1. **Memorylessness of geometric random variables**: Consider an experiment of tossing a coin with heads probability $p$ till you see a head. Given that you’ve not seen a head for the first $m$ tosses, what is the probability that you’ll need $k$ more tosses to see the first heads? How does this compare to the probability of seeing the first heads on the $k$th toss?

2. Compute the probability that a geometric random variable $X$ with probability of success $p$, is even:
   a. By direct computation
   b. By conditioning on whether $X=1$ or $X>1$

3. A fair coin is continually flipped until heads appears for the $10^{th}$ time. Let $X$ denote the total number of tosses required.
   a. Compute the probability mass function of $X$.
   b. Write $X$ as a sum of independent geometric random variables and hence compute its expectation and variance.
   c. Let $Y$ be the number of tails that occur in the process. Write $Y$ in terms of $X$ and find its pmf, expectation and variance.

   *Note*: For a geometric random variable with success probability $p$, the expectation and variance are $1/p$ and $(1-p)/p^2$, respectively.

4. Suppose we flip $n$ fair coins to obtain $X_1, X_2, ..., X_n \in \{H, T\}$. Let $Y$ be the number of pairs of consecutive heads (i.e., number of $i$’s such that $X_i = X_{i+1} = H$). We wish to compute the expectation and variance of $Y$.
   a. Let $Y_i$ be the indicator function for the event $\{X_i = X_{i+1} = H\}$ for $i = 1..n-1$. Write $Y$ in terms of $Y_i$’s.
   b. Find $E[Y_i]$ and $E[Y_i^2]$ by computing the pmf of $Y_i$
   c. Find $E[Y_i Y_j]$ for $i < j$ by computing its pmf. You might want to separate the cases $j = i + 1$ and $j > i + 1$.
   d. Compute the expectation and variance of $Y$. 

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