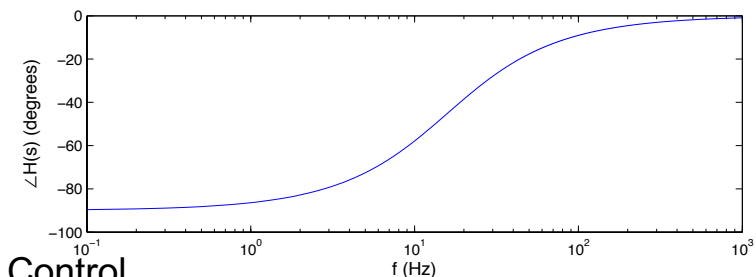
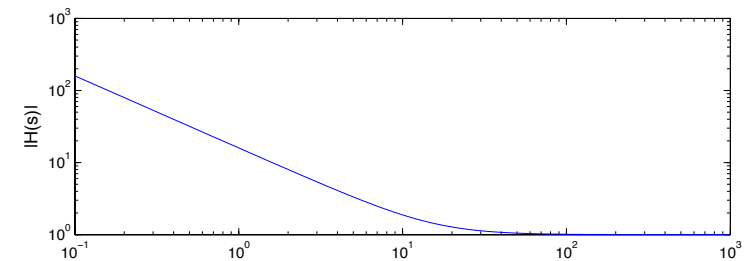
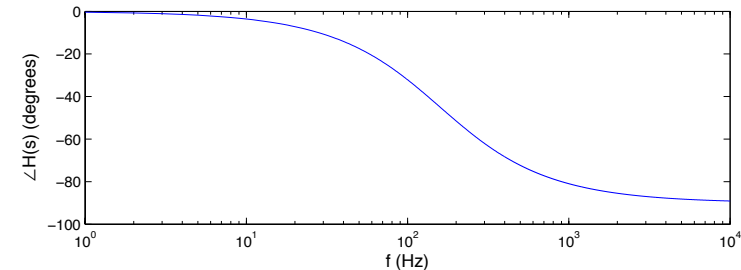
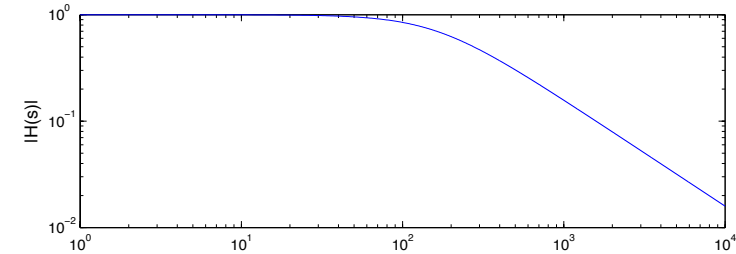
# PI Frequency Response

$p = 1, q = 100$   
pole at  $-\infty$ , zero at  $\omega = q/p$

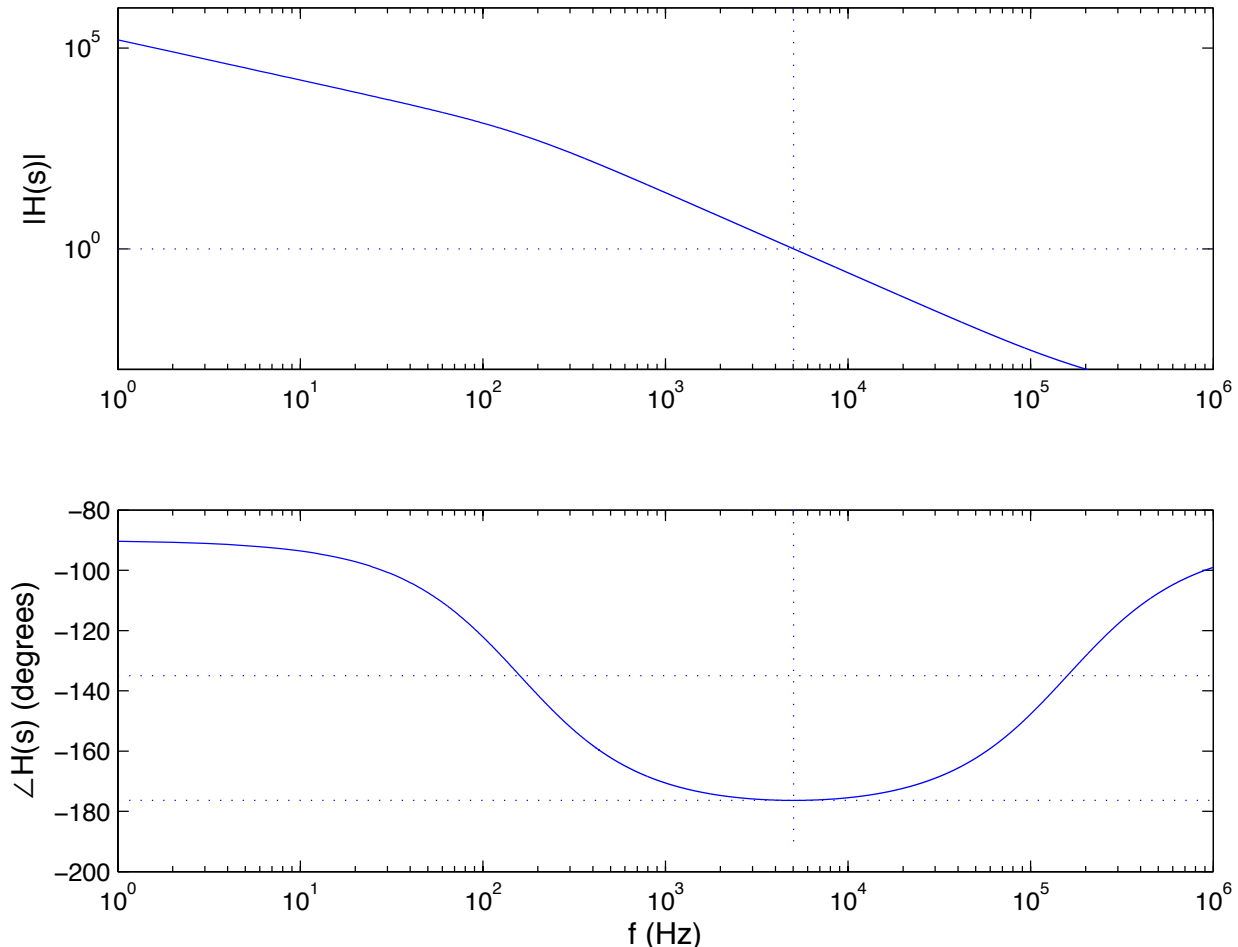


# Poles and Zeros

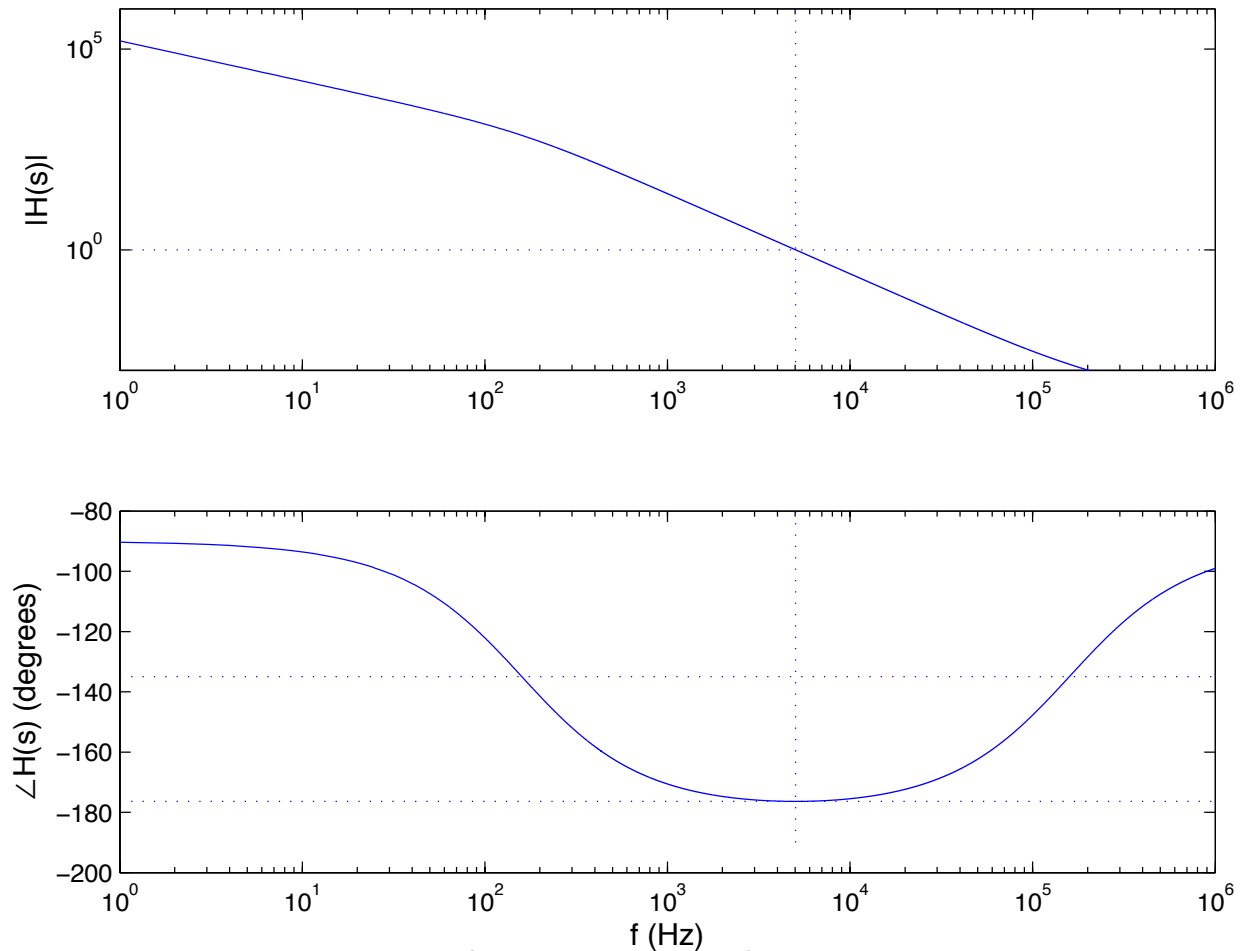
- Pole –
  - Magnitude slope decreases by 20dB/decade
  - Phase shifts negative by 90 degrees
- Zero –
  - Magnitude slope increases by 20dB/decade
  - Phase shifts positive by 90 degrees



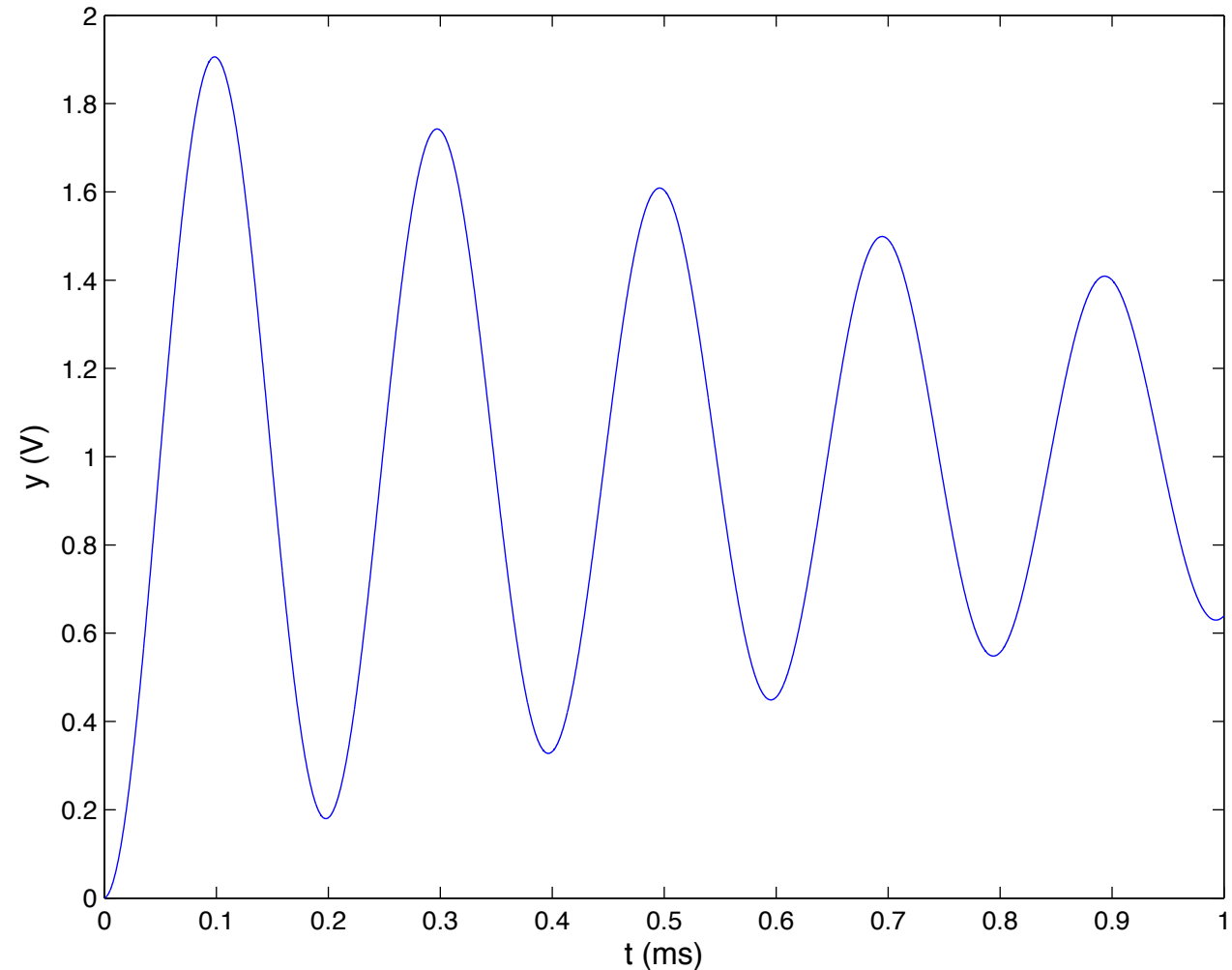
Combined Frequency Response  
poles at  $-\infty$  and  $1/RC$ , zero at  $q/p$   
Here  $R=1\text{k}\Omega$ ,  $C=1\mu\text{F}$ ,  $p=1$   $q=1\text{E}6$



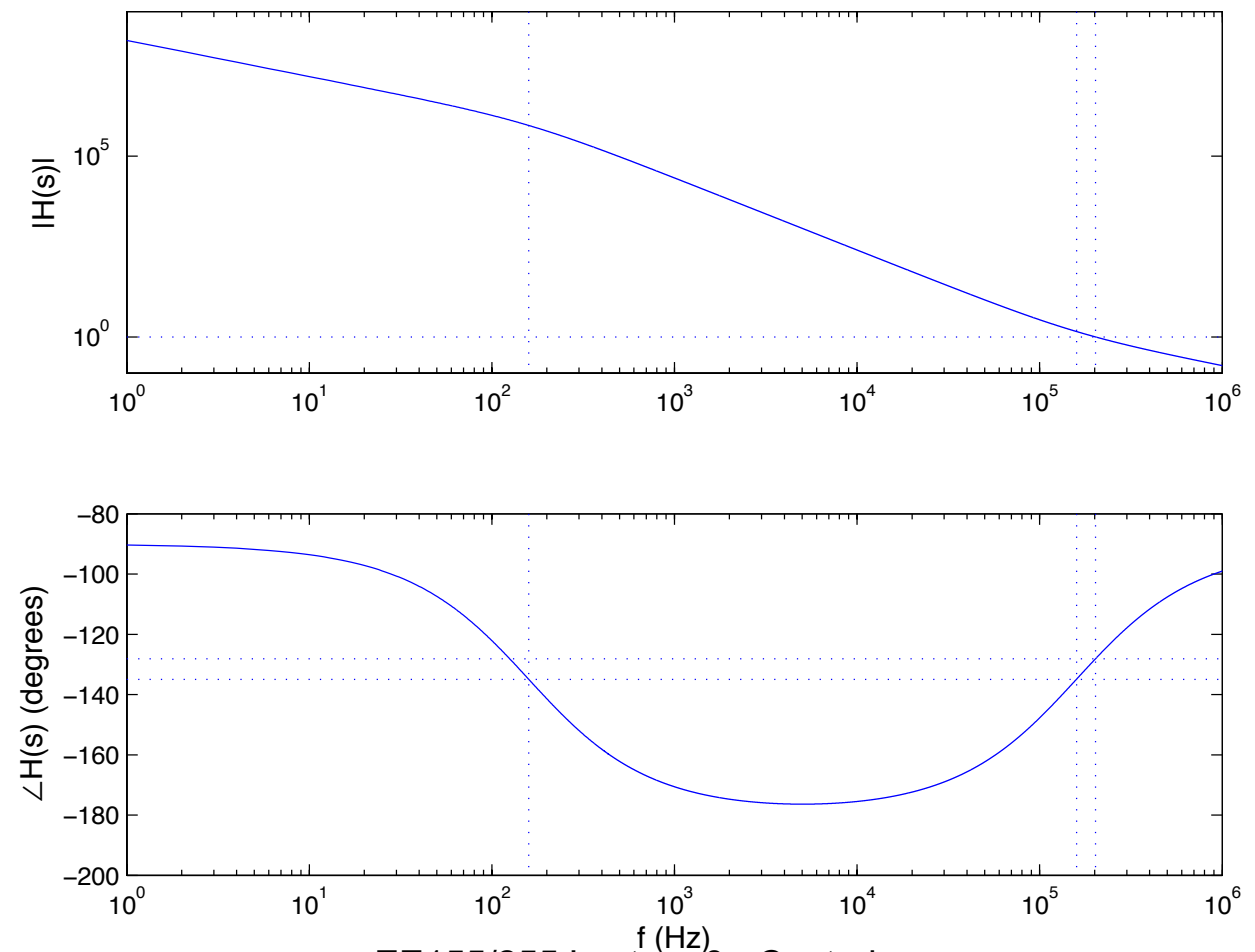
# This is unstable



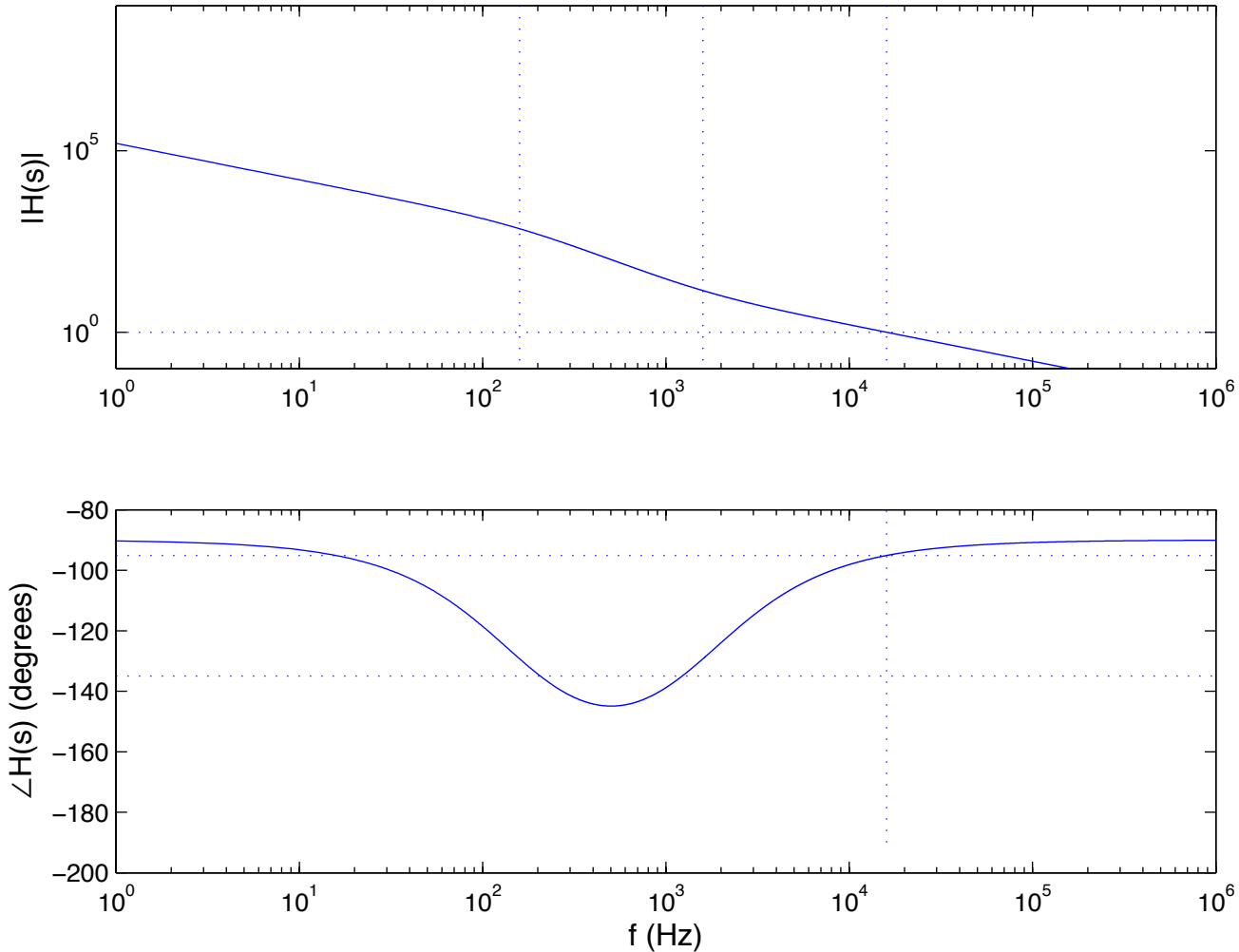
# Easier to see with the loop closed in the time domain



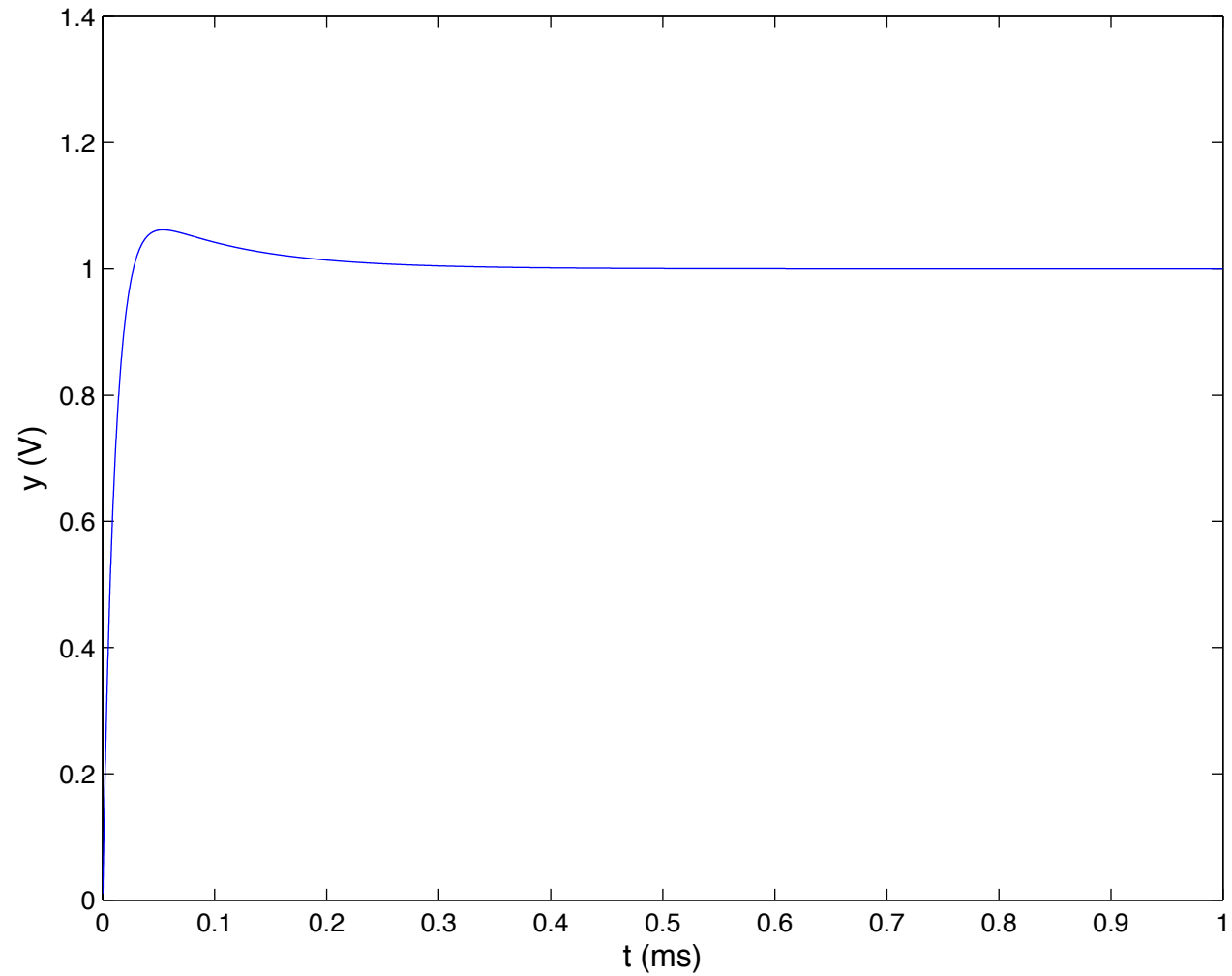
Move unity-gain frequency up  
without changing pole/zero positions  
by increasing p and q by 1000



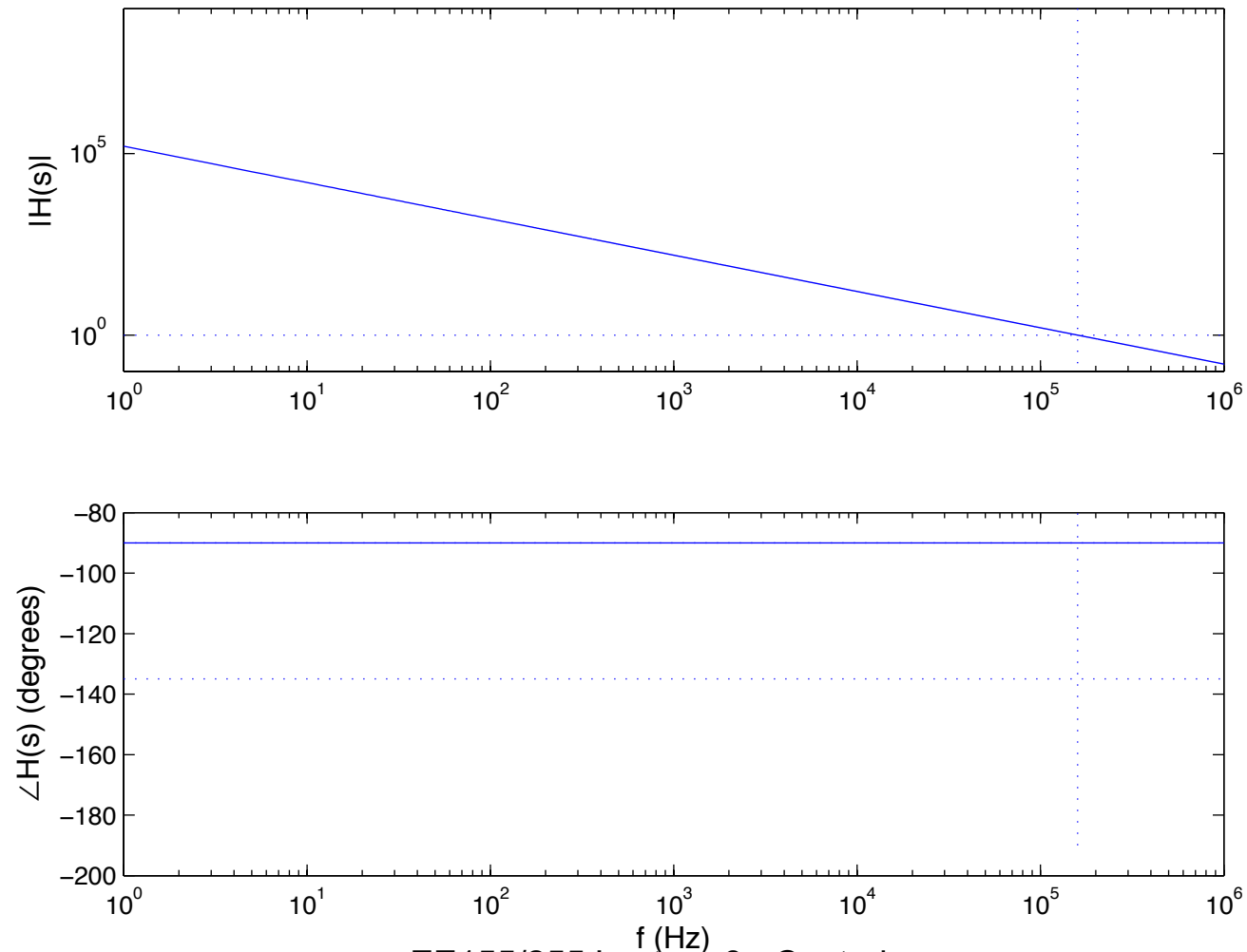
# Alternatively, move zero down by increasing $p$ by 100



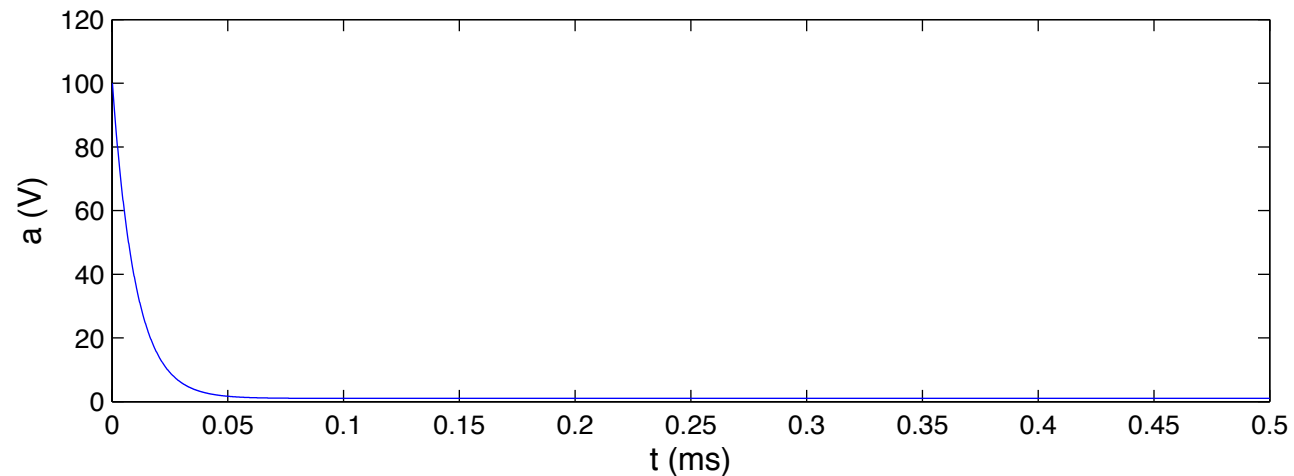
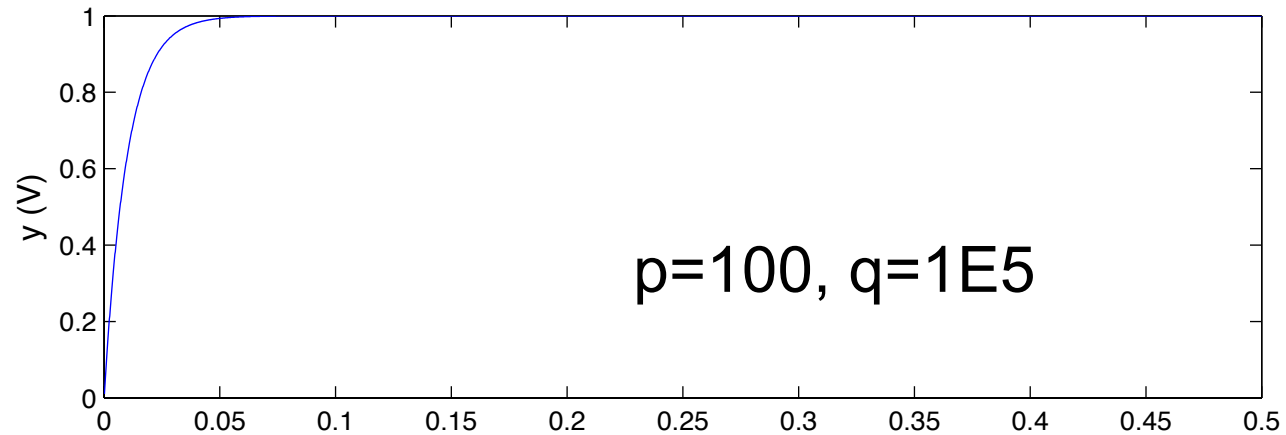
# In the time domain



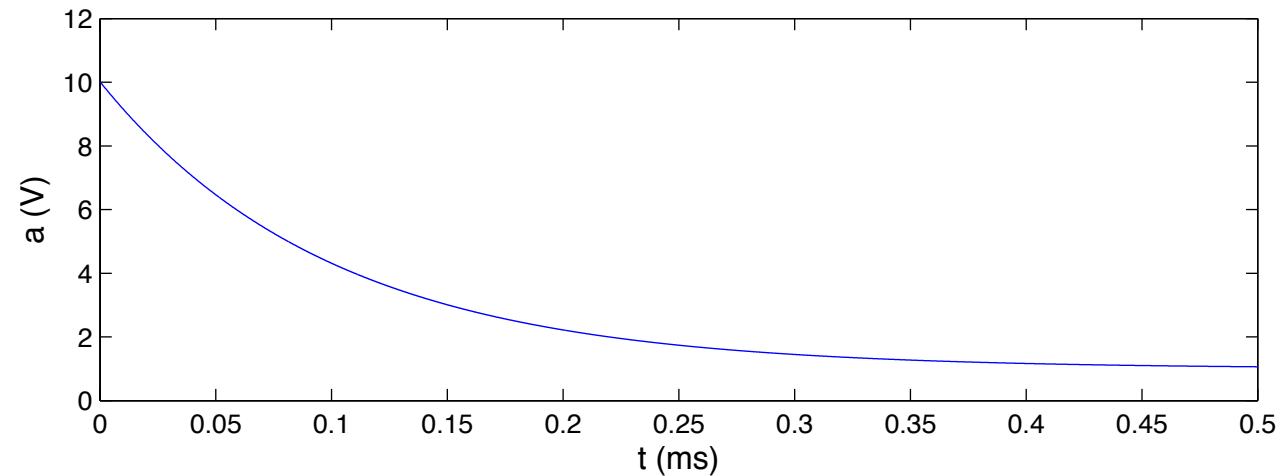
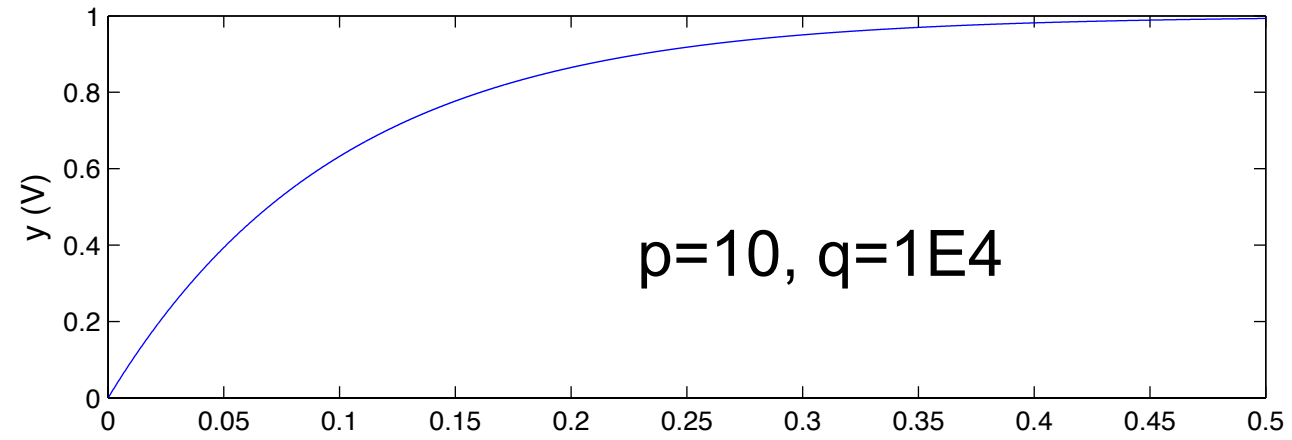
# Can completely cancel pole by putting zero on top of it ( $p=100$ , $q=1E5$ )



# We need to live within our means” in terms of p and q



# More Realistic



# A simple procedure to control first-order systems with a PI controller

1. Draw frequency response of plant (magnitude and phase)
2. Add integral gain, if needed, to make DC gain infinite
  - Eliminates residual error.
3. Place zero (q/p) as needed to give  $>45$  degrees of phase margin at unity-gain frequency.
4. Pick absolute magnitude of q and p to give fast response while staying within “range”.
5. Run Matlab to check all of this

## Chapter 5

# Controlling the Buck Converter

In Chapters 3 and 4 we saw through both analysis and simulation that the buck converter, by itself, is unstable. A transient in either duty factor or load leads to an undamped oscillation in both output voltage  $v_C$  and inductor current  $i_L$ . In many cases such oscillations would destroy the load.

A good controller modifies the dynamics of the system to make it stable, avoid overshoot, and give fast response to transients. There are many strategies for controlling a buck converter which we will visit in future chapters. In this chapter we illustrate the principles of control using a simple PD (proportional-derivative) feedback controller.

### 5.1 Controller Derivation

Figure 5.1 shows the block diagram of a PD feedback controller for the buck regulator. The output voltage  $v(s)$  is subtracted from a target reference voltage

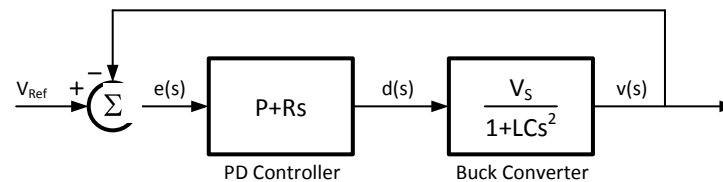


Figure 5.1: Block diagram of a feedback controller.

$$H(s) = \frac{v(s)}{e(s)}$$

DC gain is  $PV_S$

Two poles at  $\omega_0$  (10k rad/s)

$$= \frac{V_S(P + Rs)}{1 + LCs^2}$$

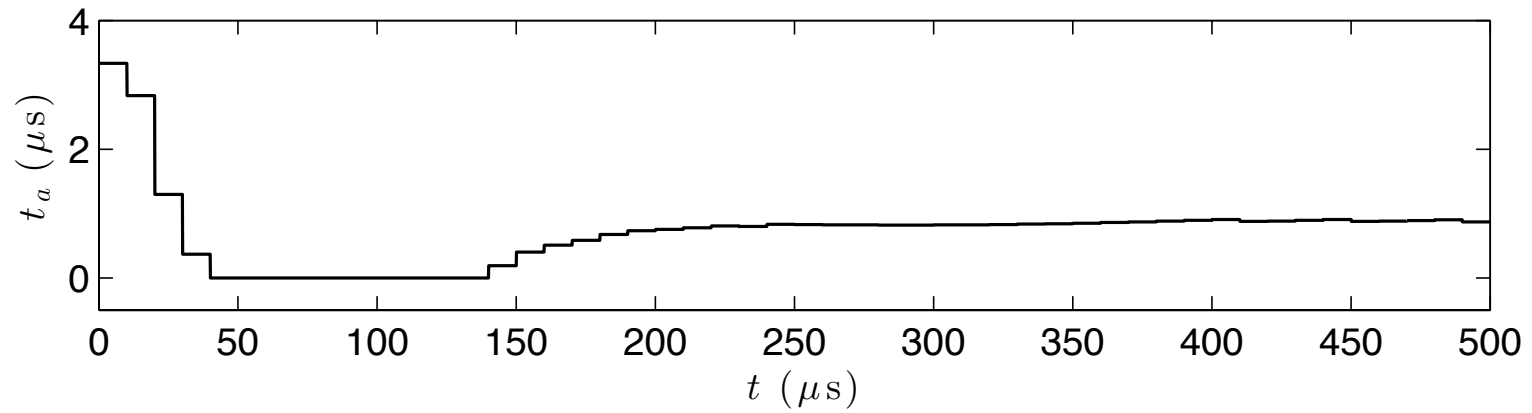
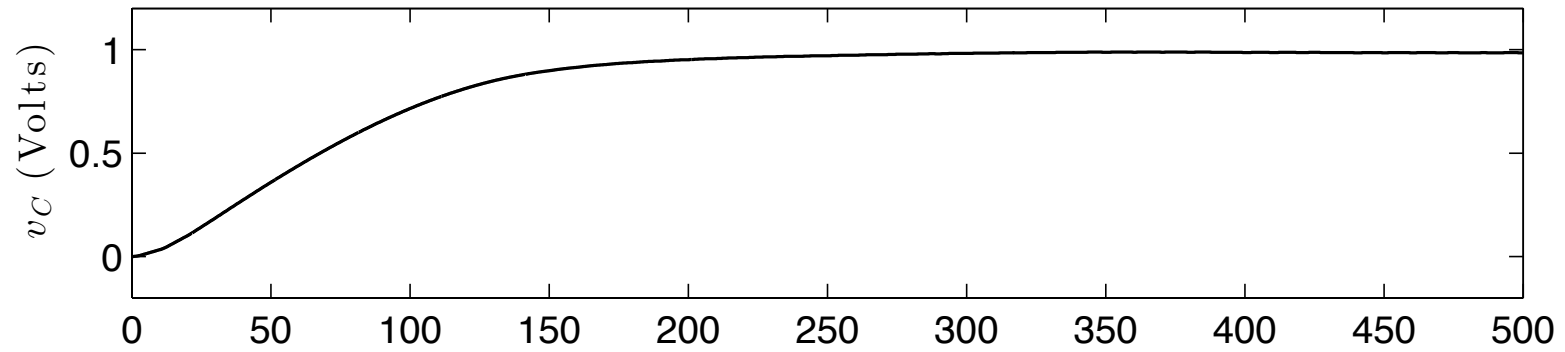
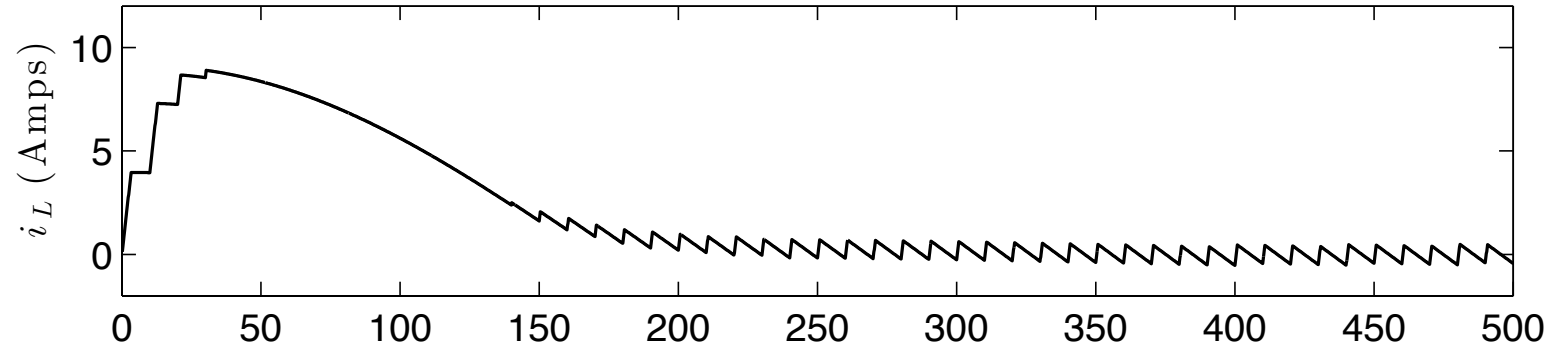
Zero at  $P/R$

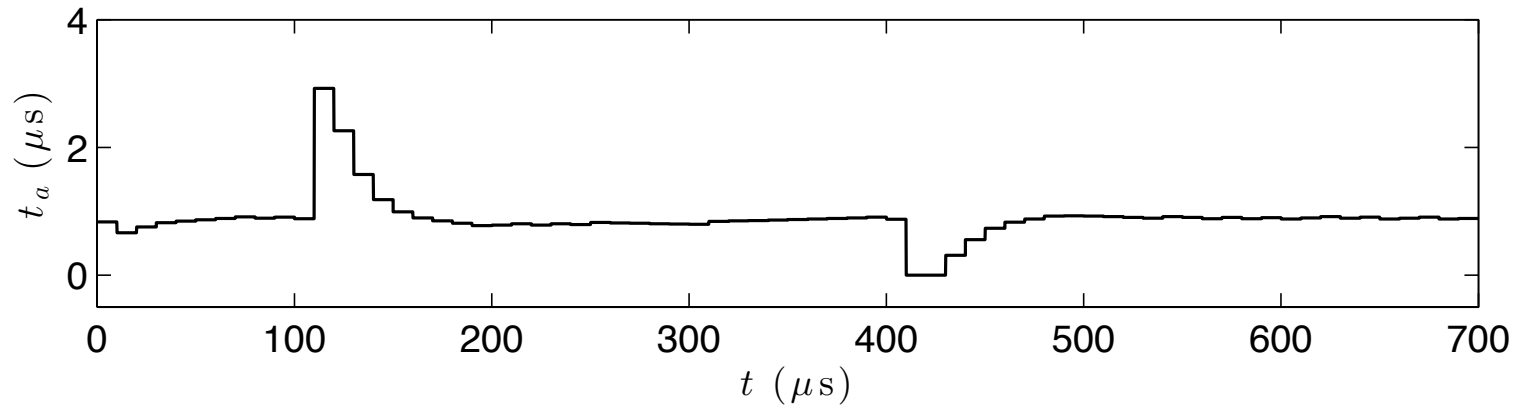
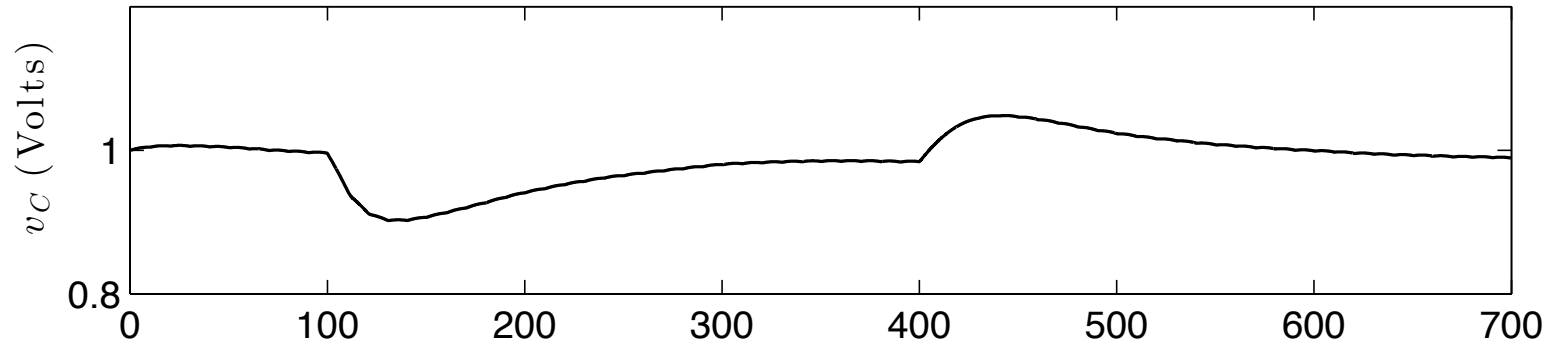
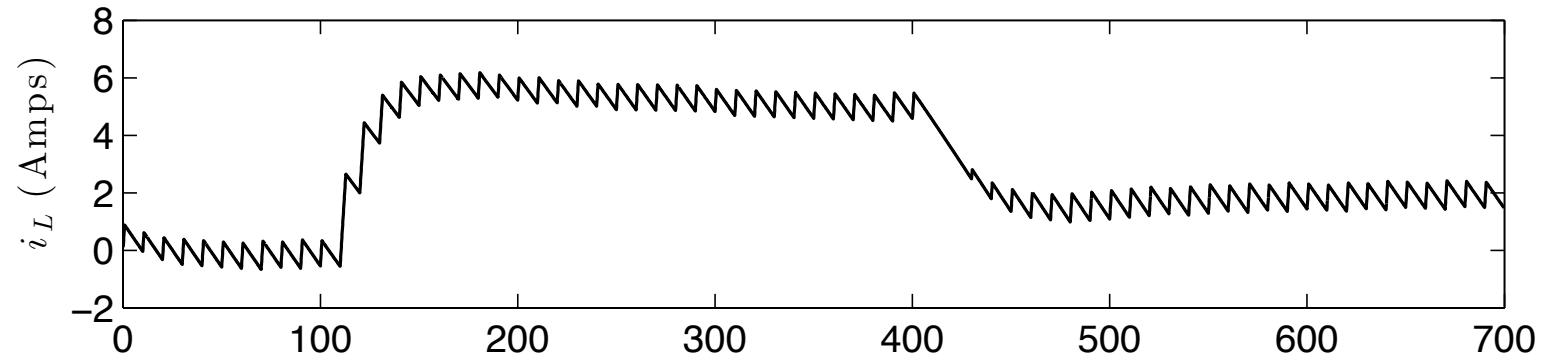
$$= \frac{(KP + KR s)}{s^2 + \frac{1}{LC}}$$

What is the unity-gain frequency w/o control?

Where would you set the gain (with P)?

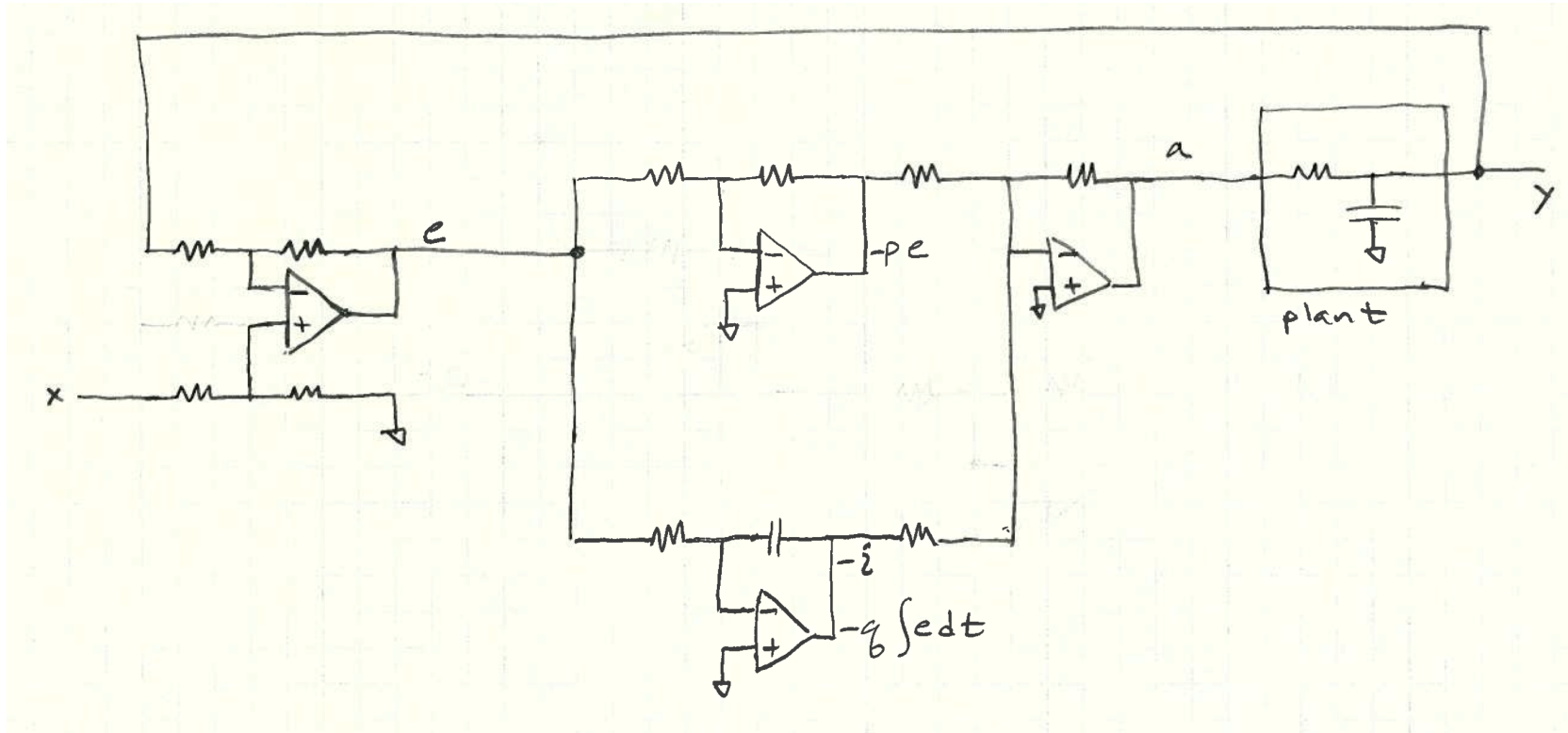
Where would you place the zero?



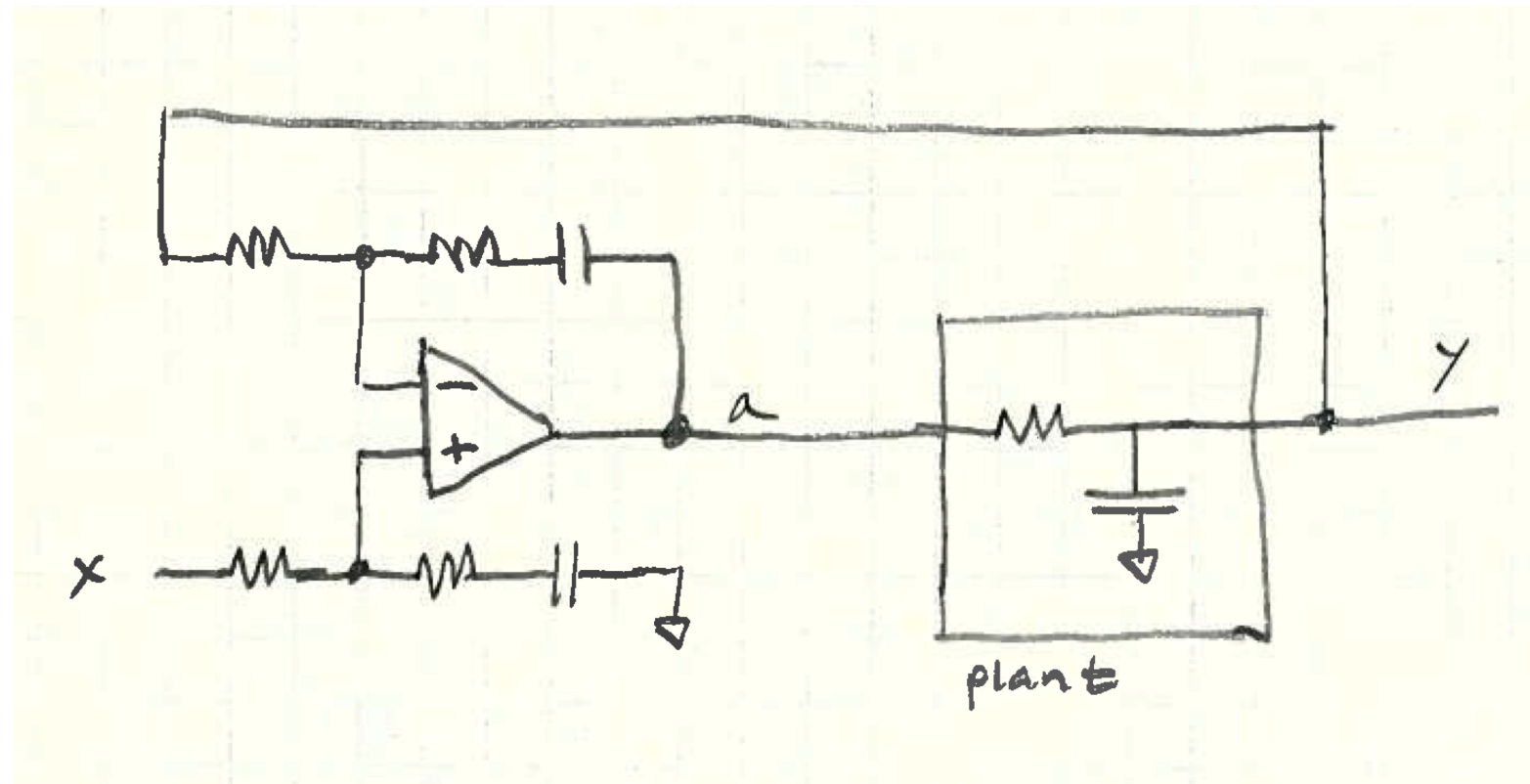


# Implementation of Controllers

# Old Analog Approach (still used for very fast loops)



# Simpler Implementation of same controller



# Digital Implementation

Each timestep:

$$\text{err} = x - y ;$$

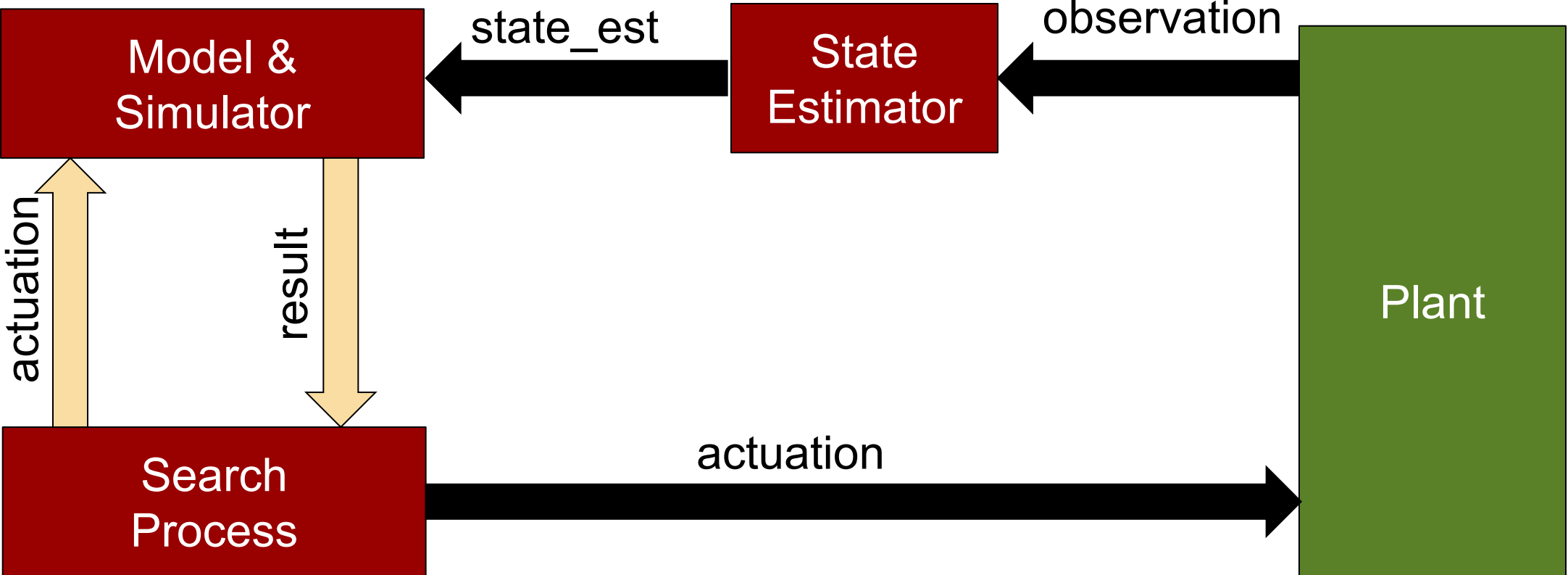
$$\text{ierr} = \text{ierr} + \text{err} * \text{dt} ;$$

$$\text{control} = p * \text{err} + q * \text{ierr} ;$$

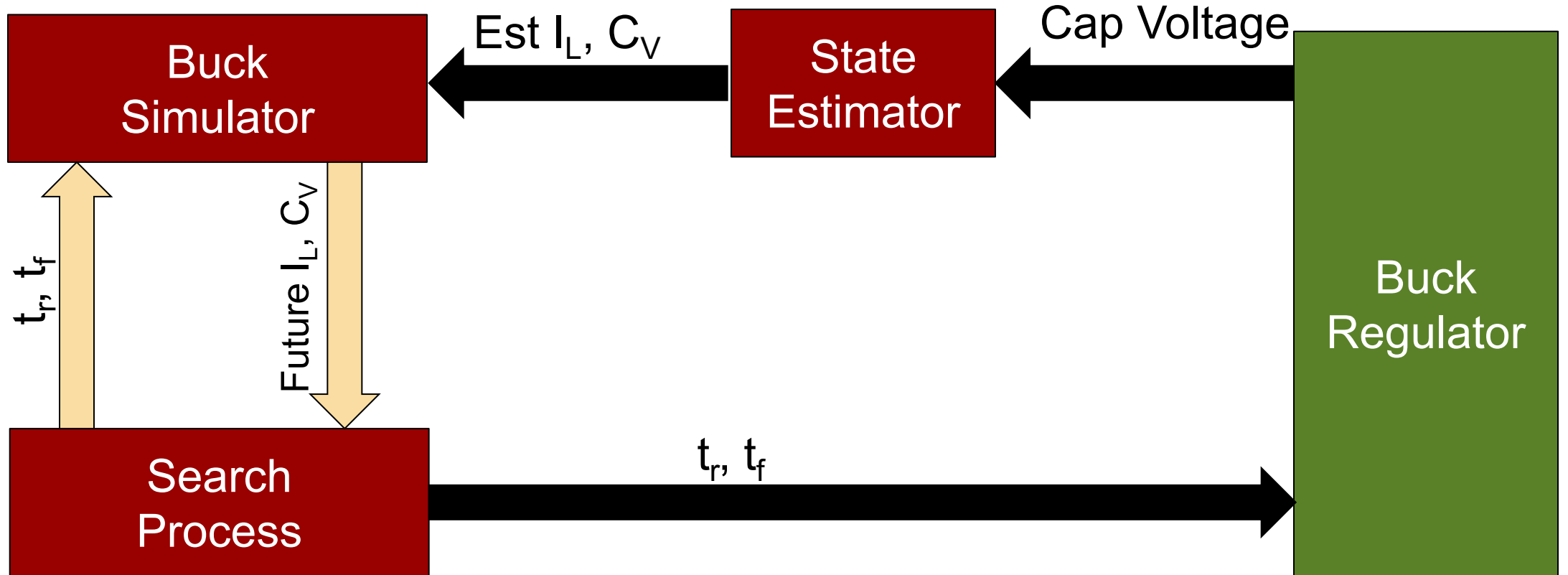
# Changing Control Laws

- Boost gain changing
- DCM gain changing
- DCM to CCM

# Model-Based Optimal Control



# Example MBOC of Buck Regulator



(12) **United States Patent**  
**Dally**

(10) **Patent No.:** **US 9,484,815 B2**  
(45) **Date of Patent:** **Nov. 1, 2016**

- (54) **ENERGY-BASED CONTROL OF A SWITCHING REGULATOR**
- (71) Applicant: **NVIDIA Corporation**, Santa Clara, CA (US)
- (72) Inventor: **William J. Dally**, Los Altos Hills, CA (US)
- (73) Assignee: **NVIDIA Corporation**, Santa Clara, CA (US)

2004/0057257 A1\* 3/2004 Sarlioglu et al. .... 363/49  
2011/0169474 A1\* 7/2011 Cuk ..... H02M 3/158  
323/311  
2011/0227413 A1\* 9/2011 Tsai ..... H02J 1/10  
307/43  
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(\*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 63 days.

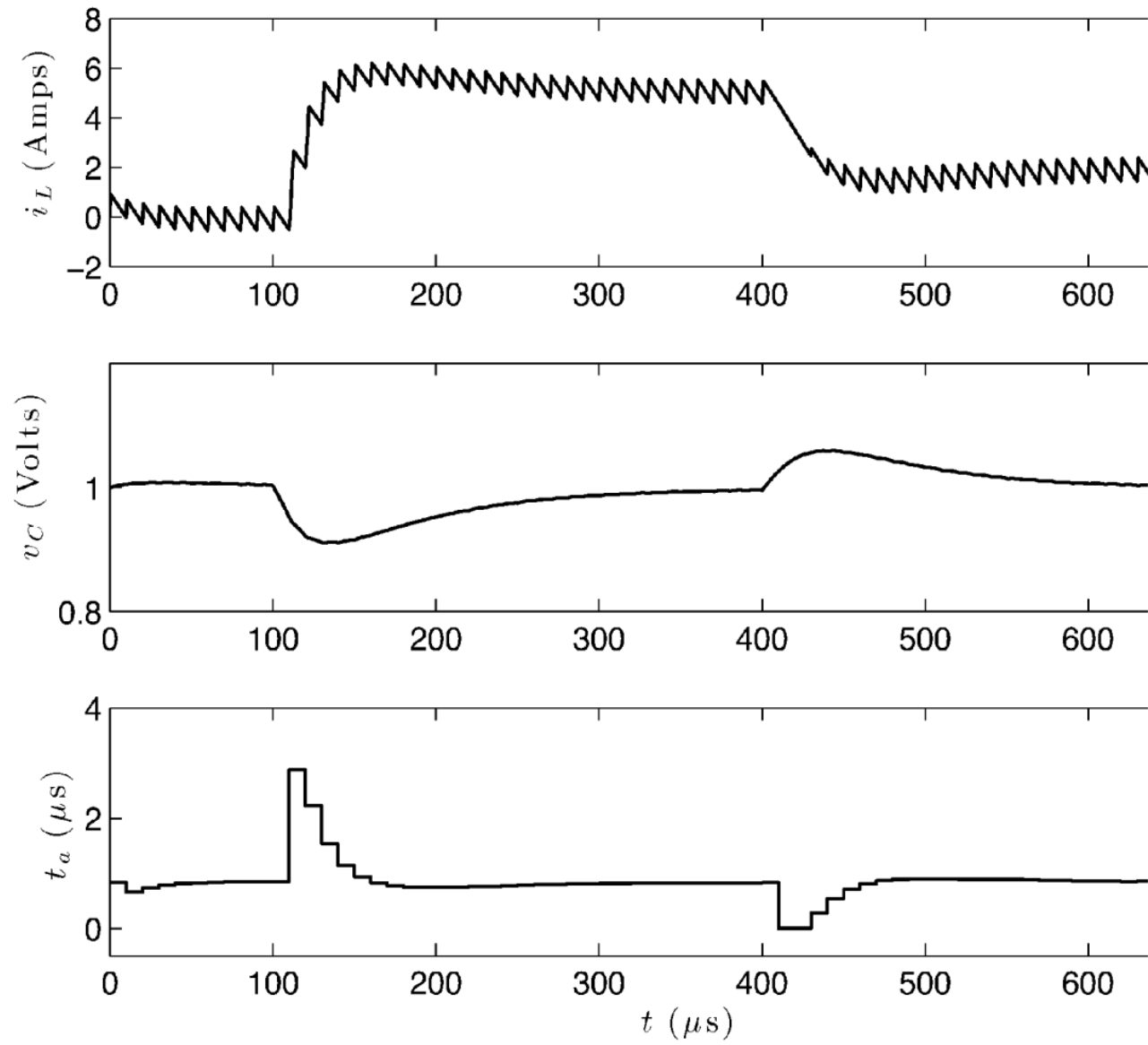
(21) Appl. No.: **14/055,819**

(22) Filed: **Oct. 16, 2013**

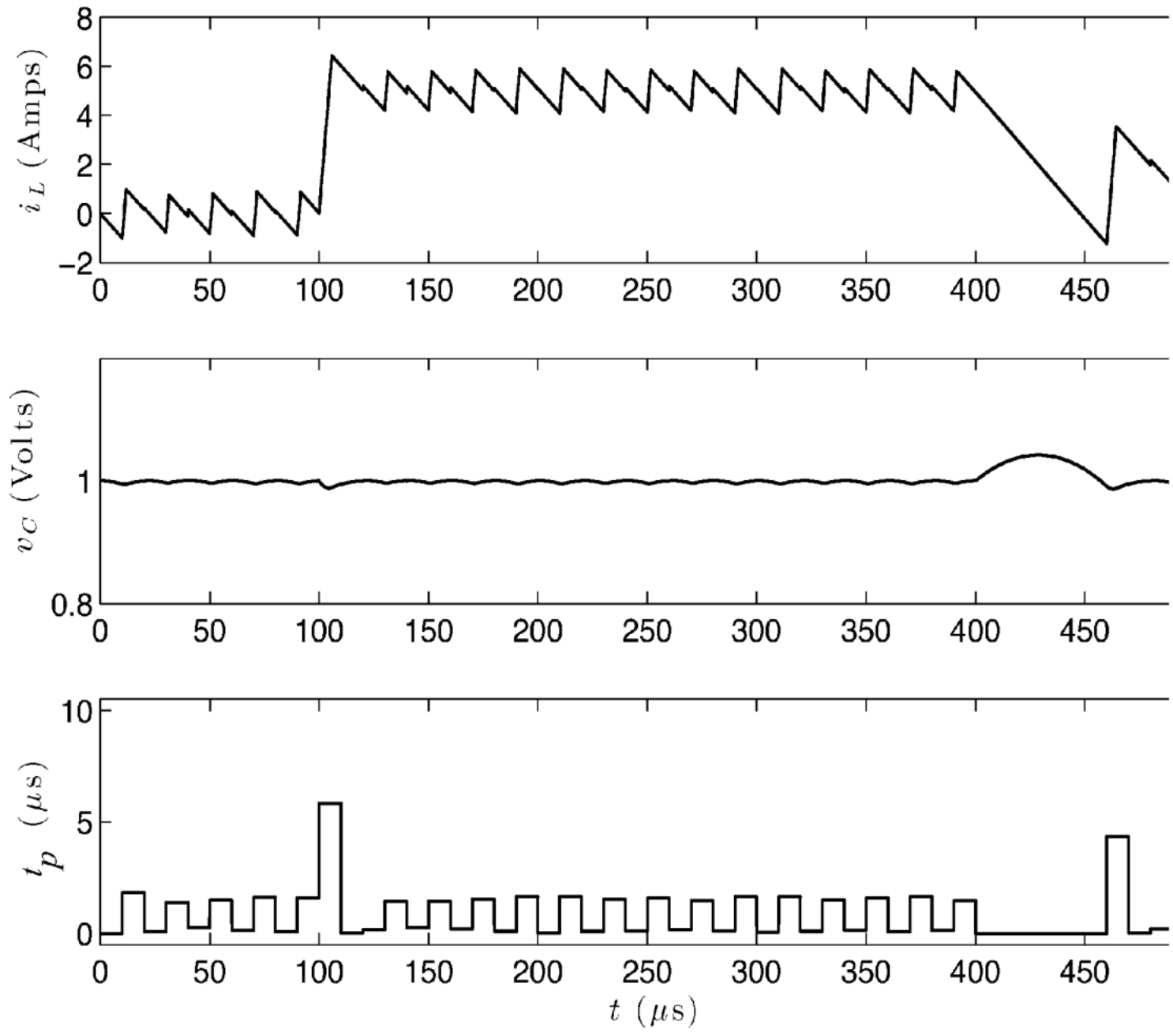
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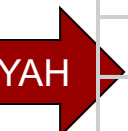
***(PRIOR ART)***  
***Fig. 1***



# Summary

- Plant is described by ODEs (possibly non-linear)
- Controller drives plant input(s) to achieve goal
- Feedback control, input is function of “error”
- Stable if
  - $\zeta \geq 1$
  - $H(s)$  has phase margin at unity gain
- PD and PI controllers
  - Derivative feedback stabilizes 2<sup>nd</sup> order system
  - Integral feedback cancels residual error (but avoid wind up)
- Motor control with current limit
- Analog and digital implementations
- Can also implement “model-based” control

$$G(s) = \frac{C(s)P(s)}{1 + C(s)P(s)} = \frac{H(s)}{1 + H(s)}$$



| No | Date     | Topic                              | HW out | HW in | Lab out | Lab ck | Lab                  | HW                      |
|----|----------|------------------------------------|--------|-------|---------|--------|----------------------|-------------------------|
| 1  | 9/25/17  | Intro (basic converters)           | 1      |       | 1       |        | Intro to ST32F3      | Periodic Steady State   |
| 2  | 9/27/17  | Embedded Prog/Power Elect.         |        |       |         |        |                      |                         |
| 3  | 10/2/17  | Power Electronics - 1 (switches)   | 2      | 1     | 2       | 1      | AC Energy Meter      | Power Devices           |
| 4  | 10/4/17  | Power Electronics - 2 (circuits)   |        |       |         |        |                      |                         |
| 5  | 10/9/17  | Photovoltaics                      | 3      | 2     | 3       | 2      | PV MPPT              | Motor control Matlab    |
| 6  | 10/11/17 | Feedback Control                   |        |       |         |        |                      |                         |
| 7  | 10/16/17 | Electric Motors                    | 4      | 3     | 4       | 3      | Motor control - Lab/ | Feedback                |
| 8  | 10/18/17 | Isolated Converters                |        |       |         |        |                      |                         |
| 9  | 10/23/17 | Solar Day                          | 5/PP   | 4     | 5       | 4      | PS                   | Isolated Converters     |
| 10 | 10/25/17 | Magnetics                          |        |       |         |        |                      |                         |
| 11 | 10/30/17 | Soft Switching                     | 6      | 5/PP  | 6       | 5      | Magnetics            | Magnetics and Inverters |
| 12 | 11/1/17  | Project Discussions                |        |       |         |        |                      |                         |
| 13 | 11/6/17  | Inverters, Grid, PF, and Batteries |        | 6     | P       | 6      | Project              |                         |
| 14 | 11/8/17  | Thermal & EMI                      |        |       |         |        |                      |                         |
| 15 | 11/13/17 | Quiz Review                        |        |       |         | C1     |                      |                         |
| 16 | 11/15/17 | Grounding, and Debugging           |        |       |         |        |                      |                         |
| Q  | 11/15/17 | Quiz - in the evening              |        |       |         |        |                      |                         |
|    | 11/20/17 | Thanksgiving Break                 |        |       |         | C2     |                      |                         |
|    | 11/22/17 | Thanksgiving Break                 |        |       |         |        |                      |                         |
| 17 | 11/27/17 | Guest Lecture                      |        |       |         |        |                      |                         |
| 18 | 11/29/17 | Martin Fornage - Enphase           |        |       |         | C3     |                      |                         |
| 19 | 12/4/17  | Colin Campbell - Tesla             |        |       |         |        |                      |                         |
| 20 | 12/6/17  | No Class                           |        |       |         |        |                      |                         |