EE155/255 Green Electronics

Magnetics: Inductors, and Transformers

10/24/16

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ENGINEERING

Course Logistics

- Lab 4 (Motor Lab 1) signed off this week
- Lab 5 is out
- HW4 was due Monday
- HW5 is out
- Project Proposal out soon

YAH

No	Date	Topic	HW out	HW in	Lab out	Lab ck	Lab	HW
1	9/25/17	Intro (basic converters)	1		1		Intro to ST32F3	Periodic Steady State
2	9/27/17	Embedded Prog/Power Elect.						
3	10/2/17	Power Electronics - 1 (switches)	2	1	2	1	AC Energy Meter	Power Devices
4	10/4/17	Power Electronics - 2 (circuits)						
5	10/9/17	Photovoltaics	3	2	3	2	PV MPPT	Motor control Matlab
6	10/11/17	Feedback Control						
7	10/16/17	Electric Motors	4	3	4	3	Motor control - Lab/	Feedback
8	10/18/17	Isolated Converters						
9	10/23/17	Solar Day	5/PP	4	5	4	PS	Isolated Converters
10	10/25/17	Magnetics						
11	10/30/17	Soft Switching	6	5/PP	6	5	Magnetics	Magnetics and Inverters
12	11/1/17	Project Discussions						
13	11/6/17	Inverters, Grid, PF, and Batteries		6	P	6	Project	
14	11/8/17	Thermal & EMI						
15	11/13/17	Quiz Review				C1		
16	11/15/17	Grounding, and Debugging						
Q	11/15/17	Quiz - in the evening						
	11/20/17	Thanksgiving Break				C2		
	11/22/17	Thanksgiving Break						
17	11/27/17	Guest Lecture						
18	11/29/17	Martin Fornage - Enphase				C3		
19	12/4/17	Colin Campbell - Tesla						
20	12/6/17	No Class						
	TBD	Project presentations			P			
	12/15/17	Project webpage due						

Magnetics

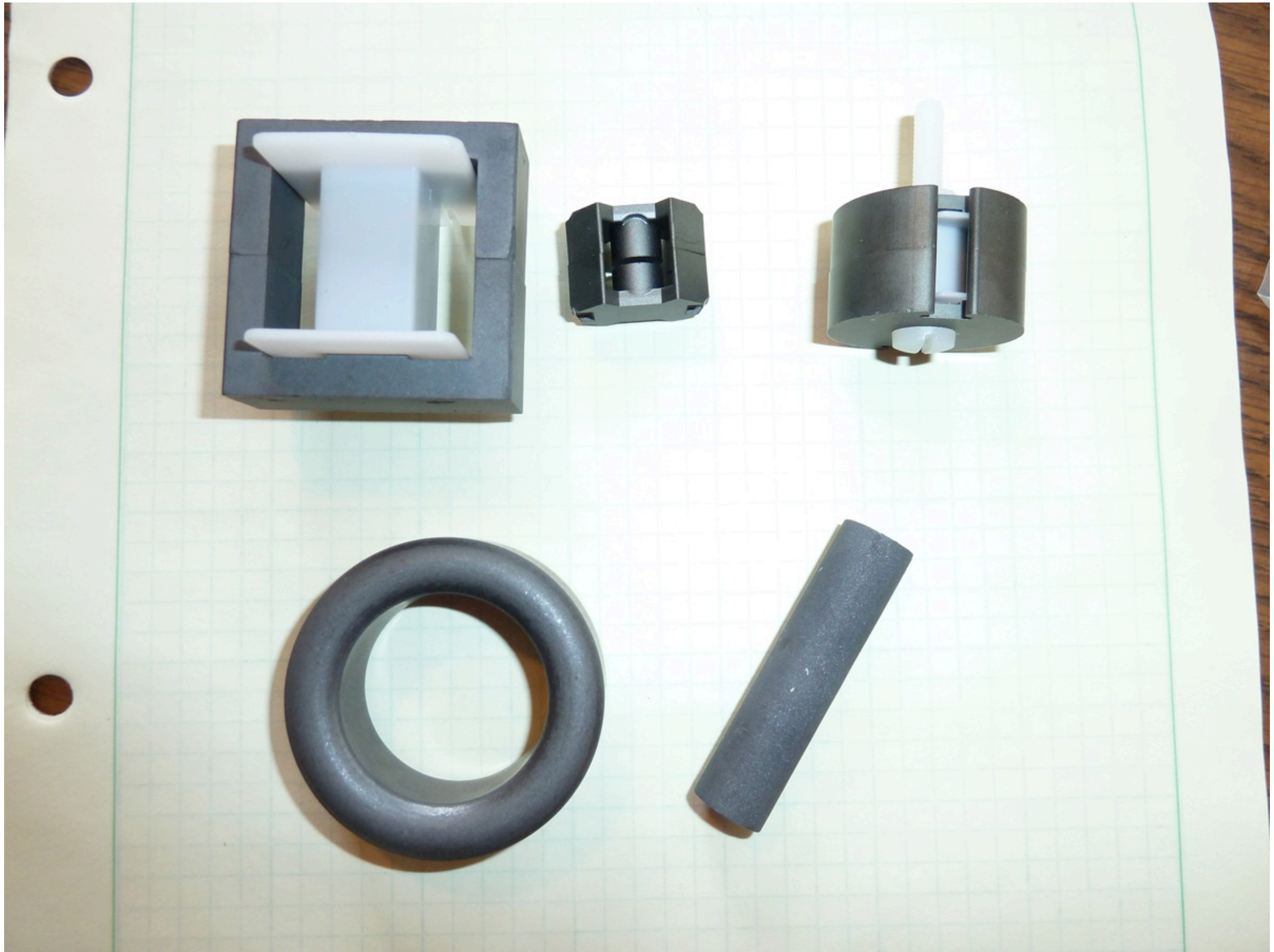
Magnetics

- Magnetic components, inductors and transformers, are key components in green electronic systems
  - Major fraction of system cost
  - Major source of power losses (core and copper)
  - Key to energy conversion process
- Based on *magnetic circuits*
  - current induces magneto-motive force ($\mathcal{F}$) which generates *flux* (ϕ) in circuit depending on reluctance ($\mathcal{R}$).
  - Voltage per turn depends on change in flux

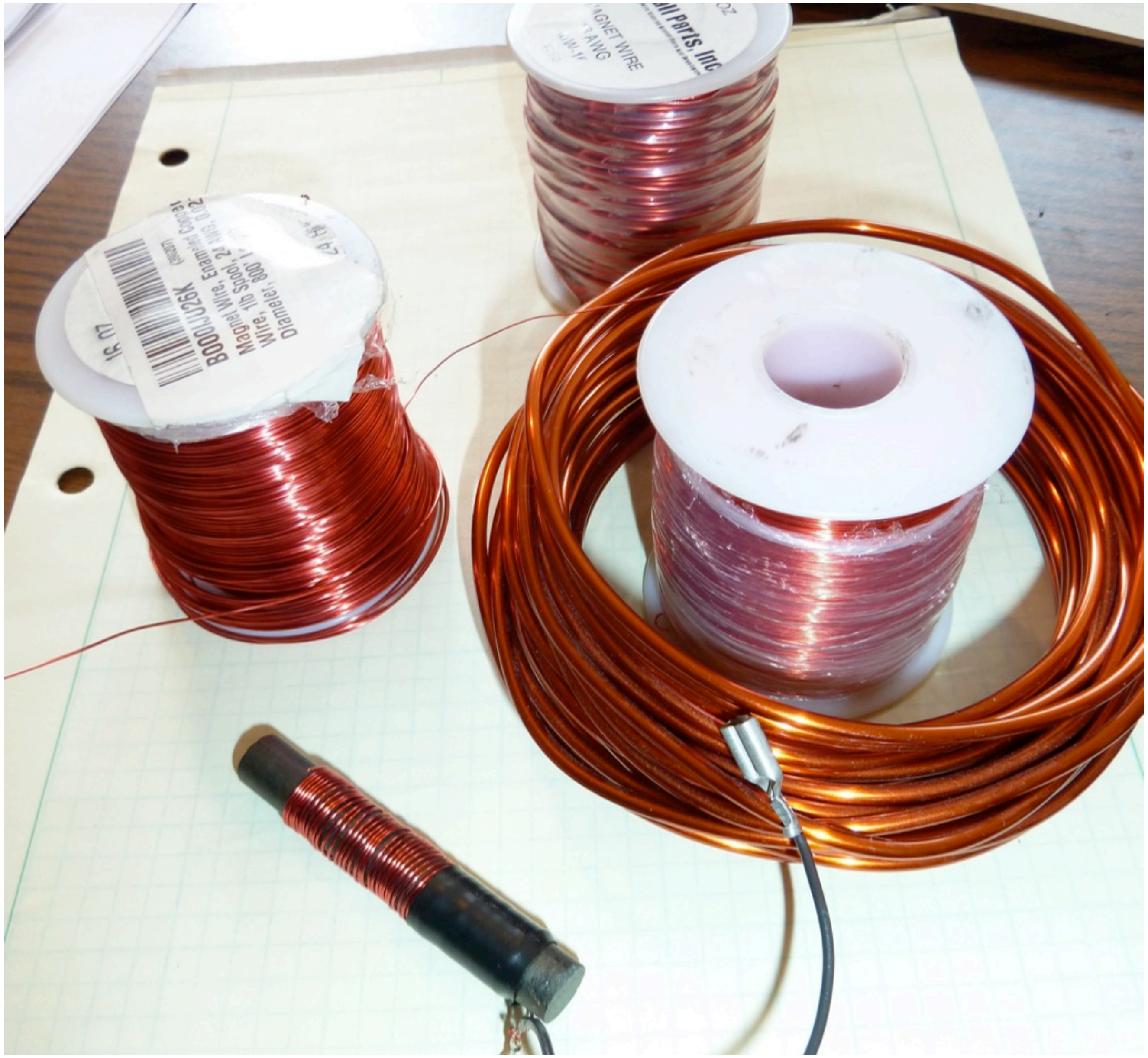
$$\phi = \frac{\mathcal{F}}{\mathcal{R}} \qquad V = N\frac{d\phi}{dt}$$

- Field in core must be kept within limits to avoid saturation and core losses
- Air gap in core sets desired inductance at desired field strength

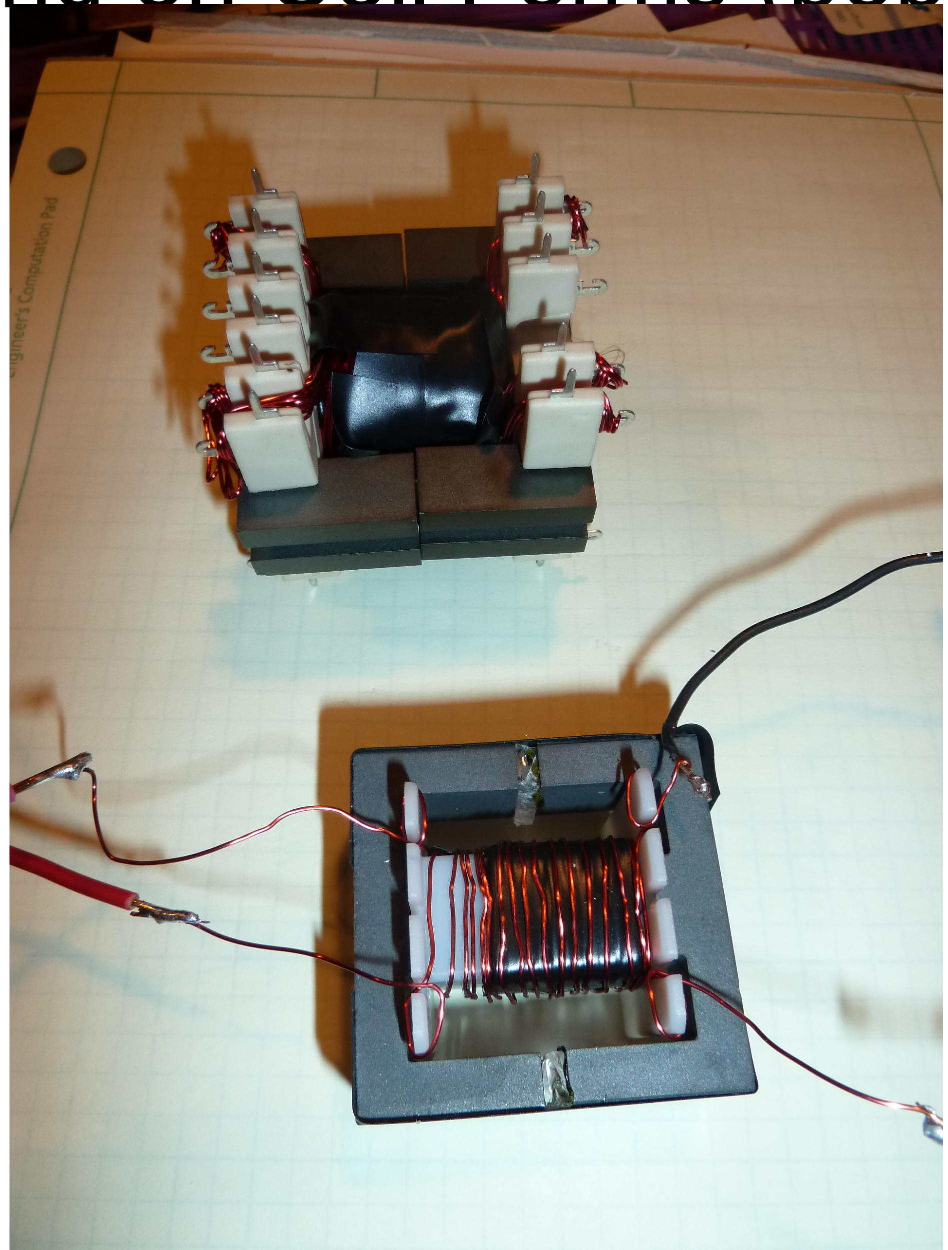
Some Example Cores



Magnet Wire



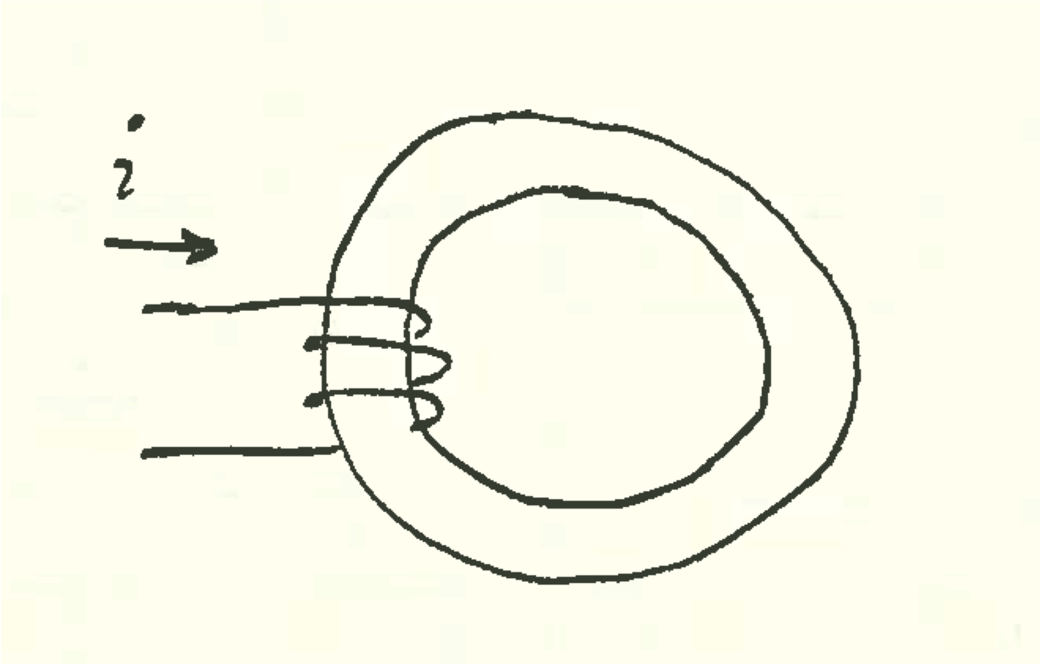
Inductor and Transformer wound on Coil Forms (bobbins)



A Magnetic Circuit

- Put N turns of wire on a magnetic core
- Pass current i through resulting coil
- Produces *Magnetomotive Force*

$$\mathcal{F} = Ni$$



A Magnetic Circuit

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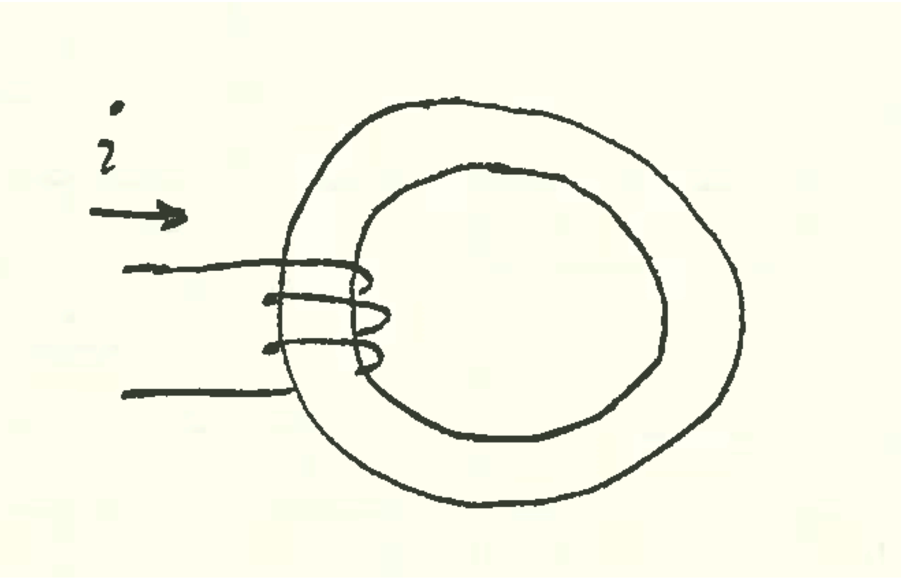
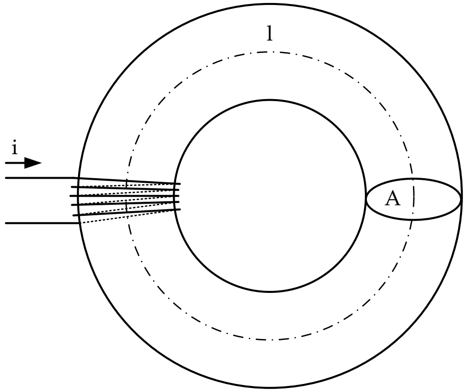


$$\mathcal{F} = Ni$$

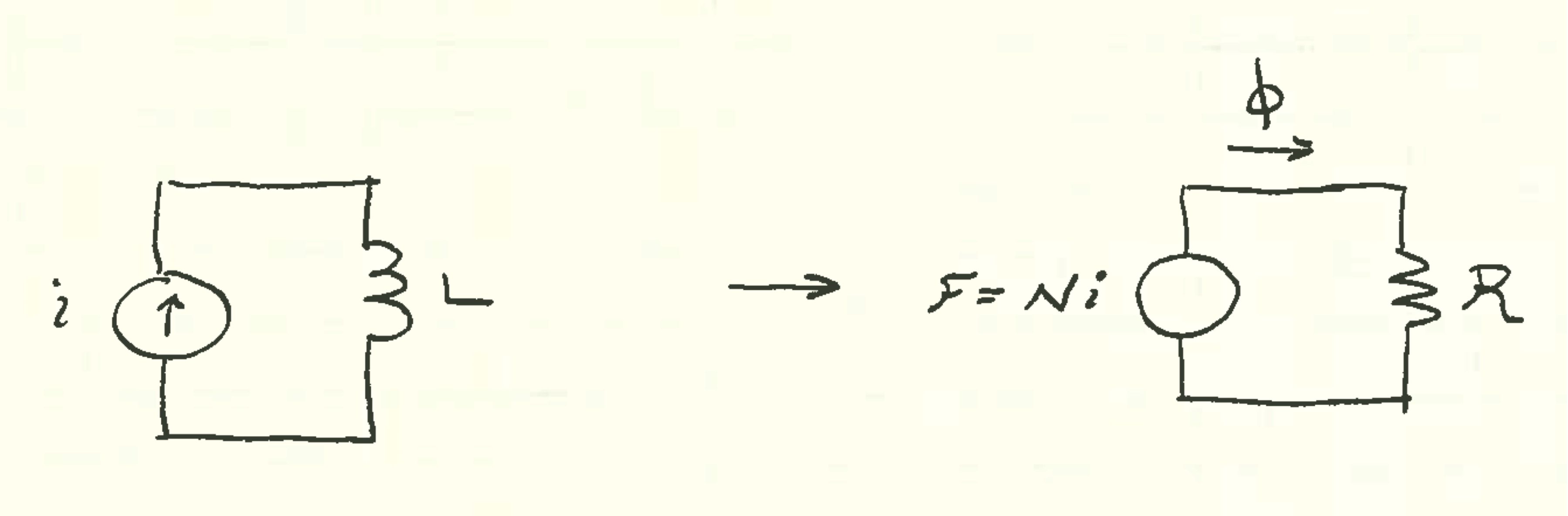
- $\mathcal{F}$ induces a flux ϕ in the core

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{Ni}{\mathcal{R}}$$



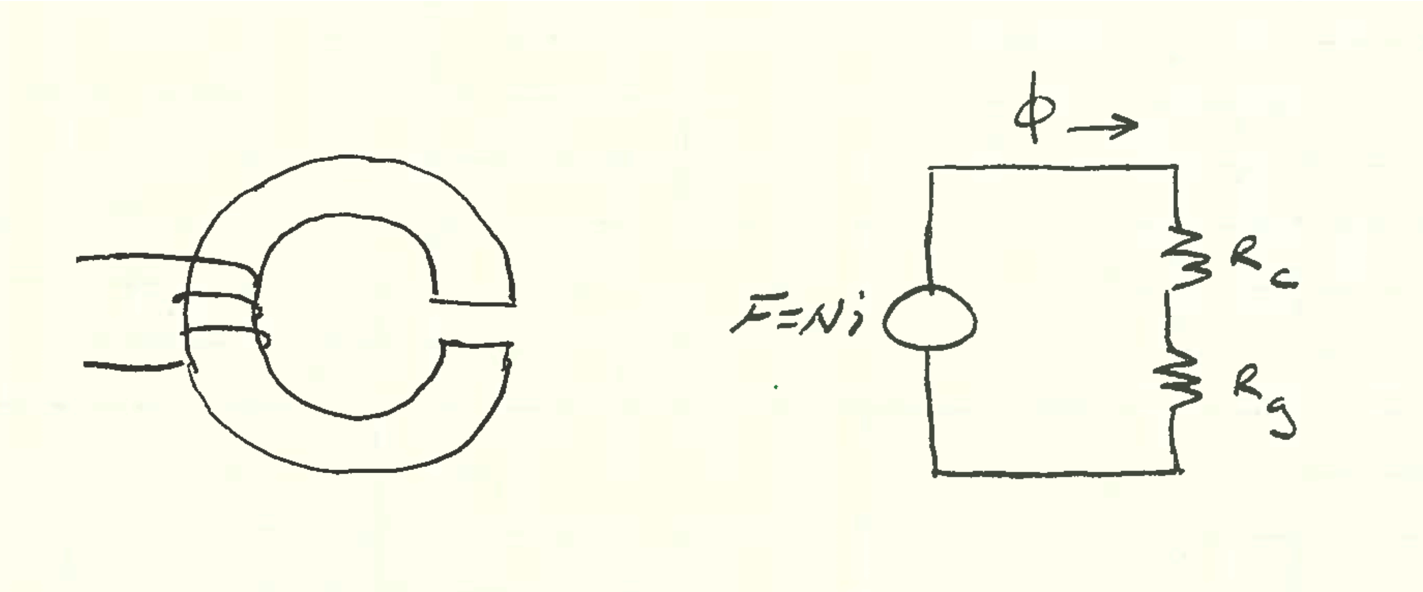
- Where $\mathcal{R}$ is the *Reluctance* of the core

Equivalent Circuit



$$\mathcal{F} = Ni \qquad \phi = \frac{\mathcal{F}}{\mathcal{R}}$$

Gapped Cores



$$R = \frac{l}{\mu A} = \frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g} \approx \frac{l_g}{\mu_g A_g}$$

Inductance

- Voltage across the coil is

$$V = N\frac{d\phi}{dt}$$

$$V = \left(\frac{N}{\mathcal{R}}\right)\frac{d\mathcal{F}}{dt}$$

$$V = \left(\frac{N^2}{\mathcal{R}}\right)\frac{di}{dt}$$

$$V = L\frac{di}{dt}$$

Inductance

$$L = \left(\frac{N^2}{\mathcal{R}} \right)$$

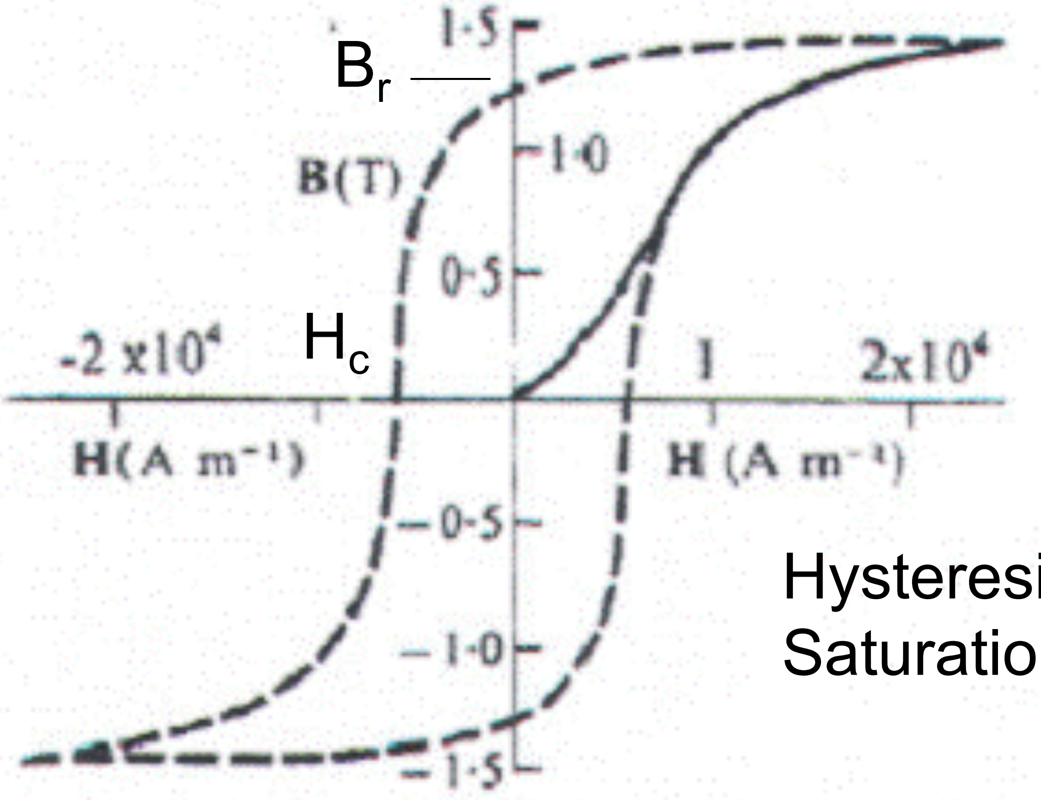
$$V = L \frac{di}{dt}$$

Magnetic Materials

- Key parameters
  - Permeability μ
  - Saturation field B_{sat}
  - Losses
- Ferrites
  - Low losses at high frequencies
- Powdered Iron
  - Higher losses at frequency, higher saturation
- Iron Alloys
  - Low frequencies only, very high saturation

Magnetic Materials
The B-H Curve

Loss per cycle is area of loop

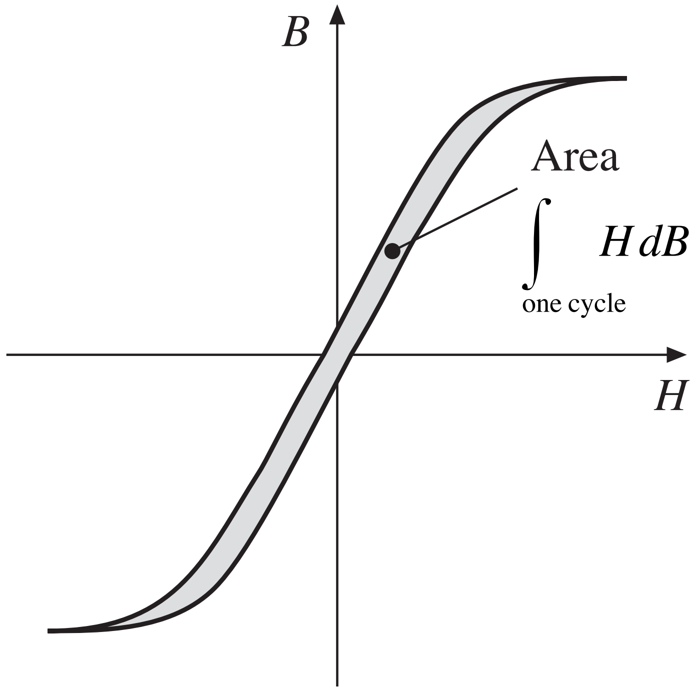


Magnetisation curve and hysteresis loop for hard magnet steel.

Hysteresis and Saturation

What happens to inductor current at core saturation?

Core Loss due to Hysteresis

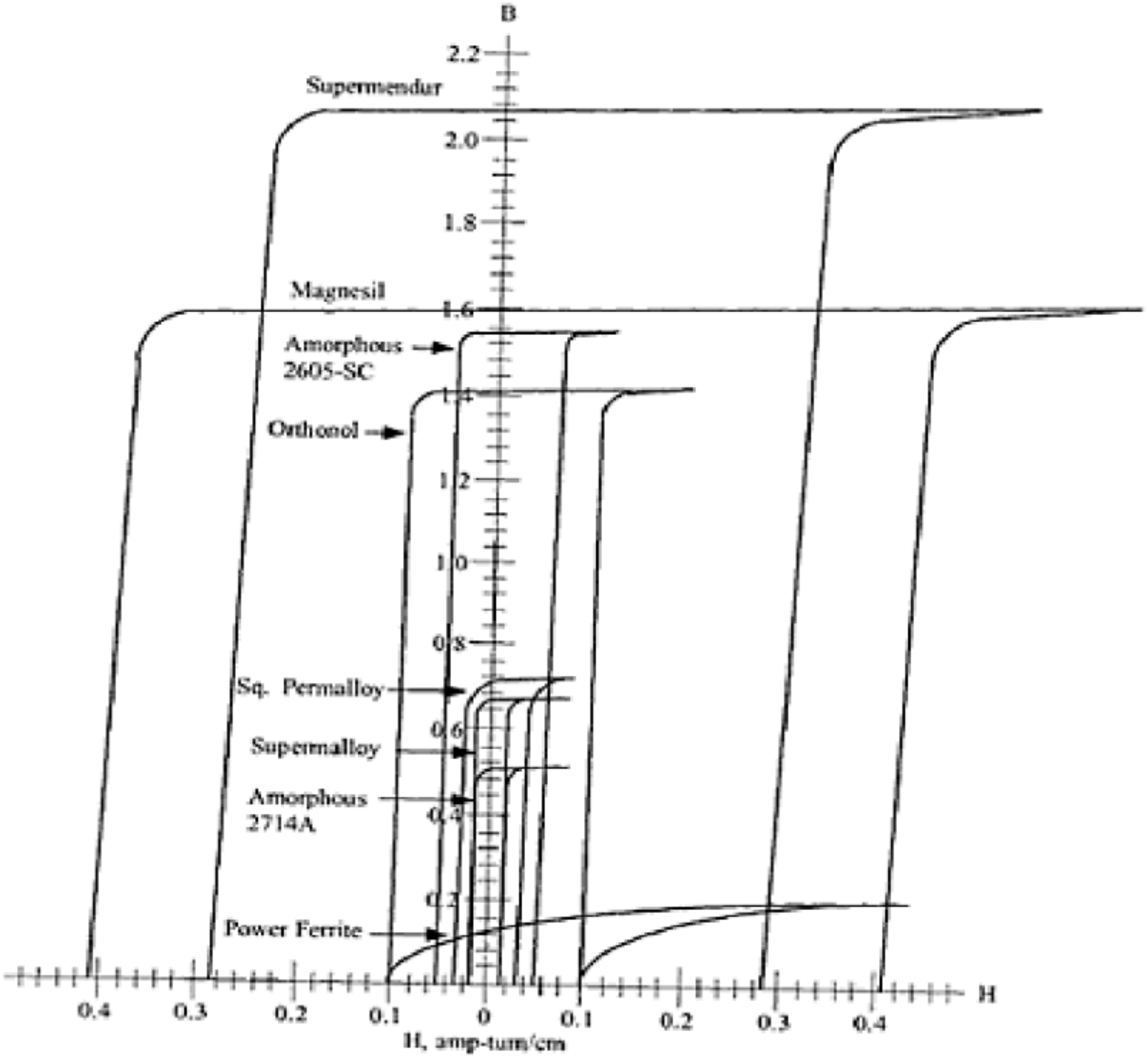


$$I = H\frac{l_c}{N}$$

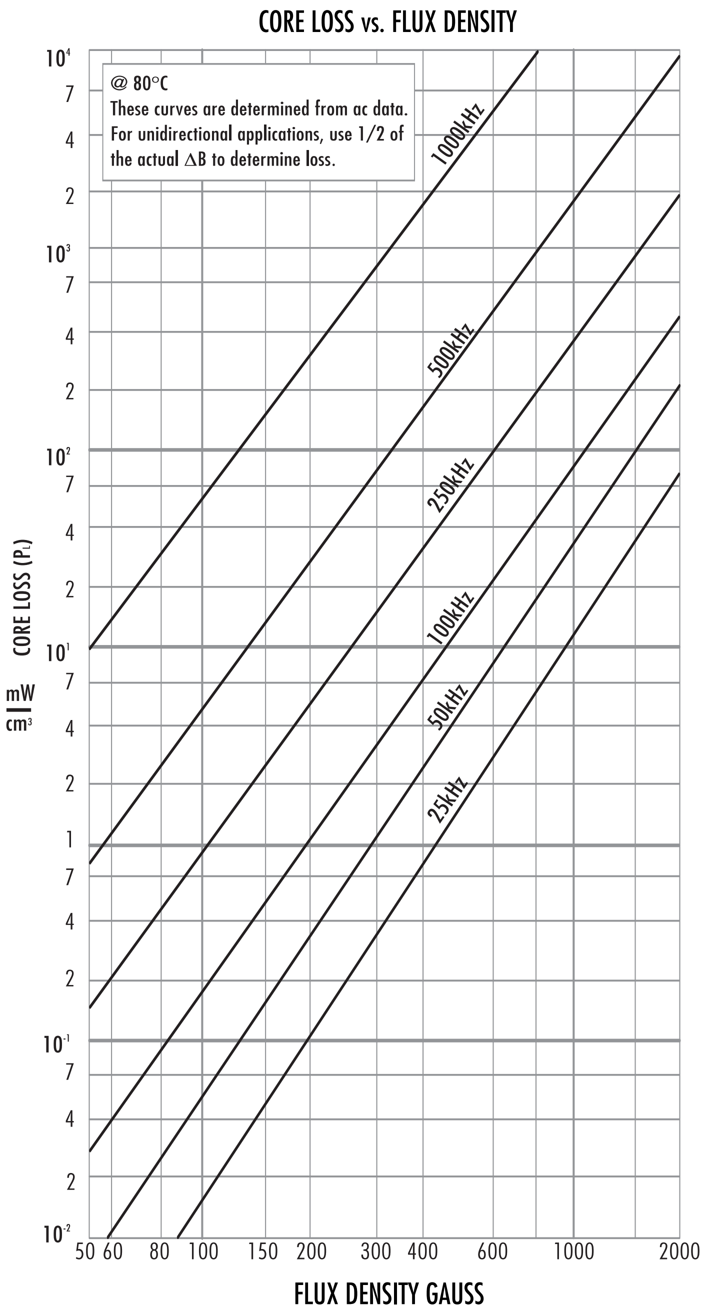
$$V = N\frac{d\phi}{dt} = NA_c\frac{dB}{dt}$$

$$E_{cy} = \int_{cy} VI\,dt = A_c l_c \int_{cy} H\,dB$$

B-H Curves of Various Materials



Magnetic Materials Loss



$$P = kVf^{1.7}B^{2.7}$$

B here is the AC component of the B field (p-p)

Steinmetz parameters k, a=1.7 and b=2.7 depend on material

3C95 SPECIFICATIONS

A low to medium frequency power material with low power losses from 25 to 100 °C for use in power transformers at frequencies up to 0.5 MHz. Especially suited for broad temperature range applications like automotive, lighting and mobile / handheld.

SYMBOL	CONDITIONS	VALUE	UNIT
μ_i	25 °C; ≤10 kHz; 0.25 mT	3000 ± 20 %	
μ_a	100 °C; 25 kHz; 200 mT	≈ 5000	
B	25 °C; 10 kHz; 1200 A/m	≈ 530	mT
	100 °C; 10 kHz; 1200 A/m	≈ 410	
P_V	25 °C; 100 kHz; 200 mT	≈ 350	kW/m^3
	100 °C; 100 kHz; 200 mT	≈ 290	
ρ	DC, 25 °C	≈ 5	Ωm
T_C		≥ 215	°C
density		≈ 4800	kg/m^3

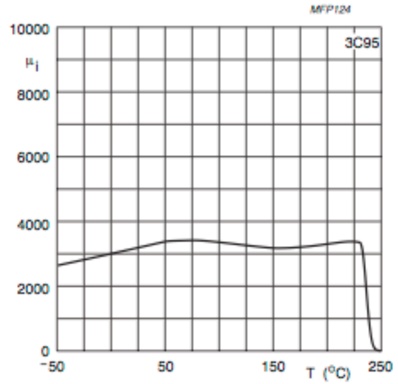


Fig.1 Initial permeability as a function of temperature.

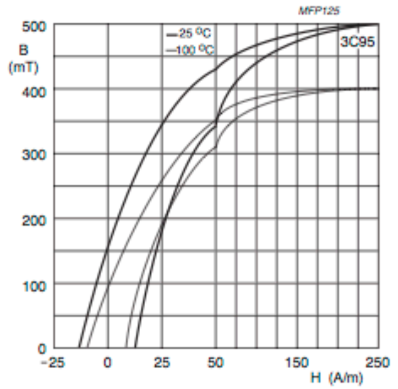


Fig.2 Typical B-H loops.

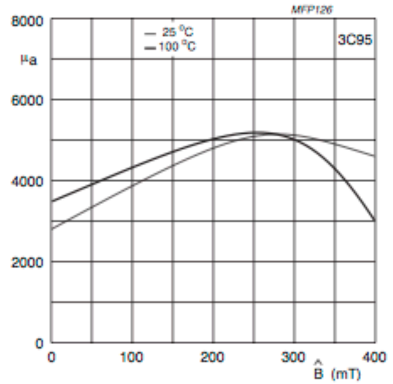


Fig.3 Amplitude permeability as a function of peak flux density.

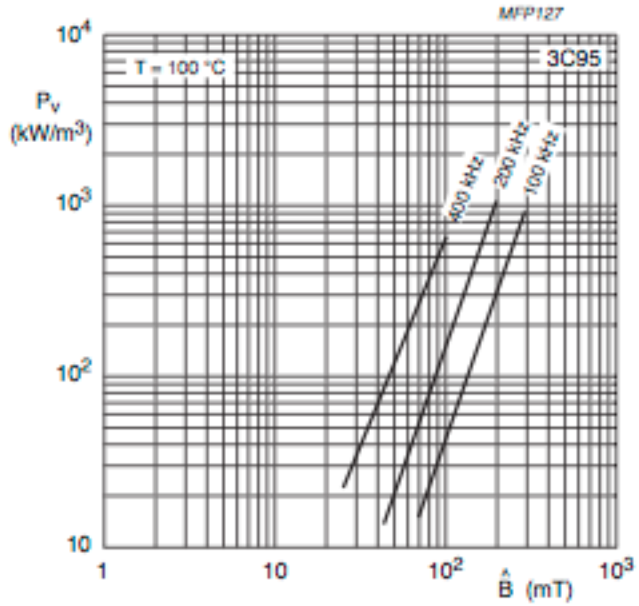
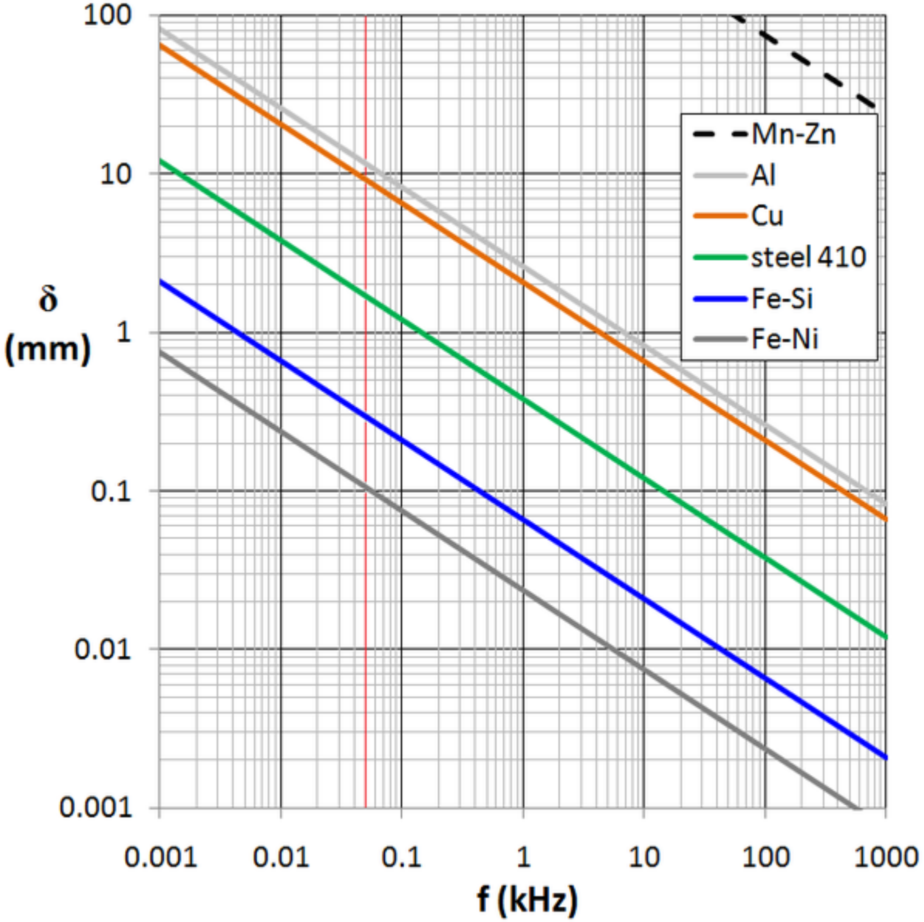


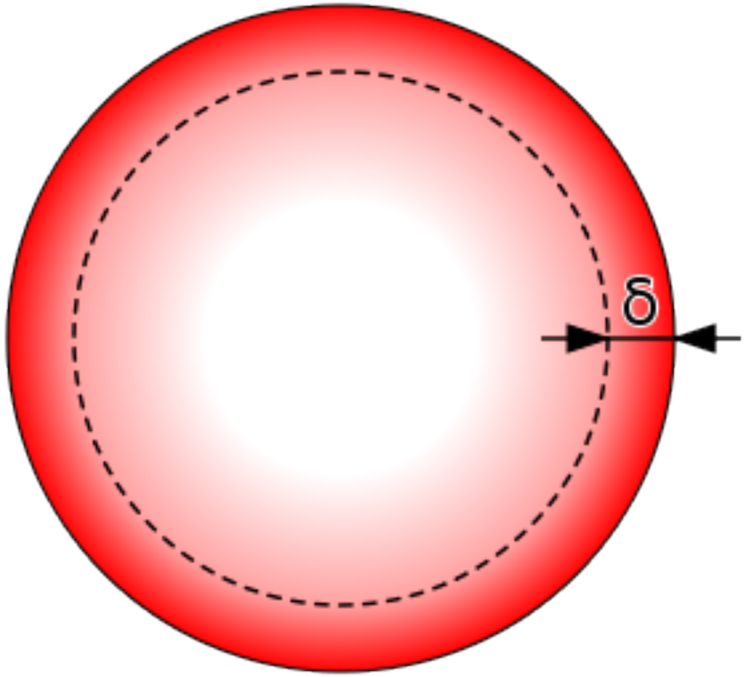
Fig.4 Specific power loss as a function of peak flux density with frequency as a parameter.

Wires

- Have resistivity ohms/m
- At high frequency suffer from skin effect
  - (and proximity effect)
- Use high enough gauge (small enough wire) to avoid skin effect

Skin Effect


100
10
1
0.1
0.01
0.001
δ
(mm)
0.001
0.01
0.1
1
10
100
1000
f (kHz)
Mn-Zn
Al
Cu
steel 410
Fe-Si
Fe-Ni


δ

Wire Gauge

AWG gauge	Conductor Diameter Inches	Conductor Diameter mm	Ohms per 1000 ft.	Ohms per km	Maximum amps for chassis wiring	Maximum amps for power transmission	Maximum frequency for 100% skin depth for solid conductor copper	Breaking force Soft Annealed Cu 37000 PSI
OOOO	0.46	11.684	0.049	0.16072	380	302	125 Hz	6120 lbs
OOO	0.4096	10.40384	0.0618	0.202704	328	239	160 Hz	4860 lbs
OO	0.3648	9.26592	0.0779	0.255512	283	190	200 Hz	3860 lbs
0	0.3249	8.25246	0.0983	0.322424	245	150	250 Hz	3060 lbs
1	0.2893	7.34822	0.1239	0.406392	211	119	325 Hz	2430 lbs
2	0.2576	6.54304	0.1563	0.512664	181	94	410 Hz	1930 lbs
3	0.2294	5.82676	0.197	0.64616	158	75	500 Hz	1530 lbs
4	0.2043	5.18922	0.2485	0.81508	135	60	650 Hz	1210 lbs
5	0.1819	4.62026	0.3133	1.027624	118	47	810 Hz	960 lbs
6	0.162	4.1148	0.3951	1.295928	101	37	1100 Hz	760 lbs
7	0.1443	3.66522	0.4982	1.634096	89	30	1300 Hz	605 lbs
8	0.1285	3.2639	0.6282	2.060496	73	24	1650 Hz	480 lbs
9	0.1144	2.90576	0.7921	2.598088	64	19	2050 Hz	380 lbs
10	0.1019	2.58826	0.9989	3.276392	55	15	2600 Hz	314 lbs
11	0.0907	2.30378	1.26	4.1328	47	12	3200 Hz	249 lbs
12	0.0808	2.05232	1.588	5.20864	41	9.3	4150 Hz	197 lbs
13	0.072	1.8288	2.003	6.56984	35	7.4	5300 Hz	150 lbs
14	0.0641	1.62814	2.525	8.282	32	5.9	6700 Hz	119 lbs
15	0.0571	1.45034	3.184	10.44352	28	4.7	8250 Hz	94 lbs
16	0.0508	1.29032	4.016	13.17248	22	3.7	11 k Hz	75 lbs
17	0.0453	1.15062	5.064	16.60992	19	2.9	13 k Hz	59 lbs
18	0.0403	1.02362	6.385	20.9428	16	2.3	17 kHz	47 lbs
19	0.0359	0.91186	8.051	26.40728	14	1.8	21 kHz	37 lbs
20	0.032	0.8128	10.15	33.292	11	1.5	27 kHz	29 lbs
21	0.0285	0.7239	12.8	41.984	9	1.2	33 kHz	23 lbs
22	0.0254	0.64516	16.14	52.9392	7	0.92	42 kHz	18 lbs
23	0.0226	0.57404	20.36	66.7808	4.7	0.729	53 kHz	14.5 lbs
24	0.0201	0.51054	25.67	84.1976	3.5	0.577	68 kHz	11.5 lbs
25	0.0179	0.45466	32.37	106.1736	2.7	0.457	85 kHz	9 lbs
26	0.0159	0.40386	40.81	133.8568	2.2	0.361	107 kH	7.2 lbs
27	0.0142	0.36068	51.47	168.8216	1.7	0.288	130 kHz	5.5 lbs
28	0.0126	0.32004	64.9	212.872	1.4	0.226	170 kHz	4.5 lbs
29	0.0113	0.28702	81.83	268.4024	1.2	0.182	210 kHz	3.6 lbs
30	0.01	0.254	103.2	338.496	0.86	0.142	270 kHz	2.75 lbs
31	0.0089	0.22606	130.1	426.728	0.7	0.113	340 kHz	2.25 lbs
32	0.008	0.2032	164.1	538.248	0.53	0.091	430 kHz	1.8 lbs
Metric 2.0	0.00787	0.200	169.39	555.61	0.51	0.088	440 kHz	
33	0.0071	0.18034	206.9	678.632	0.43	0.072	540 kHz	1.3 lbs
Metric 1.8	0.00709	0.180	207.5	680.55	0.43	0.072	540 kHz	
34	0.0063	0.16002	260.9	855.752	0.33	0.056	690 kHz	1.1 lbs
Metric 1.6	0.0063	0.16002	260.9	855.752	0.33	0.056	690 kHz	
35	0.0056	0.14224	329	1079.12	0.27	0.044	870 kHz	0.92 lbs
36	0.005	0.127	414.8	1360	0.21	0.035	1100 kHz	0.72 lbs
Metric 1.25	.00492	0.125	428.2	1404	0.20	0.034	1150 kHz	
37	0.0045	0.1143	523.1	1715	0.17	0.0289	1350 kHz	0.57 lbs
Metric 1.12	.00441	0.112	533.8	1750	0.163	0.0277	1400 kHz	
38	0.004	0.1016	659.6	2163	0.13	0.0228	1750 kHz	0.45 lbs
Metric 1	.00394	0.1000	670.2	2198	0.126	0.0225	1750 kHz	
39	0.0035	0.0889	831.8	2728	0.11	0.0175	2250 kHz	0.36 lbs
40	0.0031	0.07874	1049	3440	0.09	0.0137	2900 kHz	0.29 lbs

http://www.powerstream.com/Wire_Size.htm

Proximity Effect

- Multi-layer winding, magnetic field of one layer affects others
- Further restricts current flow

Design Example

- Start with a specification
- Pick a core
- Calculate required N and g to give required inductance with specified B field at maximum current
- Calculate number of parallel strands to give desired resistance
- Compute performance
- Iterate if needed

Design Example

Start with a specification

Symbol	Value	Description
L	22μH	Inductance
I_{Max}	10A	Peak current
f	100kHz	Operating frequency
R	0.01Ω	Resistance
B_{Max}	0.1T	Maximum DC magnetic field
ΔB_{Max}	0.05T	Maximum AC magnetic field
P	2W	Maximum loss

Two Key Equations

$$N = \frac{LI_{max}}{B_{max}A}$$

$$l_g = \frac{N^2 \mu_0 A}{L}$$

Derivation

$$\phi = BA$$

$$\frac{NI}{R} = BA$$

$$NI = BAR$$

$$NI = BA\frac{N^2}{L}$$

$$LI = BAN$$

$$N = \frac{LI}{BA}$$

Derivation (concluded)

$$R = \frac{l_g}{\mu_0 A}$$

$$l_g = \mu_0 A R$$

$$l_g = \frac{\mu_0 A N^2}{L}$$

Inductor design example

Spec

L	2.20E-05 H	Inductance
Imax	10 A	Max current
Bmax	0.1 T	Max flux density
DBMax	0.05 T	Max change in B field
Rs	0.01 Ohms	Series Resistance
f	1.00E+05 Hz	Operating Frequency

Constants

pi	3.141592654	
mu0	1.26E-06	
Rw	8.40E-02 Ohms/m	24AWG
Aw	2.00E-01 mm^2	24AWG

Core Data

Part	PQ32/30
Ac	1.67E-04 m^2
Lc	7.47E-02 m
Vc	1.25E-05 m^3
OD	27.50 mm
ID	13.50 mm
H	21.30 mm
Aw	1.49E-04 m^2
Lturn	64 mm

http://www.ferroxcube.com/appl/info/HB2009.pdf

Core Loss Data

Pc	80 kW/m^3	Core loss at 0.1T 100kHz
pcb	2.50	Exponent for B field scaling
pcf	1.65	Exponent for f scaling

Turns and Gap Computation

N	13 turns	LI/BA
Lg	1.66 mm	N^2mu0A/L

Windings

Lst	0.85 m	Length of winding
Rst	0.07 Ohms	Resistance per strand
Nst	8 strands	Parallel strands
Ast	2.11E-05 m^2	Area taken by winding
kf	0.14	Fraction of window filled
Rw	0.01 Ohms	Resistance of inductor
Pw	0.89 W	Copper loss power

Meets all specs but not using much of the window

Core Loss

Pc	0.18 W	Core loss power
Pt	1.07 W	Total losses (core plus copper)

Core Selection

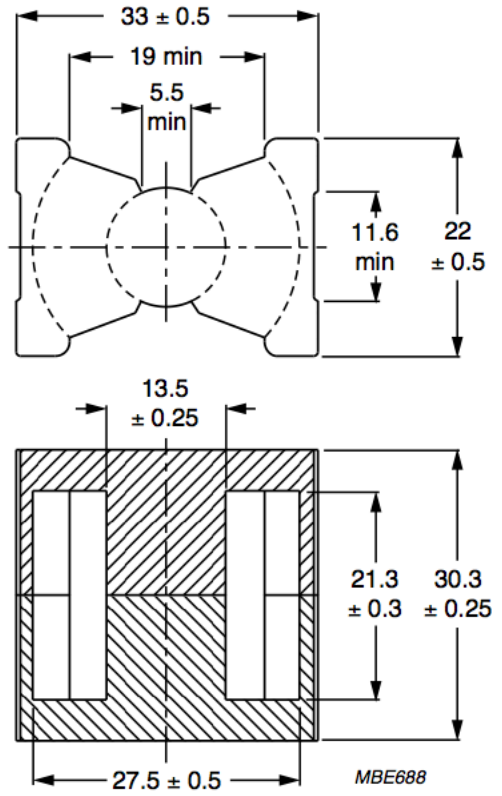
- Key properties of a core are
  - A_c – cross section area of core $\phi = BA$ (how much flux)
  - A_w – window area – $NI = JA_w$ (how much current)
  - V_c – core volume – determines loss
- Pick the core that meets the specification and for which the windings "just fit".
- Extra window area is
  - More cost
  - More volume – bigger is worse
  - More losses – since core losses are proportional to volume

PQ cores and accessories PQ32/30

CORE SETS

Effective core parameters

SYMBOL	PARAMETER	VALUE	UNIT
Σ(I/A)	core factor (C1)	0.447	mm^{-1}
V_e	effective volume	12500	mm^3
l_e	effective length	74.7	mm
A_e	effective area	167	mm^2
A_{min}	minimum area	142	mm^2
m	mass of set	≈ 57	g



Dimensions in mm.

Fig.1 PQ32/30 core set.

Inductor design example

Spec							
L	2.20E-05	2.20E-05	2.20E-05	2.20E-05	2.20E-05	H	Inductance
Imax	10	10	10	10	10	A	Max current
Bmax	0.1	0.1	0.1	0.1	0.1	T	Max flux density
DBMax	0.05	0.05	0.05	0.05	0.05	T	Max change in B field
Rs	0.01	0.01	0.01	0.01	0.01	Ohms	Series Resistance
f	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	Hz	Operating Frequency
Constants							
pi	3.141592654	3.141592654	3.141592654	3.141592654	3.141592654		
mu0	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06		
Rw	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	Ohms/m	24AWG
Aw	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	mm^2	24AWG
Core Loss Data							
Pc	80	80	80	80	80	kW/m^3	Core loss at 0.1T 100kHz
pcb	2.50	2.50	2.50	2.50	2.50		Exponent for B field scaling
pcf	1.65	1.65	1.65	1.65	1.65		Exponent for f scaling
Core Data							
Part	PQ20/16	PQ20/20	PQ26/20	PQ26/25	PQ32/30		
Ac	6.19E-05	6.26E-05	1.21E-04	1.20E-04	1.67E-04	m^2	
Lc	3.76E-02	4.57E-02	4.50E-02	5.43E-02	7.47E-02	m	
Vc	2.33E-06	2.85E-06	5.47E-06	5.82E-06	1.25E-05	m^3	
OD	18.00	18.00	22.00	22.50	27.50	mm	
ID	8.80	8.80	12.00	12.00	13.50	mm	
H	10.30	14.00	11.50	16.10	21.30	mm	
Aw	4.74E-05	6.44E-05	5.75E-05	8.45E-05	1.49E-04	m^2	
Lturn	42	42	53	54	64	mm	
Turns and Gap Computation							
N	36	35	18	18	13	turns	LI/BA
Lg	4.46	4.41	2.29	2.30	1.66	mm	N^2mu0A/L
Windings							
Lst	1.49	1.48	0.97	0.99	0.85	m	Length of winding
Rst	0.13	0.12	0.08	0.08	0.07	Ohms	Resistance per strand
Nst	13	13	9	9	8	strands	Parallel strands
Ast	9.23E-05	9.13E-05	3.28E-05	3.29E-05	2.11E-05	m^2	Area taken by winding
kf	1.95	1.42	0.57	0.39	0.14		Fraction of window filled
Rw	0.01	0.01	0.01	0.01	0.01	Ohms	Resistance of inductor
Pw	0.97	0.95	0.91	0.93	0.89	W	Copper loss power
Core Loss							
Pc	0.03	0.04	0.08	0.08	0.18	W	Core loss power
Pt	1.00	1.00	0.98	1.01	1.07	W	Total losses (core plus copper)

PQ26/20 is lower cost and lower loss

But core loss is lower than copper loss

The Art of Specification

- The specification is wrong. Its specifying L, R, I, B, f, and loss at frequency.
- What you care about is cost and total loss for a given application.
- Need freedom to vary parameters to achieve the real specification.

- Suppose we keep the original spec, but relax the specs on B and R. We don’t care what the B field or series resistance is, we care about total losses.

- We set kf=0.7 and optimize B for minimum loss

Inductor design example

Spec										
L	2.20E-05	2.20E-05	2.20E-05	2.20E-05	2.20E-05	2.20E-05	2.20E-05	2.20E-05	H	Inductance
Imax	10	10	10	10	10	10	10	10	A	Max current
f	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	Hz	Operating Frequency
Optimize to minimize power										
DBmax	0.25	0.25	0.21	0.14	0.12	0.09	0.08	0.05	T	Max delta flux density
Bmax	0.50	0.50	0.42	0.27	0.24	0.17	0.15	0.10	T	Max flux density
Constants										
pi	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14		
mu0	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06		
Rw	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	Ohms/m	24AWG
Aw	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	mm^2	24AWG
Core Loss Data										
Pc	80	80	80	80	80	80	80	80	kW/m^3	Core loss at 0.1T 100kHz
pcb	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50		Exponent for B field scaling
pcf	1.65	1.65	1.65	1.65	1.65	1.65	1.65	1.65		Exponent for f scaling
Core Data										
Part	P14/8	P14/8/I	P18/11	PQ20/16	PQ20/20	PQ26/20	PQ26/25	PQ32/30		
Ac	2.51E-05	2.99E-05	4.33E-05	6.19E-05	6.26E-05	1.21E-04	1.20E-04	1.67E-04	m^2	
Lc	1.98E-02	2.10E-02	2.58E-02	3.76E-02	4.57E-02	4.50E-02	5.43E-02	7.47E-02	m	
Vc	4.95E-07	6.28E-07	1.12E-06	2.33E-06	2.85E-06	5.47E-06	5.82E-06	1.25E-05	m^3	
OD	9.50	11.80	13.40	18.00	18.00	22.00	22.50	27.50	mm	
ID	6.00	5.90	7.60	8.80	8.80	12.00	12.00	13.50	mm	
H	5.60	5.60	7.20	10.30	14.00	11.50	16.10	21.30	mm	
Aw	9.80E-06	1.65E-05	2.09E-05	4.74E-05	6.44E-05	5.75E-05	8.45E-05	1.49E-04	m^2	
Lturn	24	28	33	42	42	53	54	64	mm	
Turns and Gap Computation										
N	18	15	12	13	15	11	12	13	turns	LI/BA
Lg	0.44	0.37	0.36	0.62	0.76	0.79	1.02	1.66	mm	N^2mu0A/L
Windings										
Lst	0.43	0.41	0.40	0.56	0.61	0.57	0.66	0.85	m	Length of winding
Rst	0.04	0.03	0.03	0.05	0.05	0.05	0.06	0.07	Ohms	Resistance per strand
At	3.50E-06	2.94E-06	2.42E-06	2.64E-06	2.92E-06	2.14E-06	2.44E-06	2.64E-06		
kf	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70		Fraction of window filled
Nst	1	3	6	12	15	18	24	39	strands	Parallel strands
Ast	3.50E-06	8.82E-06	1.45E-05	3.17E-05	4.38E-05	3.85E-05	5.86E-05	1.03E-04	m^2	Area taken by winding
Rw	3.58E-02	1.14E-02	5.59E-03	3.89E-03	3.44E-03	2.67E-03	2.31E-03	1.83E-03	Ohms	Resistance of inductor
Pw	3.58	1.14	0.56	0.39	0.34	0.27	0.23	0.18	W	Copper loss power
Core Loss										
Pc	0.39	0.50	0.57	0.39	0.36	0.29	0.23	0.18	W	Core loss power
Pt	3.97	1.64	1.13	0.78	0.70	0.56	0.46	0.36	W	Total losses (core plus copper)

Vary B field to balance core and copper

Compute number of strands to fill 0.7 of window

Can stay under 2W with a much smaller core

Hint: Use “solver” to optimize parameters

What about frequency

- Still not quite the right optimization
- We can allow frequency to vary
- $L \sim k/f$

Inductor design example

Spec										
L	1.10E-05	1.10E-05	1.10E-05	1.10E-05	1.10E-05	1.10E-05	1.10E-05	1.10E-05	H	Inductance
Imax	10	10	10	10	10	10	10	10	A	Max current
f	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	1.00E+05	Hz	Operating Frequency
Optimize to minimize power										
DBmax	0.25	0.25	0.21	0.14	0.12	0.09	0.08	0.05	T	Max delta flux density
Bmax	0.50	0.50	0.42	0.27	0.24	0.17	0.15	0.10	T	Max flux density
Constants										
pi	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14		
mu0	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06		
Rw	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	Ohms/m	24AWG
Aw	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	mm^2	24AWG
Core Loss Data										
Pc	80	80	80	80	80	80	80	80	kW/m^3	Core loss at 0.1T 100kHz
pcb	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50		Exponent for B field scaling
pcf	1.65	1.65	1.65	1.65	1.65	1.65	1.65	1.65		Exponent for f scaling
Core Data										
Part	P14/8	P14/8/I	P18/11	PQ20/16	PQ20/20	PQ26/20	PQ26/25	PQ32/30		
Ac	2.51E-05	2.99E-05	4.33E-05	6.19E-05	6.26E-05	1.21E-04	1.20E-04	1.67E-04	m^2	
Lc	1.98E-02	2.10E-02	2.58E-02	3.76E-02	4.57E-02	4.50E-02	5.43E-02	7.47E-02	m	
Vc	4.95E-07	6.28E-07	1.12E-06	2.33E-06	2.85E-06	5.47E-06	5.82E-06	1.25E-05	m^3	
OD	9.50	11.80	13.40	18.00	18.00	22.00	22.50	27.50	mm	
ID	6.00	5.90	7.60	8.80	8.80	12.00	12.00	13.50	mm	
H	5.60	5.60	7.20	10.30	14.00	11.50	16.10	21.30	mm	
Aw	9.80E-06	1.65E-05	2.09E-05	4.74E-05	6.44E-05	5.75E-05	8.45E-05	1.49E-04	m^2	
Lturn	24	28	33	42	42	53	54	64	mm	
Turns and Gap Computation										
N	9	7	6	7	7	5	6	7	turns	LI/BA
Lg	0.22	0.19	0.18	0.31	0.38	0.39	0.51	0.83	mm	N^2mu0A/L
Windings										
Lst	0.21	0.21	0.20	0.28	0.31	0.28	0.33	0.43	m	Length of winding
Rst	0.02	0.02	0.02	0.02	0.03	0.02	0.03	0.04	Ohms	Resistance per strand
At	1.76E-06	1.48E-06	1.20E-06	1.32E-06	1.46E-06	1.06E-06	1.22E-06	1.32E-06		
kf	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70		Fraction of window filled
Nst	3	7	12	25	30	37	48	79	strands	Parallel strands
Ast	5.28E-06	1.04E-05	1.44E-05	3.30E-05	4.38E-05	3.92E-05	5.86E-05	1.04E-04	m^2	Area taken by winding
Rw	6.00E-03	2.47E-03	1.39E-03	9.34E-04	8.60E-04	6.43E-04	5.79E-04	4.52E-04	Ohms	Resistance of inductor
Pw	0.60	0.25	0.14	0.09	0.09	0.06	0.06	0.05	W	Copper loss power
Core Loss										
Pc	0.39	0.50	0.57	0.39	0.36	0.29	0.23	0.18	W	Core loss power
Pt	0.99	0.74	0.71	0.49	0.45	0.36	0.28	0.22	W	Total losses (core plus copper)

Even lower power by raising frequency

What about switching losses

- A real optimization needs to optimize frequency for switching losses as well as core and copper losses.

- $P_{sw} = fE_{sw}$

Inductor design example

Spec									
L	1.10E-05	1.10E-05	1.10E-05	1.10E-05	1.10E-05	1.10E-05	1.10E-05	1.10E-05 H	Inductance
Imax	10	10	10	10	10	10	10	10 A	Max current
f	2.00E+05	2.00E+05	2.00E+05	2.00E+05	2.00E+05	2.00E+05	2.00E+05	2.00E+05 Hz	Operating Frequency
Optimize to minimize power									
DBmax	0.25	0.25	0.21	0.14	0.12	0.09	0.08	0.05 T	Max delta flux density
Bmax	0.50	0.50	0.42	0.27	0.24	0.17	0.15	0.10 T	Max flux density
Constants									
pi	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14	
mu0	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	1.26E-06	
Rw	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02	8.40E-02 Ohms/m	24AWG
Aw	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01	2.00E-01 mm^2	24AWG
Core Loss Data									
Pc	80	80	80	80	80	80	80	80 kW/m^3	Core loss at 0.1T 100kHz
pcb	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	Exponent for B field scaling
pcf	1.65	1.65	1.65	1.65	1.65	1.65	1.65	1.65	Exponent for f scaling
Core Data									
Part	P14/8	P14/8/I	P18/11	PQ20/16	PQ20/20	PQ26/20	PQ26/25	PQ32/30	
Ac	2.51E-05	2.99E-05	4.33E-05	6.19E-05	6.26E-05	1.21E-04	1.20E-04	1.67E-04 m^2	
Lc	1.98E-02	2.10E-02	2.58E-02	3.76E-02	4.57E-02	4.50E-02	5.43E-02	7.47E-02 m	
Vc	4.95E-07	6.28E-07	1.12E-06	2.33E-06	2.85E-06	5.47E-06	5.82E-06	1.25E-05 m^3	
OD	9.50	11.80	13.40	18.00	18.00	22.00	22.50	27.50 mm	
ID	6.00	5.90	7.60	8.80	8.80	12.00	12.00	13.50 mm	
H	5.60	5.60	7.20	10.30	14.00	11.50	16.10	21.30 mm	
Aw	9.80E-06	1.65E-05	2.09E-05	4.74E-05	6.44E-05	5.75E-05	8.45E-05	1.49E-04 m^2	
Lturn	24	28	33	42	42	53	54	64 mm	
Turns and Gap Computation									
N	9	7	6	7	7	5	6	7 turns	LI/BA
Lg	0.22	0.19	0.18	0.31	0.38	0.39	0.51	0.83 mm	N^2mu0A/L
Windings									
Lst	0.21	0.21	0.20	0.28	0.31	0.28	0.33	0.43 m	Length of winding
Rst	0.02	0.02	0.02	0.02	0.03	0.02	0.03	0.04 Ohms	Resistance per strand
At	1.76E-06	1.48E-06	1.20E-06	1.32E-06	1.46E-06	1.06E-06	1.22E-06	1.32E-06	
kf	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70	Fraction of window filled
Nst	3	7	12	25	30	37	48	79 strands	Parallel strands
Ast	5.28E-06	1.04E-05	1.44E-05	3.30E-05	4.38E-05	3.92E-05	5.86E-05	1.04E-04 m^2	Area taken by winding
Rw	6.00E-03	2.47E-03	1.39E-03	9.34E-04	8.60E-04	6.43E-04	5.79E-04	4.52E-04 Ohms	Resistance of inductor
Pw	0.60	0.25	0.14	0.09	0.09	0.06	0.06	0.05 W	Copper loss power
Core Loss									
Pc	1.23	1.56	1.80	1.24	1.13	0.91	0.71	0.55 W	Core loss power
Switching Loss									
Esw	2.00E-05	2.00E-05	2.00E-05	2.00E-05	2.00E-05	2.00E-05	2.00E-05	2.00E-05	
Psw	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	
Pt	5.83	5.81	5.94	5.33	5.21	4.98	4.77	4.60 W	Total losses (core plus copper plua awitching)

But if not soft switched, need to optimize including switching losses

Area Product

Product of core area and window area A_c x A_p can be used to select a core

$$A_C = \frac{LI_P}{NB_P}$$

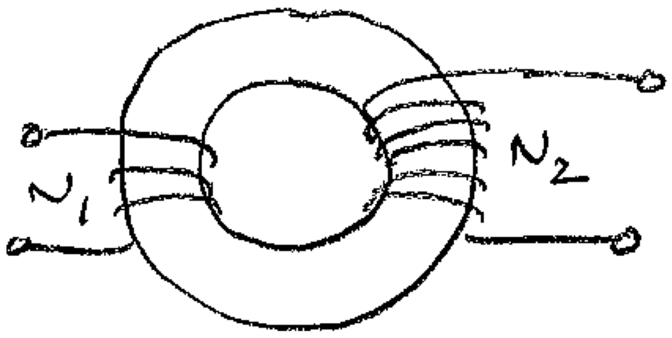
$$A_W = \frac{NI_{RMS}}{k_W J}$$

$$A_P = A_C A_W = \frac{LI_P I_{RMS}}{B_P k_W J}$$

Transformers

Transformers

- Suppose I have two windings on a core
  - One with N_1 turns, and one with N_2 turns



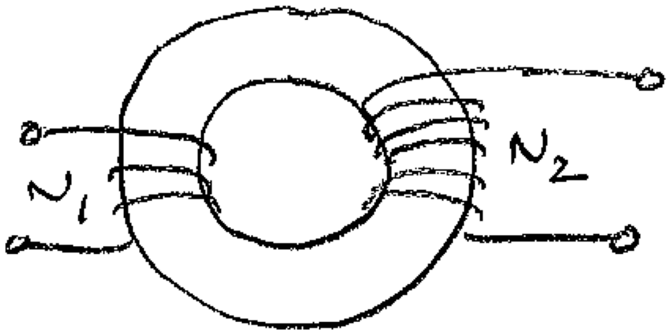
Transformers

- Suppose I have two windings on a core
  - One with N_1 turns, and one with N_2 turns
  - We know

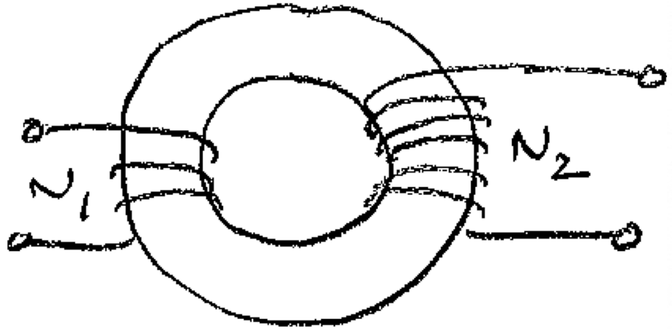
$$V_1 = N_1 \frac{d\phi}{dt}$$

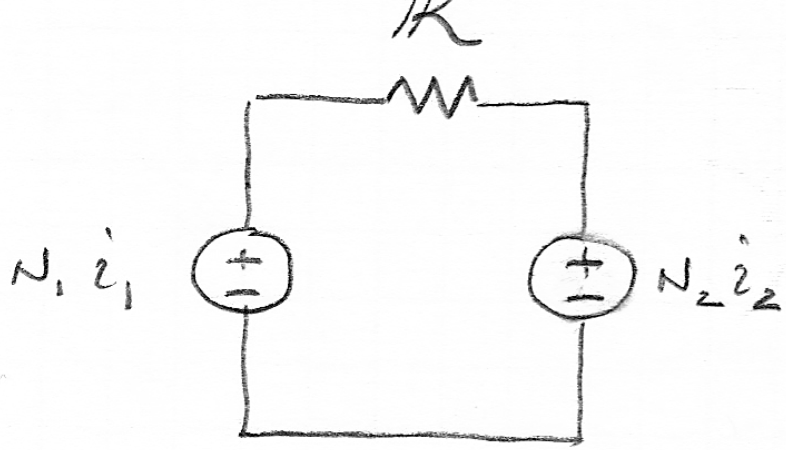
$$V_2 = N_2 \frac{d\phi}{dt}$$

$$V_2 = \frac{N_2}{N_1} V_1$$



Equivalent Magnetic Circuit



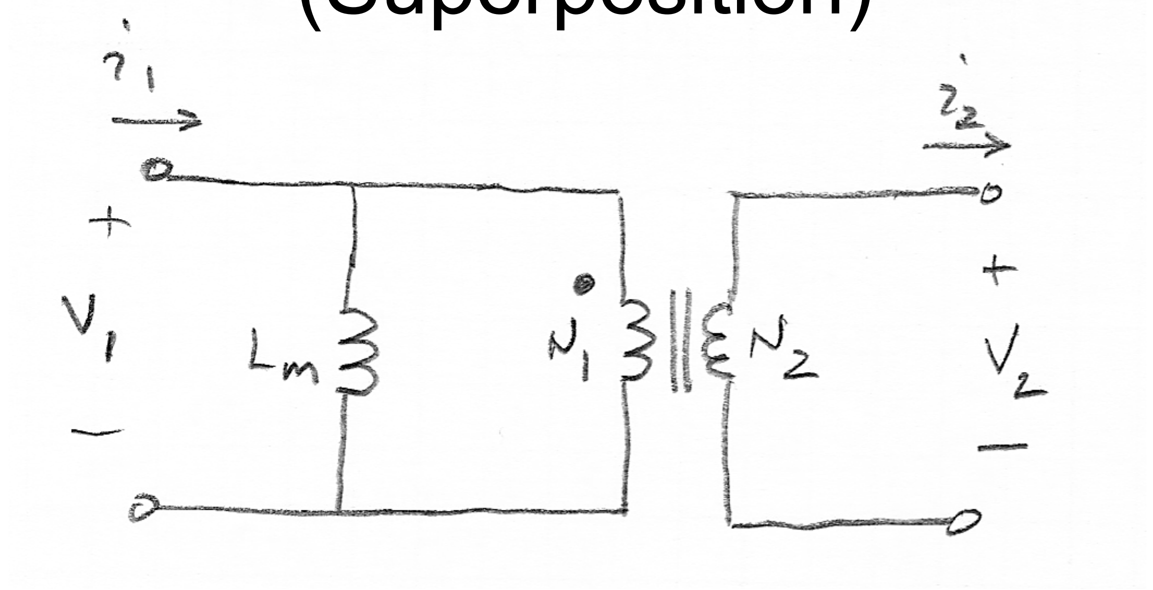


If reluctance were zero we would have

$$N_1 i_1 = N_2 i_2$$

$$i_2 = \left(\frac{N_1}{N_2} \right) i_1$$

Model as Ideal Transformer in Parallel with Magnetizing Inductance (Superposition)



$$L_m = \frac{N_1^2}{\mathcal{R}}$$

Transformer Saturation

- Transformer can pass high current from primary to secondary without saturating
- Saturation is due to magnetizing inductance
- Magnetizing flux is proportional to VT (volt-seconds)
- Choose N (number of turns) for required VT at specified max B
- VT must be *balanced* (sum to zero each cycle).

$$V = N\frac{d\phi}{dt}$$

$$VT = N\phi$$

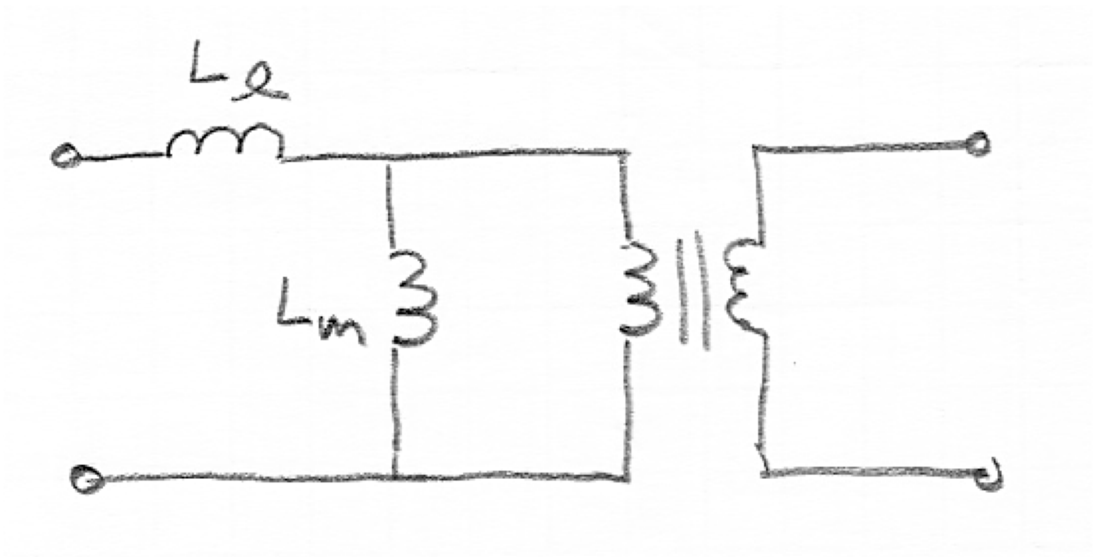
$$VT = NBA$$

$$N = \frac{VT}{BA}$$

Leakage Inductance

- Flux that is not linked by both windings results in *leakage inductance*
  - Typically 1-5% of total inductance
  - Larger for gapped cores (due to fringing)
  - Can be reduced by ‘interleaving’ windings
- Can be reflected to primary or secondary
  - Just one leakage inductance

Model with Leakage Inductance



Transformer Design Procedure

- (Does not apply to flyback transformer)
- Repeat the following steps
  1. Select core based on power to be handled
  2. Compute Np = VT/BA
  3. Compute winding and core loss
     - Distribute window area to windings to balance current density
  4. Adjust B to balance losses
  5. Repeat with different core if required

Transformer Specification

DB/B	2		Full bridge
Nratio	2:1		Turns Ratio
Vp	170	V	Primary Voltage
Tp	5	uS	Primary Pulse Width
Imax	10	A	Primary Current
F	100	kHz	Frequency
P	<20	W	Total Losses

First iteration

Transformer design example

Spec			
Nr	2		Turns ratio - primary to secondary
V	170	Volts	Primary voltage
t	5.00E-06	s	Primary pulse width
Imax	10	Amps	Max current
Bmax	0.05	T	Max flux density
DBMax	0.1	T	Max AC flux density
f	1.00E+05	Hz	Operating Frequency
Constants			
pi	3.1416		
mu0	1.26E-06		
Rw	8.40E-02	Ohms/m	24AWG
Aw	2.00E-01	mm^2	24AWG
Core Data			
Part	E41/17/12		
Ac	1.49E-04	m^2	
Lc	7.70E-02	m	
Vc	1.15E-05	m^3	
AL	5.37E-06	H	
OD	28.6	mm	
ID	12.5	m^2	
H	20.8	mm	
W	12	mm	
Aw	1.68E+02	mm^2	
Lturn	8.21E-02	m	
Core Loss Data			
Pc	80	kW/m^3	Core loss at 0.1T 100kHz
pcb	2.50		Exponent for B field scaling
pcf	1.65		Exponent for f scaling
Derived			
V*t	8.50E-04	V-s	
Np	114	Turns	Primary turns
Ns	57	Turns	Secondary turns
L	6.99E-02	H	Primary magnetizing inductance
Windings			
Kw	0.7		Fill factor
Kwp	0.35		Window fraction for primary
Awp	5.88E+01	mm^2	Area for primary
Nw	294	Turns	Number of wires that will fit
Ns	2	Strands	Parallel strands per turn
Ls	9.37E+00	m	Length of each strand
Rp	3.93E-01	Ohms	
Pp	39.34	W	
Pw	78.68	W	
Core Loss			
Pc	0.92	W	Core loss
Total Loss			
Pt	79.60		

Copper loss too high – and not balanced with core loss

Optimize B field

Transformer design example

Spec			
Nr	2		Turns ratio - primary to secondary
V	170	Volts	Primary voltage
t	5.00E-06	s	Primary pulse width
Imax	10	Amps	Max current
f	1.00E+05	Hz	Operating Frequency
Constants			
pi	3.1416		
mu0	1.26E-06		
Rw	8.40E-02	Ohms/m	24AWG
Aw	2.00E-01	mm^2	24AWG
Core Data			
Part	E41/17/12		
Ac	1.49E-04	m^2	
Lc	7.70E-02	m	
Vc	1.15E-05	m^3	
AL	5.37E-06	H	
OD	28.6	mm	
ID	12.5	m^2	
H	20.8	mm	
W	12	mm	
Aw	1.68E+02	mm^2	
Lturn	8.21E-02	m	
Core Loss Data			
Pc	80	kW/m^3	Core loss at 0.1T 100kHz
pcb	2.50		Exponent for B field scaling
pcf	1.65		Exponent for f scaling
Derived			
V*t	8.50E-04	V-s	
Bmax	0.12	T	Max flux density
DBMax	0.24	T	Max AC flux density
Np	48	Turns	Primary turns
Ns	24	Turns	Secondary turns
L	1.21E-02	H	Primary magnetizing inductance
Windings			
Kw	0.7		Fill factor
Kwp	0.35		Window fraction for primary
Awp	5.88E+01	mm^2	Area for primary
Nw	294	Turns	Number of wires that will fit
Ns	6	Strands	Parallel strands per turn
Ls	3.90E+00	m	Length of each strand
Rp	5.46E-02	Ohms	
Pp	5.46	W	
Pw	10.93	W	
Core Loss			
Pc	8.21	W	Core loss
Total Loss			
Pt	19.14		

L_M is 12mH

Copper and core loss now balanced and total loss within spec

Check core selection

Transformer design example

Spec						
Nr	2	2	2	2		Turns ratio - primary to secondary
V	170	170	170	170	Volts	Primary voltage
t	5.00E-06	5.00E-06	5.00E-06	5.00E-06	s	Primary pulse width
Imax	10	10	10	10	Amps	Max current
f	1.00E+05	1.00E+05	1.00E+05	1.00E+05	Hz	Operating Frequency
Constants						
pi	3.1416	3.1416	3.1416	3.1416		
mu0	1.26E-06	1.26E-06	1.26E-06	1.26E-06		
Rw	8.40E-02	8.40E-02	8.40E-02	8.40E-02	Ohms/m	24AWG
Aw	2.00E-01	2.00E-01	2.00E-01	2.00E-01	mm^2	24AWG
Core Data						
Part	E35/18/12	E36/21/12	E41/17/12	42/21/15		
Ac	1.00E-04	1.26E-04	1.49E-04	1.78E-04	m^2	
Lc	8.07E-02	9.60E-02	7.70E-02	9.70E-02	m	
Vc	8.07E-06	1.22E-05	1.15E-05	1.73E-05	m^3	
AL	2.98E-06	2.65E-06	5.37E-06	5.30E-06	H	
OD	24.5	24.5	28.6	29.5	mm	
ID	10.0	10.2	12.5	6.0	m^2	
H	25.0	31.5	20.8	29.6	mm	
W	12	12	12	15	mm	
Aw	1.81E+02	2.25E+02	1.68E+02	3.48E+02	mm^2	
Lturn	6.90E-02	6.94E-02	8.21E-02	7.10E-02	m	
Core Loss Data						
Pc	80	80	80	80	kW/m^3	Core loss at 0.1T 100kHz
pcb	2.50	2.50	2.50	2.50		Exponent for B field scaling
pcf	1.65	1.65	1.65	1.65		Exponent for f scaling
Derived						
V*t	8.50E-04	8.50E-04	8.50E-04	8.50E-04	V-s	
Bmax	0.14	0.12	0.12	0.09	T	Max flux density
DBMax	0.28	0.24	0.25	0.18	T	Max AC flux density
Np	62	56	46	53	Turns	Primary turns
Ns	31	28	23	27	Turns	Secondary turns
L	1.13E-02	8.31E-03	1.14E-02	1.50E-02	H	Primary magnetizing inductance
Windings						
Kw	0.7	0.7	0.7	0.7		Fill factor
Kwp	0.35	0.35	0.35	0.35		Window fraction for primary
Awp	6.34E+01	7.88E+01	5.88E+01	1.22E+02	mm^2	Area for primary
Nw	317	394	294	609	Turns	Number of wires that will fit
Ns	5	7	6	11	Strands	Parallel strands per turn
Ls	4.24E+00	3.89E+00	3.78E+00	3.77E+00	m	Length of each strand
Rp	7.13E-02	4.66E-02	5.30E-02	2.88E-02	Ohms	
Pp	7.13	4.66	5.30	2.88	W	
Pw	14.26	9.33	10.60	5.76	W	
Core Loss						
Pc	8.20	8.76	8.87	5.99	W	Core loss
Total Loss						
Pt	22.46	18.09	19.46	11.75	W	71.77

E36/21 core has lower loss than E41/17 core due to larger window area

Magnetics Summary

- Inductors and transformers are key components of Green Electronic systems
- Governed by Lentz's and Ampere's Laws

$$V = N\frac{d\phi}{dt} \qquad \phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{NI}{\mathcal{R}} \qquad L = \frac{N^2}{\mathcal{R}}$$

- Gap cores to give required reluctance while avoiding saturation
- Magnetic materials
  - Characterized by μ, B_{sat}, and losses
  - Use ferrites for high-frequency applications, Si Steel for low
- Wires have resistivity and suffer from skin effect
- Inductor design

$$N = \frac{LI_{max}}{B_{max}A} \qquad l_g = \frac{N^2\mu_0 A}{L}$$

  - Iterate on core selection and parameters to minimize losses
- Transformers
  - Have magnetizing inductance and leakage inductance
  - Volt-seconds must be balanced over each cycle

$$N = \frac{VT}{BA}$$

No	Date	Topic	HW out	HW in	Lab out	Lab ck	Lab	HW
1	9/25/17	Intro (basic converters)	1		1		Intro to ST32F3	Periodic Steady State
2	9/27/17	Embedded Prog/Power Elect.						
3	10/2/17	Power Electronics - 1 (switches)	2	1	2	1	AC Energy Meter	Power Devices
4	10/4/17	Power Electronics - 2 (circuits)						
5	10/9/17	Photovoltaics	3	2	3	2	PV MPPT	Motor control Matlab
6	10/11/17	Feedback Control						
7	10/16/17	Electric Motors	4	3	4	3	Motor control - Lab/	Feedback
8	10/18/17	Isolated Converters						
YAH 9	10/23/17	Solar Day	5/PP	4	5	4	PS	Isolated Converters
10	10/25/17	Magnetics						
11	10/30/17	Soft Switching	6	5/PP	6	5	Magnetics	Magnetics and Inverters
12	11/1/17	Project Discussions						
13	11/6/17	Inverters, Grid, PF, and Batteries		6	P	6	Project	
14	11/8/17	Thermal & EMI						
15	11/13/17	Quiz Review				C1		
16	11/15/17	Grounding, and Debugging						
Q	11/15/17	Quiz - in the evening						
	11/20/17	Thanksgiving Break				C2		
	11/22/17	Thanksgiving Break						
17	11/27/17	Guest Lecture						
18	11/29/17	Martin Fornage - Enphase				C3		
19	12/4/17	Colin Campbell - Tesla						
20	12/6/17	No Class						
	TBD	Project presentations			P			
	12/15/17	Project webpage due						