EE152 F14 Midterm Solutions

Name: (please print)________________________________

In recognition of and in the spirit of the Stanford University Honor Code, I certify that I will neither give nor receive unpermitted aid on this exam.

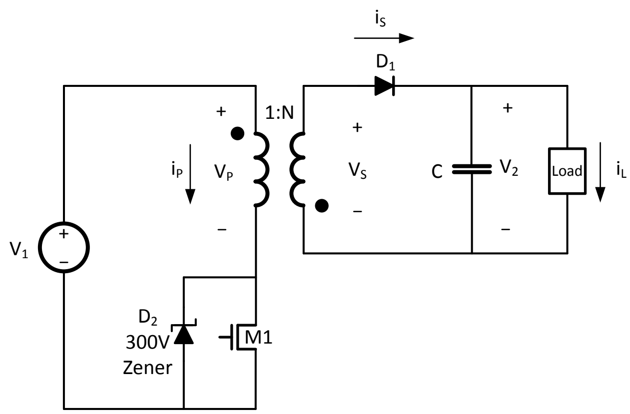
Signature:____

You may not, collaborate in any manner on this exam. This exam is open notes limited to one 8.5 x 11 inch sheet of notes. You have 120 minutes to complete the exam. Please do all of your work on the exam itself. Attach any additional pages as necessary.

Before starting, please check to make sure that you have all 9 pages.

1		**20**
2		**15**
3		**15**
4		**15**
5		**15**
6		**20**
7		**20**
Total		**120**

Problem 1: Periodic Steady State Analysis [20 Points]



Consider the flyback converter shown in the Figure above operating with a switching cycle time of t_{cy} = 4μs. The Zener diode, D_2 prevents the voltage across FET M_1 from exceeding 300V.

(a) [4 Points] Ignoring leakage inductance, what is the relationship between output voltage V_2 and input voltage V_1 as a function of the duty factor D of the FET M_1 in the periodic steady state.

To get volt-second balance across magnetizing inductance

$$\frac{V_2}{V_1} = N\left(\frac{D}{1-D}\right)$$

(b) [8 Points, 4 Points Each] Ignoring leakage inductance, write expressions for the change in magnetizing inductor current ΔI and the change in capacitor voltage ΔV over one cycle when not in the periodic steady state.

Referenced to the primary side, the magnetizing inductance sees V_1 for Dt_{cy} and $-V_2/N$ for $(1-D)t_{cy}$ so we have.

$$\Delta I = \left(\frac{t_{cy}}{L}\right)\left(DV_1 - (1-D)\frac{V_2}{N}\right)$$

The magnetizing current is scaled down by N and only applied to the capacitor during the time the diode is conducting. The load current is applied to the capacitor all the time so we have

$$\Delta V = \left(\frac{t_{cy}}{C}\right)\left(\left(\frac{1-D}{N}\right)I_M - I_L\right)$$

(c) [8 Points] Consider a leakage inductance of 1uH. Further assume V_1=V_2=100V, i_L=5A, and the transformer has a winding ratio of 1:1. Recall that the Zener diode clamps the drain of M1 at 300V. Assume the magnetizing inductance is large enough that it can be considered to be a current source. Under these conditions what is the duty factor D in the periodic steady state.

During the turn-on transient, the flyback converter delays the application of V_1 to the primary until the leakage inductance matches the magnetizing current. During this period there is 200V across L_L: 100V from V_1 plus 100V from V_2 reflected through the transformer. Hence the commutation time is:

$$t_{com-on} = \frac{L_L I_M}{V_1 + V_2 / M} = \frac{(1\mu H)(5A)}{200V} = 25ns$$

During turn-off there is no delay in applying the reflection of V2 across the magnetizing inductance. Therefore the turn-off commutation time does not affect duty factor.

From part (a) we know that the duty factor without commutation delay would be 50% for an on-time of t_a = 2us. To allow for the turn-on delay we need to increase this to 2.025us giving a duty factor of 2.025/4 = 50.6%

Problem 2: Motors [15 Points]

Suppose you have a four-phase brushless permanent magnet motor with phases 0, 1, 2, and 3. The stator winding for phase *i* is centered on an angle of 90*i* degrees. One terminal of each stator winding is connected to the motor controller. The other terminal of each stator winding is connected to a common point. You have perfect knowledge of the rotor angle θ (θ = 0 corresponds to the north pole being at 0 degrees). The motor has a constant K_E = 1V-s/rad and a series resistance of 0.01Ω.

To save power you decide that after starting you will disconnect phases 1 and 3 and drive only phases 0 and 2.

(a) [4 Points] What voltage should you drive on phase 0 and what voltage should you drive on phase 2 (as a function of θ and ω) to deliver a torque of 1N-m averaged over one revolution?

We know we need to drive phase 0 with Xsin(θ) and phase 2 with -Xsin(θ). To solve for X we observe that the torque from phase 0 is

$$\tau_0 = \frac{K_M}{R}\left(X - K_M\omega\right)\sin^2\theta$$

Averaged over the cycle, $\sin^2\theta=0.5$, so we have

$$X = \frac{R}{K_M} + K_M\omega$$

So the voltages that give the desired torque are

$$V_0 = \left(\frac{R}{K_M} + K_M\omega\right)\sin\theta$$

$$V_2 = -\left(\frac{R}{K_M} + K_M\omega\right)\sin\theta$$

(b) [4 Points] What is the instantaneous torque as a function of θ?

From (a) we know

$$\tau = 2\tau_0 = 2\sin^2\theta = 1 - \cos 2\theta$$

(c) [4 Points] Now suppose you choose to drive all four phases, what voltage should you drive on each phase (as a function of θ and ω) to deliver a torque of 1N-m averaged over one revolution?

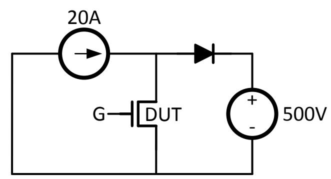
Here we want each phase to contribute 0.25N-m so the value of X above is halved and each phase drives

$$V_i = \left(\frac{0.5R}{K_M} + K_M\omega\right)\sin\left(\theta - \phi_i\right)$$

(d) [3 Points] For the four-phase drive of part c, what is the instantaneous torque as a function of θ?

$$\tau = \sin^2\theta + \cos^2\theta = 1$$

Problem 3: FET Losses [15 points]



Consider the boost converter shown above operating in the periodic steady state with a 200kHz switching frequency f_{cy}. Suppose the MOSFET has an Ron of 100mΩ and switches with a linear current ramp of 1A/ns for both turn-on and turn-off. Assume that the capacitance on the drain of the MOSFET is negligible and that duty factor D=0.5. Also assume that the diode, inductor, and capacitor are ideal and that ripple current is negligible. Compute the switching loss and conduction loss of this converter.

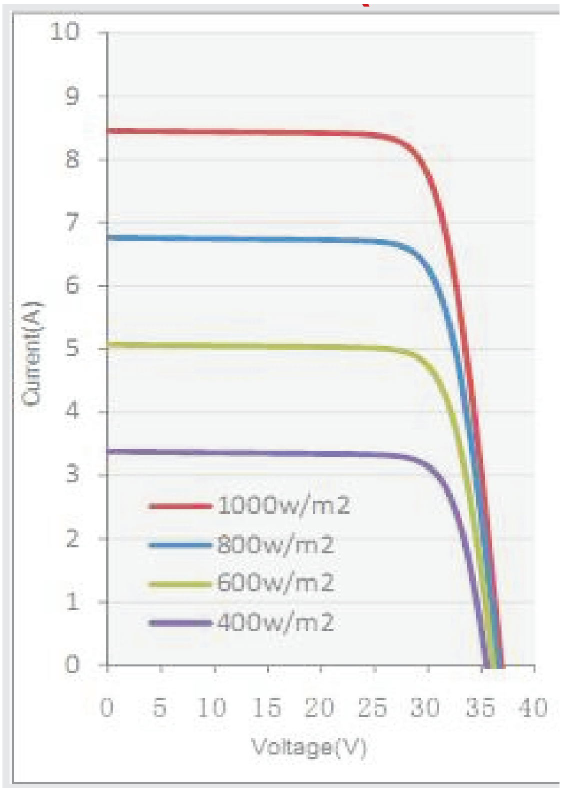
The MOSFET is only conducting when it is on so:

$$P_{conduction} = D * I^2 * R = (.5)(20A)^2(100m\Omega) = 20W$$

For switching losses, the voltage will remain at 500V until the entire current is drawn through the MOSFET. With a slew of 1A/ns, the switch takes 20ns. The voltage would then fall instantly meaning that the power is just the area of the current triangular waveform multiplied by 500V and the switching frequency. The turn off energy is the same as the turn on so the total switching power is doubled:

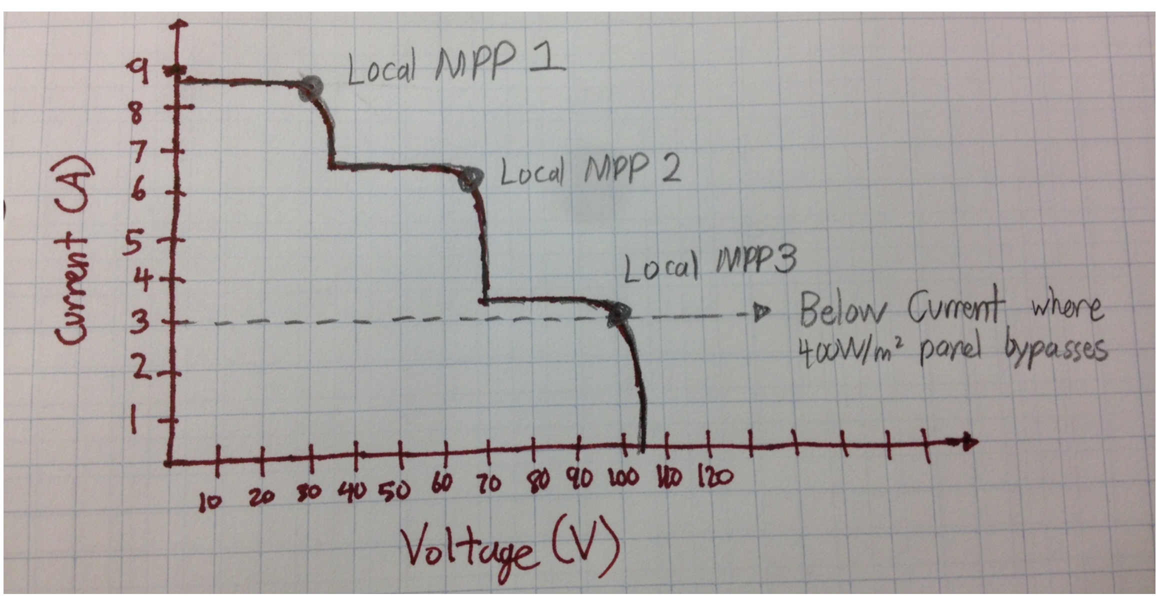
$$P_{sw} = \frac{1}{2} I * t_{switching} * V * f_{sw} * 2 = 2*(.5)(20A)(500V)(20ns)(200kHz) = 40W$$

Problem 4: Photovoltaics [15 Points, 3 Points Each]



Suppose I have a string of 3 PV modules. The first has I-V characteristics given by the 1000W/m² line on the figure above. The second follows the 800W/m² line and the third follows the 400W/m² line. The three modules are connected with bypass diodes across each module. Answer each of the following questions about this three module configuration.

IV Curve of Panels in String



(a) What is the approximate open-circuit voltage of the 3-panel configuration?

(35+35+35) = 105V

(b) What is the approximate short-circuit current of the 3-panel configuration?

8.5A

(c) At a current just below the current at which the 400W/m^2 panel bypasses, what are I and V?

At lower current, panel hasn't bypassed yet, so all panels on. Operates at around Local MPP3
V = (35+35+30) = 100V
I = 3A

(d) What is the approximate maximum power point MPP of the three-panel configuration?

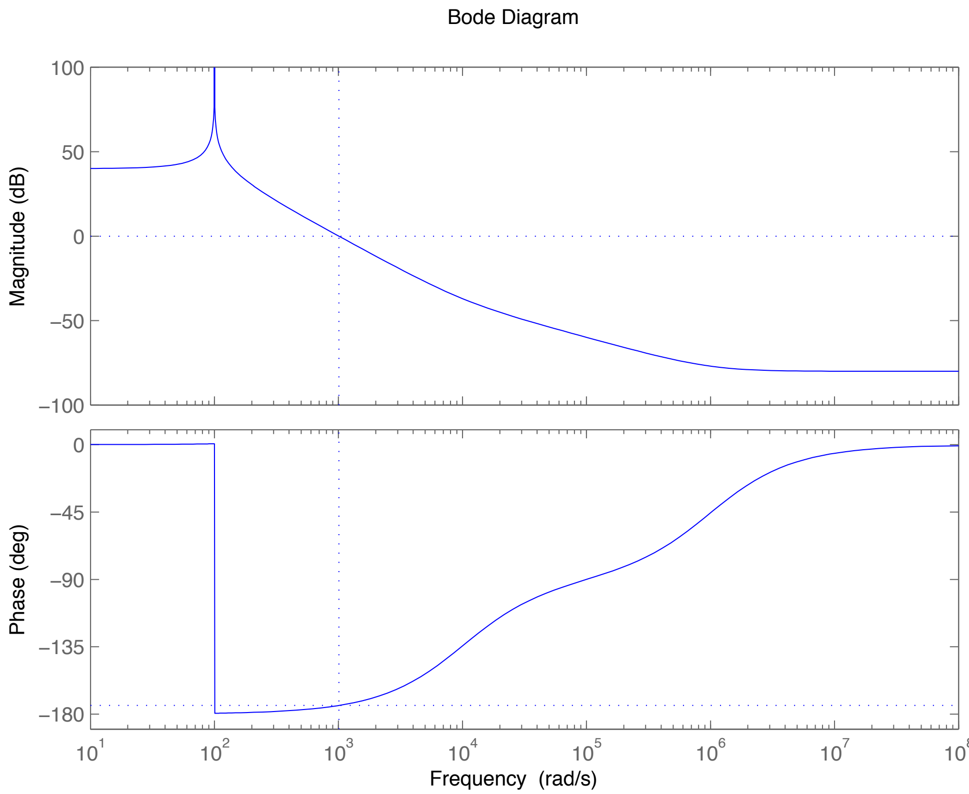
*Local MPP1 = 30*8 = 240W*
*Local MPP2 = 65*6.5 = 423W*
*Local MPP3 = 100*3 = 300W*

MPP = 423W.

(e) Can the maximum power point be found by gradient search (hill climbing) from both the open-circuit and short-circuit configurations? (yes or no)

If you start from open-circuit you will hill climb to Local MPP1.
If you start from closed-circuit you will hill climb to Local MPP3.
Neither are the global MPP, so no.

Problem 5: Feedback Control [15 Points]



You have a plant plus controller with the open-loop frequency response as shown in the Bode plot above. Suppose you close a feedback loop around this system. Answer the following questions about the resulting system:

(a) [3 Points] Is the system adequately damped, i.e., will any ringing after an abrupt transition die out in at most a cycle or two? (yes/no)

No, there is not adequate phase margin.

(b) [3 Points] At what frequency in (rad/s) will any ringing occur?

At 1k rad/s – the unity-gain frequency.

(c) [3 Points] At what frequency in (rad/s) are the poles and zeros of this system?

Two poles at 10^2, One zero at 10^4, One zero at 10^6

(d) [6 Points] Which of the following will increase the phase margin, and hence the damping of the system (circle all that will):

i) Increase the DC gain by 10 (20dB).
ii) Decrease the DC gain by 10 (20dB).
iii) Move the first zero up (higher frequency) by 10.
iv) Move the first zero down (lower frequency) by 10.

i and iv both will

Problem 6: Inductor Design [20 Points, 4Points Each]

The relevant properties for a Ferroxcube E20/10/5 core are shown in the table below. In the 3C96 material this core has a μ_r=1530 and A_L=1400nH. The window area is A_W=100mm^2, and the length of the average turn is L_{turn} = 38mm. Recall the premeability of free space $\mu_0 = 4\pi \times 10^{-7}$.

SYMBOL	PARAMETER	VALUE	UNIT
Σ(I/A)	core factor (C1)	1.37	mm^{-1}
V_e	effective volume	1340	mm^3
l_e	effective length	42.8	mm
A_e	effective area	31.2	mm^2
A_{min}	minimum area	25.2	mm^2
m	mass of core half	≈ 4.0	g

(a) What is the inductance of an inductor made by winding 10 turns of magnet wire on this core with no gap?

$$L = N^2 * A_L = (10)^2(1400nH) = 140\mu H$$

(b) At what current will your 10-turn inductor from part (a) saturate the core with B=0.5T.

$$I_{max} = \frac{N * A * B_{max}}{L} = \frac{(10)(31.2 * 10^{-6} m^2)(.5T)}{140 * 10^{-6} H} = 1.11A$$

(c) To double the inductance from part (a) would require approximately how many turns (round to nearest integer).

Inductance varies with N squared so to get twice the inductance, there would have to be

$10 * \sqrt{2} \approx 14$ turns

(d) What gap should you use to realize a 1uH inductor that can carry 5A with B=0.2T using this core? (Ignore fringing fields and assume all of the reluctance is in the air gap).

N must be an integer so it must be solved for first and rounded

$$N = \frac{L * I_{\max}}{B_{\max} * A} = \frac{(1uH)(5A)}{(.2T)(31.2 * 10^{-6} m^2)} \approx 1$$

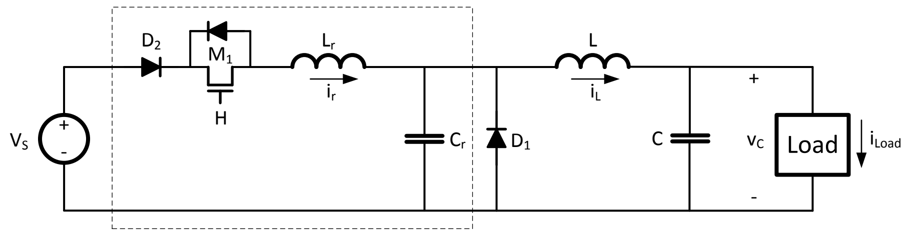
$$l_g = \frac{N^2 * \mu_0 * A}{L} = \frac{(4\pi * 10^{-7})(31.2 * 10^{-6} m^2)}{1\mu H} = 39.2\mu m$$

(e) What number of turns does your inductor from part (d) require?

Once again, N must be an integer so it is rounded

$$N = \frac{L * I_{\max}}{B_{\max} * A} = \frac{(1uH)(5A)}{(.2T)(31.2 * 10^{-6} m^2)} \approx 1$$

Problem 7: Soft Switching [20 Points, 5Points Each]



Consider the quasi-resonant ZCS buck converter shown above where V_S=100V, V_C=50V, and i_L=10A. Suppose Lr=1uH and Cr=100nF. Assume L is large enough that it can be considered to be a current source.

(a) How much time elapses from the time switch M_1 turns on until diode D_1 turns off?

L sees constant current of 10A. Current through Ir + Current through D1 = 10A. Current through Lr ramps from 0 to 10A, D1 ramps from 10A to 0A.

$$t_{ramp} = \frac{L_r \Delta I_r}{V_{Lr}} = \frac{1e-6*10}{100} = 100ns$$

(b) Approximately how much time elapses from D_1 turning off until diode D_2 turns off? For the purpose of analyzing the tank circuit formed by L_r and C_r, you may ignore the effects of the load current i_L – i.e., calculate the time for the voltage on C_r to reach its peak value if there were no load current.

When D1 turns off and we ignore current to load, we see Vs, Lr, and Cr act in resonance. V_s=V_L+V_c. When current through Lr is 0, the diode turns off. At this point, the voltage across Cr is approximately double Vs. The lowest value of the voltage sinusoid is 0V while the highest value of the voltage sinusoid is 200V, and this takes half the resonant cycle.

The resonant frequency is $\frac{1}{2\pi\sqrt{L_r C_r}}$. *Half the resonant period =* $\pi\sqrt{L_r C_r} = 993ns$.

(c) How much time elapses from diode D_2 turning off until diode D_1 turns back on?

Cr must discharge from 200V to 0V through L since D_2 is off. 10A is constantly being flowing through Cr.

$$t_{fall} = \frac{C_r \Delta V_c}{I_L} = \frac{100e-9*200}{10} = 2us$$

(d) Approximating the voltage on C_r as a triangle wave (which its not), what period will this regulator have in the periodic steady state?

Volt-seconds across L must be 0 over one cycle to be in periodic steady state.

The positive side of L will see the triangle voltage wave. It's peak voltage is 200V and the length of the triangle is 2us + 993ns = 2.993us.

The negative side of L will see the constant output voltage, which is 50V.

*.5*200V*2.993e-6 = 50*tcyc*
Tcyc = 5.986us