1. **DSB-SC modulator** (Lathi & Ding 4.2-3). You are asked to design a DSB-SC modulator to generate a modulated signal \( km(t) \cos(\omega_c t + \theta) \), where \( m(t) \) is a signal bandlimited to \( B \) Hz. Figure P4.2-3 shows a DSB-SC modulator available in the stockroom. The carrier generator available generates not \( \cos \omega_c t \) but \( \cos^3 \omega_c t \). Explain whether you would be able to generate the design using only this equipment. You may use any kind of filter you like.

   ![DSB-SC modulator diagram](image)

   a. What kind of filter is required in Fig. P4.2.3?
   
   b. Determine the signal spectra at points \( b \) and \( c \), and indicate the frequency bands occupied by these spectra.
   
   c. What is the minimum usable value of \( \omega_c \)?
   
   d. Would this scheme work in the carrier generator output were \( \sin^3 \omega_c t \)? Explain.
   
   e. Would this scheme work in the carrier generator output were \( \cos^n \omega_c t \) for any integer \( n \geq 2 \)?

2. **Audio scrambler** (Lathi & Ding 4.2-8). The system shown in Fig. P4.2-8 is used for scrambling audio signals. The output \( y(t) \) is the scrambled version of the input \( m(t) \).

   ![Audio scrambler diagram](image)

   a. Find the spectrum of the scrambled signal \( y(t) \).
   
   b. Suggest a method for descrambling \( y(t) \) to obtain \( m(t) \).

3. **AM signal** (Lathi & Ding 4.3-1). In an amplitude modulation system, the message signal is given by Fig. P4.3-1 and the carrier frequency is 1 KHz. The modulator output is

   \[
   s_{AM}(t) = 2(b + 0.5m(t)) \cos \omega_c t.
   \]

   a. Determine the average message power.
b. If \( b = 1 \), determine the modulation index and the modulation power efficiency.

c. Sketch the modulated signal of part (a) in the time domain.

d. If \( b = 0.5 \), repeat parts (a) and (b).

4. **Hilbert transform** (Lathi & Ding 4.4-5). Given that \( m_h(t) \) is the Hilbert transform of \( m(t) \), do the following.

   a. Show that the Hilbert transform of \( m_h(t) \) is \( -m(t) \).

   b. Show that the energies of \( m(t) \) and \( m_h(t) \) are identical.

5. **FM detection.**

   This problem illustrates design choices and limitations for certain FM detector designs. Consider an FM system where the modulated signal is

   \[
   s(t) = 10 \cos \left( 2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau \right),
   \]

   where the carrier frequency is \( f_c = 100 \) MHz. The modulating signal is \( m(t) = 10 \cos(2\pi f_m t) \) where \( f_m = 3 \) KHz.

   a. What is the maximum value of \( k_f \) such that \( s(t) \) can be demodulated using an ideal differentiator followed by an envelope detector?

   For the remainder of the problem assume that \( k_f = 10 \).

   b. What is the approximate bandwidth of \( s(t) \)? Is this NBFM or WBFM?

   c. Find the instantaneous frequency \( f_i(t) \) of \( s(t) \). What are the maximum and minimum values of \( f_i(t) \)?

   d. Suppose that you demodulate \( s(t) \) using an ideal differentiator followed by an envelope detector. Assume a standard envelope detector as shown below, where the capacitor has capacitance \( C = 10^{-9} \) F. Propose values for the source resistance \( R_s \) and load resistance \( R_l \) such that the output of the envelope detector is approximately equal to \( c_1 + c_2 m(t) \) for some constants \( c_1 \) and \( c_2 \). Is it possible to use this detection method if \( f_c \approx f_m \)? Why or why not?

   ![Envelope Detector Diagram]

   e. Suppose that you use a zero-crossing detector for \( s(t) \). Find an expression for the minimum interval \( T \) for a zero-crossing detector such that there are at least four zero crossings in every interval \( T \). Evaluate this expression.