1. Let A and B be independent events. Using the definition of independence, show that the events
   $\ A$ and $B^c$ are independent.

   **Note:** In general, if $A_1, A_2, ..., A_n$ are independent events, then so are $B_1, B_2, ..., B_n$, where each
   $B_i$ can be either $A_i$ or $A_i^c$.

2. Prove that if $E_1, E_2, E_3, ..., E_n$ are independent events, then
   \[ P(E_1 \cup E_2 \cup ... \cup E_n) = 1 - \left(1 - P(E_1)\right)\left(1 - P(E_2)\right)...\left(1 - P(E_n)\right) \]

3. In this problem, we consider a circuit consisting of components. Each component functions
   properly with probability $p$, independent of the other components. The circuit works if there is a
   path from A to B with all components working.

   ![Circuit Diagram](image)

   (a) Compute the probability that a circuit with two sub-circuits in series works, if the two
   sub-circuits have probabilities $p_1$ and $p_2$ of working.

   (b) Compute the probability that a circuit with $n$ sub-circuits in parallel works, if the sub-
   circuits have probabilities $p_1, p_2, ..., p_n$ of working.

   (c) Compute the probability that the circuit shown in the figure works by decomposing it
   into sub-circuits.

   (d) Given that the circuit shown in the figure works, find the probability that the component
   marked by star functions properly.

4. There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased
   coin that comes up heads 75 percent of the time. When one of the 3 coins is selected uniformly
   at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

5. We deal from a well-shuffled 52-card deck. Calculate the probability that the 3rd card is the first
   king to be dealt.