EE152 F13 Midterm 2

Name: (please print)___SOLUTION____________________________

In recognition of and in the spirit of the Stanford University Honor Code, I certify that I will neither give nor receive unpermitted aid on this exam.

Signature:____

You may not, collaborate in any manner on this exam. This exam is open notes, open book. You have 90 minutes to complete the exam. Please do all of your work on the exam itself. Attach any additional pages as necessary.

Before starting, please check to make sure that you have all 8 pages.

1		24
2		18
3		20
4		15
5		11
6		12
Total		100

Problem 1: Inductor [24 Points – 4 points each]

The relevant properties for a Ferroxcube E38/8/25 core are shown in the table below. In the 3C95 material this core has a μ_r=2060 and A_L=9600nH. The window area is A_W=100mm^2, and the length of the average turn is L_{turn} = 90mm.

Effective core parameters of a set of E cores

SYMBOL	PARAMETER	VALUE	UNIT
$\Sigma(l/A)$	core factor (C1)	0.272	mm^{-1}
V_e	effective volume	10200	mm^3
l_e	effective length	52.4	mm
A_e	effective area	194	mm^2
A_{min}	minimum area	194	mm^2
m	mass of core half	≈25	g

Suppose you have an inductor wound on this core with 10 turns of 24AWG wire.

(a) What is the inductance of this 10-turn inductor?

$L = N^2 A_L = \boxed{960\,\mu\text{H}}$

(b) At what current will this 10-turn inductor reach saturation, defined as B = 0.5T?

$I_{max} = B_{max} A_{eff} N / L = \boxed{1.01\text{ A}}$

(c) To double the inductance would require roughly how many turns?

14 (4 more turns) since L grows as N^2

(d) Now consider the same 10-turn inductor on the E38/8/25 core but with a 1mm air gap. You may assume all of the reluctance of the magnetic path is in the gap and ignore fringing. What is the inductance of this inductor?

$L = N^2/\mathcal{R}$, where $\mathcal{R} = l_g/\mu_0 A_{eff}$ *L = 24.4μH*

(e) At what current does the inductor of part (d) saturate (reach B=0.5T)?

$I_{max} = B_{max} A_{eff} N / L = \boxed{39.8\text{ A}}$

(f) To double the inductance of the inductor of part (d) (keeping N=10) would require roughly what gap length?

L grows as the inverse of gap length so l_g = 0.5mm

Problem 2: Transformer [18 Points – 6 points each]

Consider a transformer created by taking the inductor from Problem 1 (part a – without the gap) and add a secondary winding with 20 turns of 24AWG wire.

(a) Starting from B=0, how many volt-seconds can be applied to the primary of this transformer before reaching saturation, B=0.5T?

$$VT = NB_{max}A_{eff} = \boxed{0.97\,\text{Vms}}$$

(b) With a fill factor of $k_w = 0.64$, our core's window can hold 320 strands of 24AWG wire. Devoting half of the window to the primary gives 160 strands, or 16 strands per turn for N=10. At 90mm per turn, this gives a primary resistance of 4.7mΩ. At 50A this gives a copper loss in the primary of 11.7W.

If we hold copper loss at 11.7W and the fill factor at 0.64 but cut the number of turns in the primary and secondary in half, i.e., use N_1=5 turns for the primary and N_2=10 turns for the secondary, how much current can our transformer now carry?

Halving the turns doubles the number of parallel strands and halves the length of each strand. Thus it quarters the resistance. Since $P = I^2R$, the 5 turn transformer can handle twice the current or 100A.

(c) For the transformer of part (b) with N_1=5 and N_2=10, how many volt-seconds can be applied to the primary before reaching saturation at B=0.5T.

$$VT = NB_{max}A_{eff} = \boxed{0.485\,\text{Vms}}$$

Problem 3: Isolated Converters [20 points – 10 points each]

Consider a flyback converter built with the transformer of Problem 2 but with a 1mm gap in the core, i.e., an E38/8/25 3C95 core with l_g=1mm, N_1=10, and N_2=20. Assume the converter is operating in continuous conduction mode (CCM) and the input voltage is 10V.

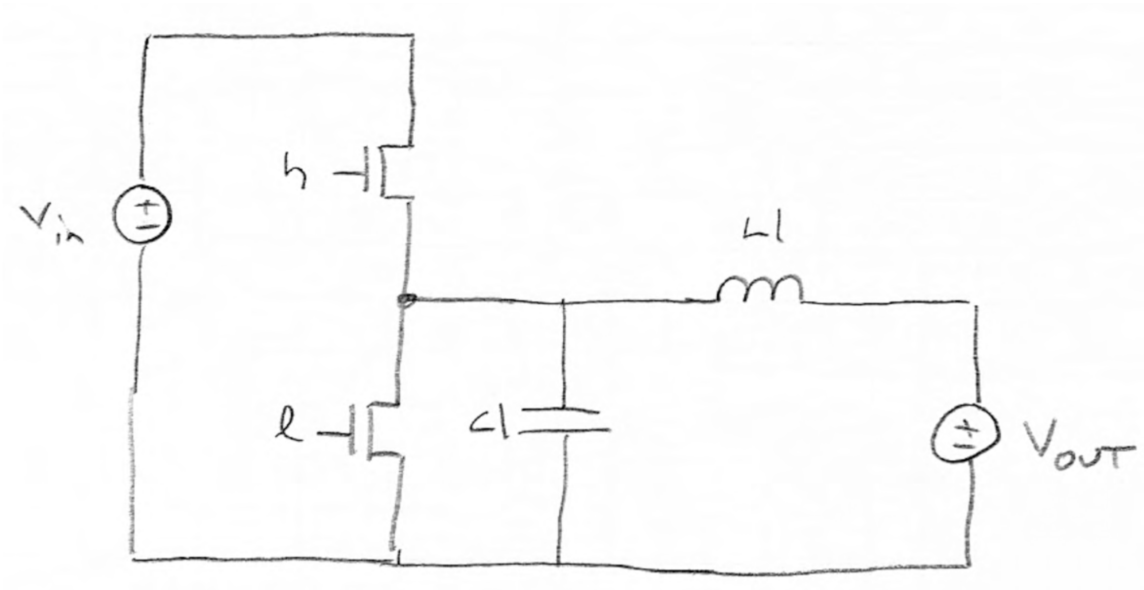
(a) If the duty factor of the primary-side switch is 0.4, what is the average output voltage in the periodic steady state?

$V_{out} = \frac{D}{1-D}\frac{N_2}{N_1}V_{in} = \boxed{13.3\,\mathrm{V}}$

(b) What is the lowest frequency that this converter can be operated at while keeping the change in B field over a cycle to no more than ΔB=0.3T.

$VT = V_{in}D/f_{sw} = N\Delta B A_{eff} \rightarrow f_{sw} = V_{in}D/N\Delta B A_{eff} = \boxed{6.87\,\mathrm{kHz}}$

Problem 4: Soft Switching [15 Points, 5 Points Each]



Consider the quasi-square wave soft-switched buck converter shown above with V_{in}=100V, V_{out}=12V, C_1=200nF (includes parasitic drain capacitance), and L_1=20μH.

(a) What is the minimum reverse current that must be reached in L_1 to ensure that the high-side switch can turn-on with zero voltage across it (ZVS)?

Conservation of energy: $\frac{1}{2}LI_0^2 = \frac{1}{2}C_1V_{C1,f}^2 \rightarrow \boxed{I_0 = 10\text{ A}}$

(b) To minimize dissipation during startup – when the converter is hard switching – a switch is placed in series with C_1 and is open during startup. Once the converter reaches steady state, this switch should be closed during which part of the switching cycle (i.e., when does this switch close relative to the openings and closings of the high-side and low-side switches)?

When the low-side switch l is closed.

(c) If the low-side switch turns off after having reached only half of the current you calculated in part (a) how high a voltage does the left side of the inductor reach before starting to fall?

Half the current gives half the voltage – 50V

Problem 5: Power Factor [11 points]

Suppose a device has a current waveform that is a square wave that is in phase with the voltage waveform – i.e., any time the AC voltage is positive the current is +1A. Any time the voltage is negative, the current is -1A. What is the power factor of this device. Hint1, the Fourier series for a square wave is:

$$f(x) = \frac{4}{\pi} \sum_{n\ odd} \frac{1}{n} \sin(nx)$$

Hint2: You need not use the Fourier series to solve this problem.

We are given: $V(\phi) = V_p \sin\phi$, $I(\phi) = \sin\phi / |\sin\phi| = \frac{4}{\pi} \sum_{i \in \text{Odd}} \frac{1}{n} \sin n\phi$

$$V_{rms} = V_p/\sqrt{2}, \qquad I_{rms} = 1 \qquad \rightarrow \qquad P_{apparent} = V_{rms} I_{rms} = V_p/\sqrt{2}$$

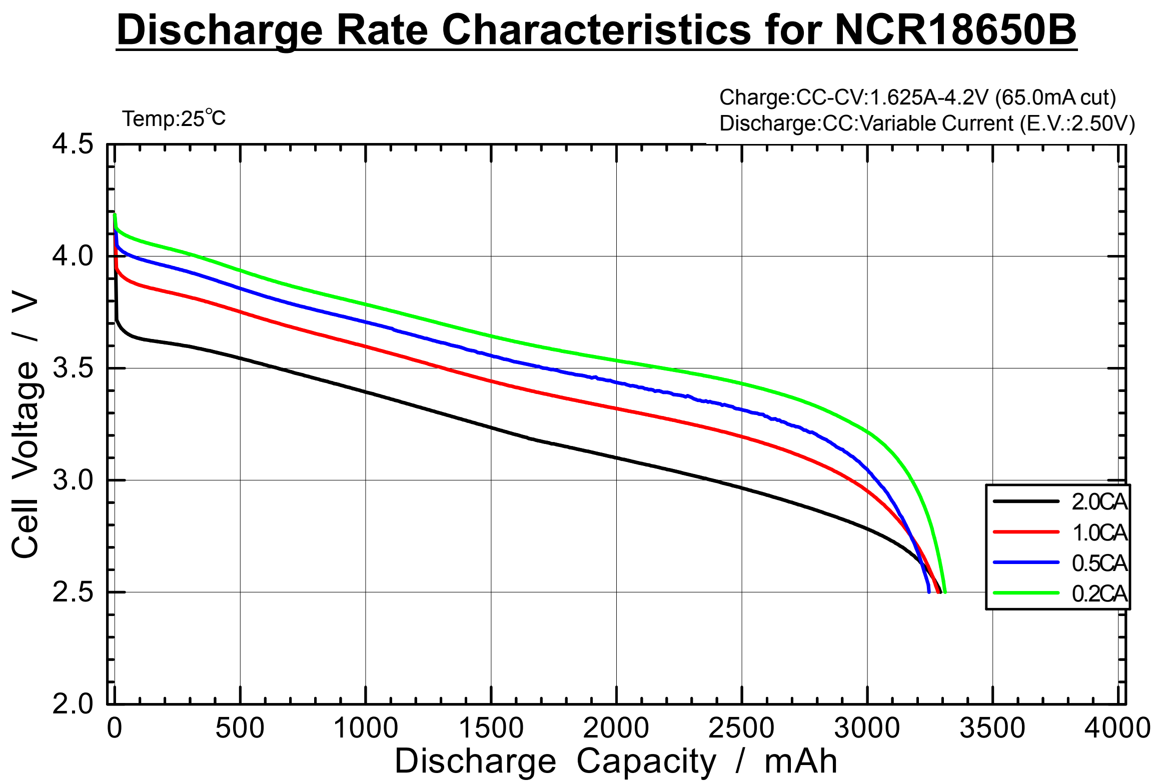
$$\langle P \rangle = \frac{1}{2\pi} \int_0^{2\pi} V(\phi) I(\phi)\, \mathrm{d}\phi = \frac{V_p}{2\pi} \frac{4}{\pi} \int_0^{2\pi} \sin\phi \sum_{i \in \text{Odd}} \frac{1}{n} \sin n\phi\, \mathrm{d}\phi$$

We know $\int_0^{2\pi} \sin n\phi \sin m\phi = \pi \delta_{nm}$, so the above integral evaluates to π.

$$\langle P \rangle = \frac{V_p}{2\pi} \frac{4}{\pi} \cdot \pi = 2V_p/\pi$$

$$k = P_{actual}/P_{apparent} = \frac{V_p \cdot 2/\pi}{V_p/\sqrt{2}} = \boxed{0.900}$$

Problem 6: Batteries [12 Points]



Suppose the open circuit voltage for the NCR18650 battery is a straight line starting at 4.2V and 0mAh and running parallel to the lines shown in the figure above out to 3200mAh. Write an expression for the fraction of charge remaining on the battery as a function of open circuit voltage.

We have two data points: $(0\,\text{mAh}, 4.2\,\text{V})$ and $(3200\,\text{mAh}, 3.3\,\text{V})$

We can rewrite them in terms of state of charge instead of depth of discharge: $(3200\,\text{mAh}, 4.2\,\text{V})$ and $(0\,\text{mAh}, 3.3\,\text{V})$

$$\boxed{\text{SoC} = \frac{1}{0.9\,\text{V}}(V_{oc} - 3.3\,\text{V})}$$