Question 1, 2016 Midterm Solutions EE155/255

a) To solve for the capacitor voltage, we will make use of our favorite property of magnetic elements in period-steady state... volt-second balance!

$$V_1 = D_1 V_{c1} + D_2 V_{c2} + D_3(0)$$

$$1 = D_1 V_{c1} + 2D_2$$

$$V_{c1} = \boxed{\frac{1 - 2D_2}{D_1}}$$

b) The change in inductor current over a cycle when not in the periodic steady state can be derived from the basic inductor relationship for each position of the switch.

$$\frac{\Delta I}{\Delta t} = \frac{V}{L}$$

$$\frac{\Delta I_1}{D_1 t_{cy}} = \frac{V_1 - V_{c1}}{L} \quad , \quad \frac{\Delta I_2}{D_2 t_{cy}} = \frac{V_1 - V_{c2}}{L} \quad , \quad \frac{\Delta I_3}{D_3 t_{cy}} = \frac{V_1}{L}$$

$$\Delta I = \Delta I_1 + \Delta I_2 + \Delta I_3 = \frac{t_{cy}}{L}((1 - V_{c1})D_1 + (1 - 2)D_2 + D_3)$$

$$\Delta I = \boxed{\frac{t_{cy}}{L}((1 - V_{c1})D_1 - D_2 + D_3)}$$

The change in capacitor voltage is somewhat easier to calculate. We can assume current in the inductor is relatively constant over a single cycle of the switch, and use the basic capacitor equation to calculate the change in voltage.

$$\Delta V_{c1} = \frac{Q_{c1}}{C1} = \boxed{\frac{I_L D_1 t_{cy}}{C1}}$$

c) To calucalte the natural frequency of the response on C1, we start with our knowledge of the natural freqnecy response of a standard LC circuit, which has a frequency $\omega = \sqrt{\frac{1}{LC}}$. In this case, we have to account for the switch between the inductor and capacitor which has a DF of D_1. Essentially, the resonance is "paused" whenever the switch is not in the position connecting C1 with the inductor, so the natural frequency will be scaled down by the DF. so $\omega = D_1\sqrt{\frac{1}{LC}}$.

Question 2, 2016 Midterm Solutions EE155/255

a) The voltage waveform at $\omega = 4$ rad/s will be both twice the frequency and twice the amplitude since the back-EMF isproportional to rotational speed. Thus the waveform will be $v(t) = 2\sqrt{2}sin(4t)$.

b) In order to keep the rotor at a steady speed, we must apply a torque equal in magnitude and opposite in direction to the 2 N-m that is being applied to the shaft. From part a), we have a motor constant $K_m = 1/2$, so at $\omega = 10$ rad/s, we have a back EMF of 5V RMS. Given our $K_m = 1/2$ we need 4A to achieve 2 N-m ot torque, and so the resistance must be 5V/4A $= 1.25\ \Omega$.

c) The torque with respect to current is a sine wave (with no phase offset since we defined $\theta = 0$ as the vertical position). Since $sin(30°) = 1/2$ and $sin(90°) = 1$, the torque at 90° will be twice that at 30° , for 2 N-m total.

d) Since torque is proportinal to current, doubling the current from part c) will double the torque from part c), for 4 N-m total.

To calculate the total losses through the FET in this boost configuration we need to consider the conduction and switching losses of the FET.

To calculate the conduction losses we consider the R_{on} of the FET. This is calculated by considering the current passing through it and the duty cycle.

$$P_{\text{cond}} = DI_D^2R_{\text{on}} = (0.5)(20\text{A})^2(100m\Omega) = \boxed{20\text{W}}$$

To consider the switching loss we need to calculate the energy lost per switching transition. We first assume linear current and voltage waveforms. Looking at the voltage and current waveforms shown in figure 1 we can derive expressions for the dissipated energy. These plots show two valid approximations for the switching waveforms.

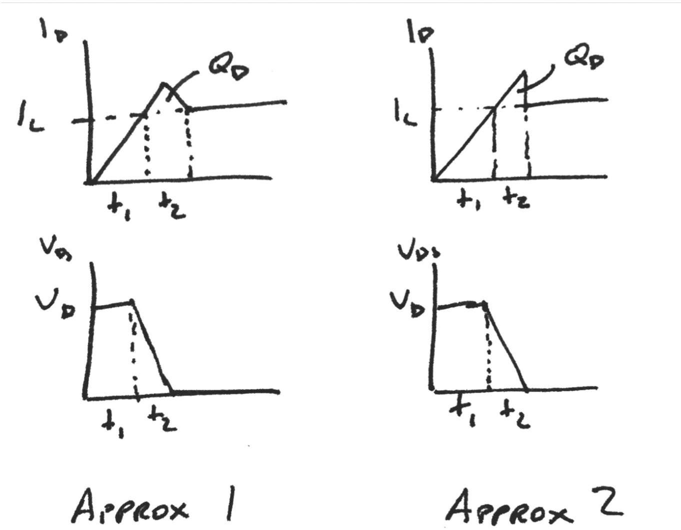

Figure 1: FET turn on waveforms

Considering the energy dissipated during t_1 we have the same expression for both approximations.

$$t_1 = \frac{I_L}{(dI/dt)} = \frac{20\text{A}}{1\text{A/ns}} = 20\text{ns}$$

$$E_1 = \int_0^{t_1} V_D I_D \text{dt} = \frac{1}{2}V_D[(1\text{A/ns})(t_1)]t_1 = 100\mu\text{J}$$

The calculation for E_2 depends on what approximation you used with approximation 1 yielding a larger loss than approximation 2. The additional area covered by the current waveform must equal the charge to be removed from the drain capacitor, Q_D.

Approximation 1:

$$
\begin{aligned}
Q_D &= C_D V_D && (1)\\
\frac{1}{2}(\frac{t_2}{2}(dI/dt))t_2 &= Q_D && (2)\\
t_2 &= \sqrt{\frac{4Q_D}{(dI/dt)}} && (3)\\
&= \sqrt{\frac{4(100\text{pF})(500\text{V})}{1\text{A/ns}}} && (4)\\
&= 10\sqrt{2}\text{ns} \approx 14.1\text{ns} && (5)
\end{aligned}
$$

$$
\begin{aligned}
E_2 &= \frac{1}{2}V_D I_L t_2 + \frac{1}{2}C_D V_D^2 && (6)\\
&= \frac{1}{2}(500\text{V})(20\text{A})(14.1\text{ns}) + \frac{1}{2}100\text{pF}(500\text{V})^2 && (7)\\
&\approx 83.2\mu\text{J} && (8)
\end{aligned}
$$

$$
\begin{aligned}
E_{\text{sw}} &= E_1 + E_2 && (9)\\
&= 100\mu\text{J} + 83.2\mu\text{J} = 183\mu\text{J} && (10)\\
P_{\text{sw}} &= E_{\text{sw}}(f_{\text{sw}}) = (183\mu\text{J})(100\text{kHz}) = \boxed{18.3\text{W}} && (11)\\
P_{tot} &= P_{\text{cond}} + P_{\text{sw}} = \boxed{38.3\text{W}} && (12)
\end{aligned}
$$

Approximation 2:

$$
\begin{aligned}
Q_D &= C_D V_D && (13)\\
\frac{1}{2}(dI/dt)(t_2)^2 &= Q_D && (14)\\
t_2 &= \sqrt{\frac{2Q_D}{(dI/dt)}} && (15)\\
&= \sqrt{\frac{2(100\text{pF})(500V)}{1\text{A/ns}}} && (16)\\
&= 10\text{ns} && (17)
\end{aligned}
$$

$$
\begin{aligned}
E_2 &= V_D I_L \frac{t_2}{2} + \frac{1}{3} C_D (V_D)^2 && (18)\\
&= (500\text{V})(20\text{A})\frac{10\text{ns}}{2} + \frac{1}{3}(100\text{pF})(500\text{V})^2 && (19)\\
&\approx 58.3\mu\text{J} && (20)
\end{aligned}
$$

$$
\begin{aligned}
E_{\text{sw}} &= E_1 + E_2 && (21)\\
&= 100\mu\text{J} + 53.3\mu\text{J} = 158\mu\text{J} && (22)\\
P_{\text{sw}} &= E_{\text{sw}}(f_{\text{sw}}) = (158\mu\text{J})(100\text{kHz}) = \boxed{15.8\text{W}} && (23)\\
P_{tot} &= P_{\text{cond}} + P_{\text{sw}} = \boxed{35.8\text{W}} && (24)
\end{aligned}
$$

Note that this is larger than would be expected from just the energy stored in the capacitor. That is because there is additional energy being dissipated in the switch while the capacitor discharges due to the load current. The choice in linear approximation makes a difference because the capacitor discharges at different rates. To see the full derivations of the energy dissipated as the capacitor discharges see below.

Approximation 1: In this approximation we are assuming that the current forms an equilateral triangle above the load current. Then we can split the energy into two sections.

$$
\begin{aligned}
E_{2,1} &= \int_0^{t_2/2} (I_L + (dI/dt)t)(V_D - \frac{V_D}{t_2}t)dt && (25)\\
&= V_D I_L t + \frac{1}{2} V_D (dI/dt) t^2 - \frac{1}{2}\frac{V_D I_L}{t_2} t^2 - \frac{1}{3}\frac{V_D}{t_2}(dI/dt)t^3 && (26)\\
&= \frac{1}{2} V_D I_L t_2 + \frac{1}{8} V_D (dI/dt)(t_2)^2 - \frac{1}{8} V_D I_L t_2 - \frac{1}{24} V_D (dI/dt)(t_2)^2 && (27)
\end{aligned}
$$

To solve for $E_{2,2}$ we can integrate starting from time t_2 and go until $t_2/2$ to make the integral a little simpler.

$$
\begin{aligned}
E_{2,2} &= \int_0^{t_2/2} (I_L + (dI/dt)t)(\frac{V_D}{t_2}t)dt && (28)\\
&= \frac{1}{2} I_L \frac{V_D}{t_2} t^2 + \frac{1}{24} V_D (dI/dt)(t_2)^2 && (29)
\end{aligned}
$$

Then noting that $\frac{(t_2)^2}{4}(dI/dt) = C_D V_D$ we can write E_2 as

$$
\begin{aligned}
E_2 &= E_{2,1} + E_{2,2} && (30)\\
&= V_D I_L \frac{t_2}{2} + \frac{1}{8} V_D (dI/dt)(t_2)^2 && (31)\\
&= V_D I_L \frac{t_2}{2} + \frac{1}{2} C_D (V_D)^2 && (32)
\end{aligned}
$$

Approximation 2:
The derivation for E_2 under the second approximation is similar except that now $\frac{(t_2)^2}{2}(dI/dt) = C_D V_D$.

$$
\begin{aligned}
E_2 &= \int_0^{t_2} (I_L + (dI/dt)t)(V_D - \frac{V_D}{t_2}t)dt && (33)\\
&= I_L V_D t - \frac{1}{2}\frac{V_D}{t_2} I_L t^2 + \frac{1}{2}V_D(dI/dt)t^2 - \frac{1}{3}V_D(dI/dt)t^3 && (34)\\
&= V_D I_L t_2 - \frac{1}{2}V_D I_L t_2 + \frac{1}{2}V_D(dI/dt)(t_2)^2 - \frac{1}{3}V_D(dI/dt)(t_2)^2 && (35)\\
&= V_D I_L \frac{t_2}{2} + C_D(V_D)^2 - \frac{2}{3}C_D(V_D)^2 && (36)\\
&= V_D I_L \frac{t_2}{2} + \frac{1}{3}C_D(V_D)^2 && (37)
\end{aligned}
$$

a) No, the system is not adequately damped. There will be lots of ringing as phase margin is very close to 0 degrees.

b) It looks like the frequency when gain = 1 (ω_{co}) is at around $\boxed{120 rad/sec}$. This is where much ringing will occur.

c) A PD controller is equivalent to adding a pure zero somewhere in our frequency range. If we add our zero at ω_{co}, we will get around 45 degrees of added phase at 120 rad/sec, however this will also push our ω_{co} up as there is added gain from the zero, giving us a total phase margin of a bit more than 45 degrees. To hit our target of 60 degrees properly, we'll put our zero slightly behind 120 rad/sec at 100 rad/sec. This will give us more room for phase to grow to 60 degrees. Anything slower than 75 rad/sec will give us far too much phase margin than our desired of 60 degrees.

Our controller is in the form:

$$(\frac{s}{100} + 1)$$

Notice the fractional form to introduce no DC gain. Our PD controllers are of the form:

$$K_p + K_d s$$

Factoring, we get:

$$K_p(\frac{s}{\frac{K_p}{K_d}} + 1)$$

Since $K_p = 1$, our $\boxed{K_d = \frac{1}{100}}$.

d) From 4.c, we know our zero will be at $\boxed{100 rad/sec}$.

Question 5, 2016 Midterm Solutions EE155/255

a) Use the turn inductance to calculate the primary side magnetizing inductance.

$$L_P = A_L(N_P)^2 = (1.4\mu\text{H/turns}^2)(5\text{turns})^2 = \boxed{35\mu\text{H}}$$

b)

$$\begin{aligned} \Delta BAN &= L_M I_{\max} & (38)\\ \Delta BAN &= L_M \frac{VT}{L_M} & (39)\\ \Delta BAN &= VT & (40)\\ (0.5T)(31.2\text{mm}^2)(5\text{turns}) &= \boxed{7.8 \times 10^{-5} Vs} & (41) \end{aligned}$$

c)

$$\begin{aligned} \Delta BAN &= VT & (42)\\ \frac{10^{-4} Vs}{\Delta BA} &= N = 6.41 \approx \boxed{7 \text{ turns}} & (43) \end{aligned}$$

d) We can approach this problem by either mirroring the primary magnetizing inductance to the secondary or directly calculating the secondary magnetizing inductance from the number of turns.

$$\begin{aligned} N &= 8 & (44)\\ L_{M,S} &= N^2 L_{M,P} & (45)\\ &= 35\mu\text{H}(64) = \boxed{2.24\text{mH}} & (46) \end{aligned}$$

$$A_L(N_S)^2 = 1.4\mu\text{H/turns}^2(40 \text{ turns})^2 = \boxed{2.24\text{mH}} \quad (47)$$

a) We use energy and power to calculate how long we want to charge the inductor (amount of energy built up) and therefore the duty factor we desire. We don't care about how long it takes to discharge its current/energy.

$$OutputEnergy = 100V * 1A * T_{cy} = 1mJ/cycle$$
$$InputEnergy = \int V_L I_L dt = 50V \int I_L dt$$

Notice that we are supplying power the entire cycle, not just the discharge period, because our device...needs power constantly...
Current over the inductor during charging slowly ramps up linearly. The ramp up time is DT_{cy}, and the peak current is $\frac{50V}{L_{mag}} * DT_{cy}$. We use only the magnetizing inductance for charging since we're ignoring the charging of leakage inductance.

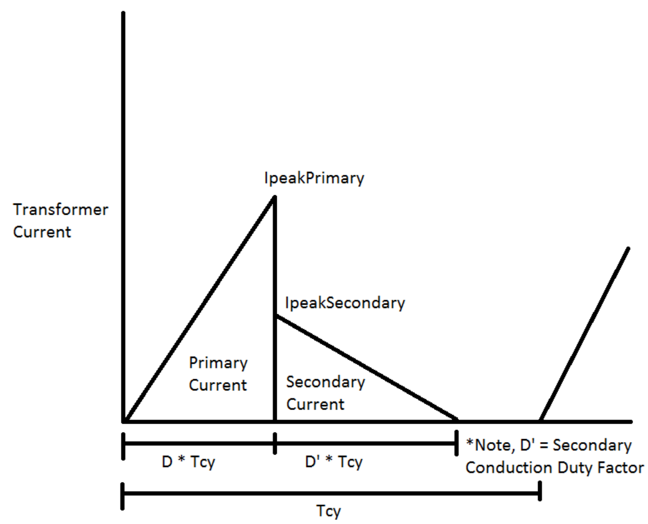

Figure 2: Transformer Current Waveform

From Figure 2, we see that the primary side charges to a primary peak current with time DT_{cy}, afterwards, it takes $D'T_{cy}$ time to discharge from the secondary. Note that $D + D' \neq 1$, but using D and D' would be the correct volt-seconds balance times. After a short period of no current, we restart the cycle and charge up the primary side again. To solve for primary side volts-seconds balance, the charging voltage (50V) * charging time is equal to the reflected secondary voltage (turns ratio * 100V) * discharging time.

$$50V * D * T_{cy} = \frac{1}{2} * 100V * D' * T_{cy}$$
$$D = D'$$

The total energy charged is the integral of primary current, which waveform is a triangle with base (ramp up time) and height (peak primary current). The energy therefore is:

$$50V * \frac{1}{2} * DT_{cy} * \frac{50V}{L_{mag}} * DT_{cy} = \frac{D^2}{160} = 1mJ$$
$$\boxed{D = 0.4}$$

One can also solve this with the inductor energy equation

$$\frac{1}{2}L_{mag}I_{peak}^2 = 1mJ$$

b) Given our analysis above, this problem is simple:

$$I_{peak} = \frac{50V}{L_{mag}} * DT_{cy} = \boxed{10A}$$

c) With the leakage inductance, our magnetizing inductance will take longer to charge up to the required energy output. Since energy stored is a function of peak current, we must ensure our magnetizing inductance (and leakage inductance since it is in series) still charges up to 10A. Our new inductance is the two inductors in series, which is an inductor of $21\mu H$.

$$I_{peak} = \frac{50V}{L_{mag+leakage}} * D_{new}T_{cy} = 10A$$
$$D_{new} = \frac{21}{50} = \boxed{0.42}$$

d) If you calculated the amount of energy lost from the leakage inductance every cycle, we gave you full points.

$$\frac{1}{2}L_{leakage}I_{peak}^2 = \frac{1}{2}1\mu H(10A)^2 = 50\mu J$$

But that is not what the question was asking. The question asked how much energy is dissipated in the zener diode, which is a function of not only the leakage inductance energy, but also some energy stolen from the secondary during commutation.

If we reflect the secondary circuit to the primary side during commutation, we get this:

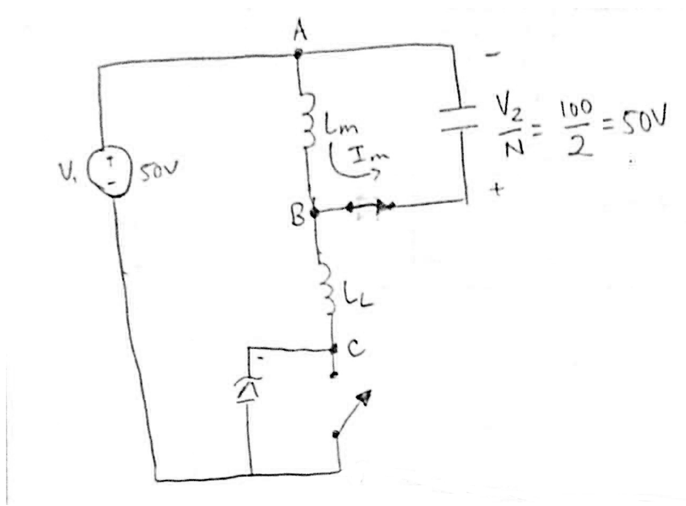

Figure 3: Reflected Flyback Circuit

If we reference all voltages to primary side ground, we have node A at 50V, and node B must be at 100V. Node C must be at 300V since this is a 300V zener and it pegs C at 300V. Therefore, we have a 200V drop across the leakage inductance ramping down linearly from $I_{peak} = 10A$ (waveform on next page)

$$Energy_{zener} = \int V_{zener} I_{zener} dt$$

$$V_{zener} = 300V$$

$$Energy_{zener} = 300V \int I_{zener} dt$$

$$\int I_{zener} dt = \int I_{leakage} dt = \frac{1}{2} I_{peak} T_{discharge}$$

$$T_{discharge} = 10A / \frac{200V}{1\mu H} = 50ns$$

$$\int I_{zener} dt = 0.25\mu A * s$$

$$Energy_{zener} = 300V \int I_{zener} dt = \boxed{75\mu J}$$

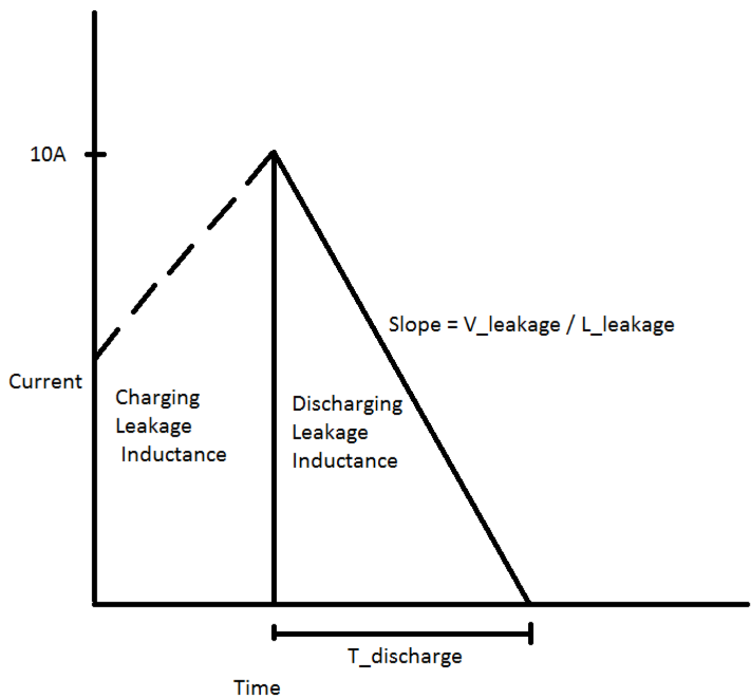

Figure 4: Leakage Inductor Current Waveform

a) We can determine the approximate open-circuit voltage of the three series connected panels by summing the open vircuit voltages of each. From the graph, the open circuit voltages are about 37v, 37v, and 35v, giving a toal voltage of around ≈110V.

b) The short circuit current of the three series-connected modules will be about 8.5A (the I_{sc} of the two stronger modules). This is due to the bypass diode on the weak module allowing the full I_{sc} of the strong modules to bypass it.

c) The $400W/m^2$ module will bypass when the string current is higher than its I_{sc}, so the lowest current where it bypasses is around 3.4A. At this current, the bypassed module contributes no voltage to the string, and each of the other modules contributes about 35V, giving a total voltage of 70V at 3.4A.

d) There are two potential options for the MPP of the three-module configuration. The MPP will be either at the MPP of the stronger two modules, with the weaker one bypassed, or it will be at the MPP of the weaker module, with the stronger modules contributing power but at less than their MPP. The first point occurs at 8A, with each strong module at 30V, giving a total power of 480W. The second option, at ≈3A, gives 100V (35V+35V+30V), for 300W total. Clearly, the first local optima is the global optima, so the MPP for the three-panel configuration is 480W.

e) No, the gradient searching will only find the global optimum starting from short circuit. Starting from the open circuit condition, the gradient search algorithm will find the 300W local optimum, and then stay there without exploring the curve further. From I_{sc}, gradient-search will find the global optimum first, and will stay there. This result demonstrates why most commercial inverters use gradient-search algorithms that are modified to break out of local optima (e.g. by sweeping the full curve, jumping to a new operating point, etc.).