Homework #1
Due Friday, October 13

1. *Signal estimation.* In Lecture 3, we said that if we want to estimate a signal vector $c\mathbf{x}$ given a measurement $\mathbf{g}$ with an unknown error $\mathbf{e}$

$$\mathbf{g} = \mathbf{cx} + \mathbf{e}$$

that the value of c that minimizes the error is

$$c = \frac{< \mathbf{g}, \mathbf{x} >}{< \mathbf{x}, \mathbf{x} >}$$

Show that this is true by starting with the error

$$||e||^2 = (\mathbf{g} - \mathbf{cx}) \cdot (\mathbf{g} - \mathbf{cx})$$

expand it, differentiate with respect to c, set it to zero, and solve for c.

2. *Correlation coefficient.* The correlation coefficient for two signals $\mathbf{x(t)}$ and $\mathbf{y(t)}$ is

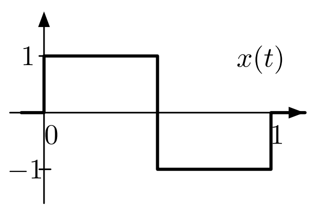
$$\rho = \frac{< \mathbf{x}, \mathbf{y} >}{||\mathbf{x}||\ ||\mathbf{y}||} = \frac{1}{\sqrt{E_x E_y}} \int_{-\infty}^{\infty} x(t) y^*(t) dt$$

This is a measure of how similar the *shapes* of two signals are.

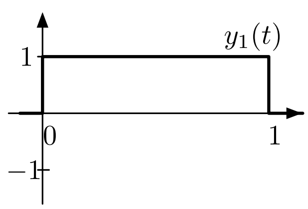
a. Show that ρ is independent of the amplitude of either $x(t)$ or $y(t)$, and that

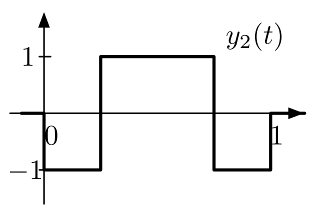
$$-1 \leq \rho \leq 1.$$

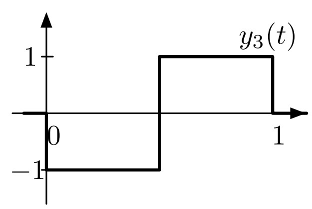
b. We want to choose two waveforms for a binary communication system. One, $x(t)$, looks like



The alternatives for the other signal, $y(t)$, are







which one is the *least* similar (has the largest change in ρ).

3. *Stretching impulses.* It is often useful to scale impulses in time.

a. Show that

$$\delta(at) = \frac{1}{|a|}\delta(t).$$

Start by evaluating

$$\int_{-\infty}^{\infty} f(t)\delta(at)dt$$

and then argue the conclusion.

b. Show that

$$\delta(\omega) = \frac{1}{2\pi}\delta(f)$$

where $\omega = 2\pi f$

c. Show that the transforms for $\cos(\omega t)$ in ω and $\cos(2\pi f t)$ in $2\pi f$ are consistent.

4. *Capacity of AWGN channel.* The capacity in bits per second of an additive white Gaussian noise (AWGN) channel is

$$C = B \log_2 \left(1 + \frac{P}{N_0 B}\right),$$

where P is the received signal power, B is the signal bandwidth, and $N_0/2$ is the noise power spectral density (PSD). (The total noise power is $N_0 B$.) Consider a wireless channel where received power falls off with distance d according to the formula $P(d) = P_t(d_0/d)^3$. Given $d_0 = 10$m, transmitter power $P_t = 1$W, noise PSD $N_0 = 10^{-9}$ W/Hz, and channel bandwidth $B = 30$ KHz, find the capacity of this channel for transmitter-receiver distances of 100m and 1km.

5. *Cross correlation.* The cross correlation operation is used for matched filtering. If $x(t)$ is a signal we are looking for in another signal $y(t)$, we perform the operation

$$(x \star y)(t) = \int_{-\infty}^{\infty} x(\tau) y^*(\tau - t) dt = \int_{-\infty}^{\infty} x(\tau + t) y^*(\tau) dt$$

Derive a simple Fourier transform theorem for the cross correlation $\mathcal{F}\{(x \star y)(t)\}$, and the autocorrelation $\mathcal{F}\{(x \star x)(t)\}$.