Communication Channels

- wires (PCB trace or conductor on IC)
- optical fiber (attenuation 4dB/km)
- broadcast TV (50 kW transmit)
- voice telephone line (under -9 dbm or 110 μW)
- walkie-talkie: 500 mW, 467 MHz
- Bluetooth: 20 dBm, 4 dBm, 0 dBm
- Voyager: X band transmitter, 160 bit/s, 23 W, 34m dish antenna

`http://science.time.com/2013/03/20/humanity-leaves-the-solar-system`
`-35-years-later-voyager-offically-exits-the-heliosphere/`

Communication Channel Distortion

The linear description of a channel is its impulse response $h(t)$ or equivalently its transfer function $H(f)$.

$$y(t) = h(t) * x(t) \quad \Longleftrightarrow \quad Y(f) = H(f)X(f)$$

Note that $H(f)$ both attenuates ($|H(f)|$) and phase shifts ($\angle H(f)$).

Channels are subject to impairments:

- Nonlinear distortion (e.g., clipping)
- Random noise (independent or signal dependent)
- Interference from other transmitters
- Self interference (reflections or multipath)

Channel Equalization

Linear distortion can be compensated for by *equalization*.

$$H_{\text{eq}}(f) = \frac{1}{H(f)} \Rightarrow \hat{X}(f) = H_{\text{eq}}(f)Y(f) = X(f)$$

The equalization filter accentuates frequencies attenuated by channel.

However, if $y(t)$ includes noise or interference,

$$y(t) = x(t) + z(t)$$

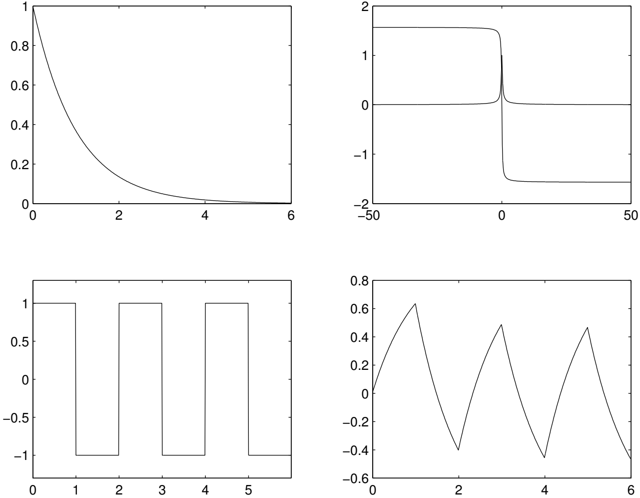
then

$$H_{\text{eq}}(f)Y(f) = X(f) + \frac{Z(f)}{H(f)}$$

Equalization may accentuate noise!

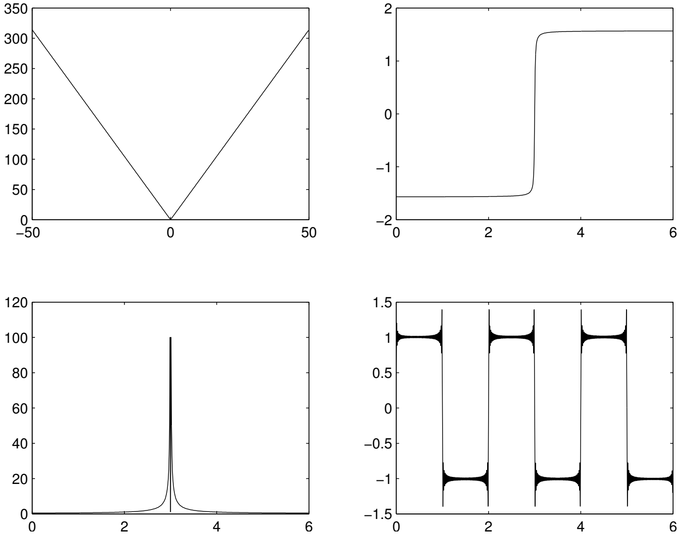
Channel Equalization Example

$h(t) = u(t)e^{-t}$, $x(t)$ is square wave, $y(t) = h(t) * x(t)$.



Channel Equalization Example (cont.)

Equalizing filter has transfer function $1 + j2\pi f$, approximates differentiator.



Signal Energy and Energy Spectral Density

Parseval's theorem for an energy signal $g(t)$ is

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |G(f)|^2 df$$

Signal has same total energy E_g in the time domain frequency domain.

The *essential bandwidth* is the range of frequencies with "most" of the signal's energy of the signal. The definition of "most" depends on the application. One choice might be 90%.

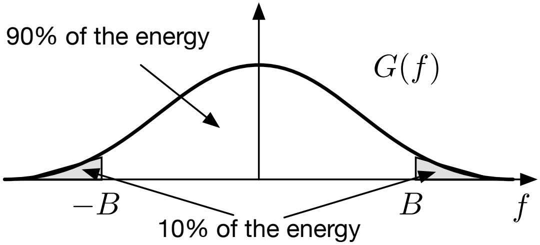
If $G(f)$ is a lowpass signal, and E_B is the energy from $-B$ to B, then

$$E_B = \int_{-B}^{B} |G(f)|^2 df$$

Then the essential bandwidth is the B such that

$$E_B/E_g = 0.9$$

This is illustrated below:



Other definitions of width

- 95% or 99% energy
- Half amplitude width
- Half power width
- 50% energy

Autocorrelation and Energy Spectral Density

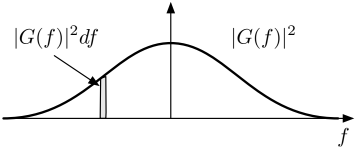
The autocorrelation of a signal $g(t)$ is

$$\psi_g(t) = \int_{-\infty}^{\infty} g(\tau) g^*(t+\tau)\, d\tau$$

You'll show in your homework that

$$\mathcal{F}\{\psi_g(t)\} = |G(f)|^2 = \Psi(f)$$

This is the *energy spectral density* or ESD. It reflects where the energy of the signal is located.

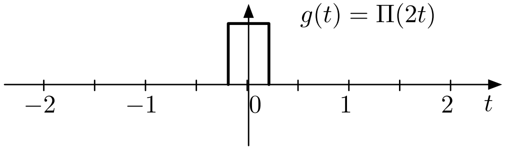


Note that

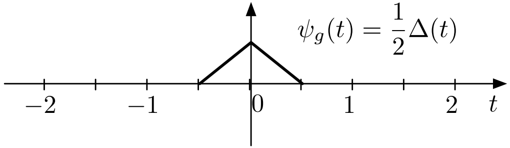
$$E_g = \int_{-\infty}^{\infty} |G(f)|^2 df = \int_{-\infty}^{\infty} \Psi(f) df$$

Energy Spectral Density Example

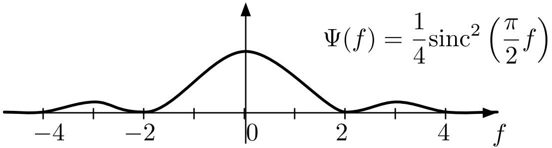
Let $g(t) = \Pi(2t)$



The autocorrelation $\psi(t)$ is



The energy spectral density is then



Autocorrelation and Power Spectral Density

For power signals, we normalize the ESD by the duration, to produce the *power spectral density* or PSD.

The autocorrelation for a power signal $g(t)$ is defined as

$$\mathcal{R}_g(t) = \lim_{T\to\infty} \frac{1}{T} \int_{T/2}^{T/2} g(\tau) g^*(t+\tau) d\tau$$

This has the Fourier transform

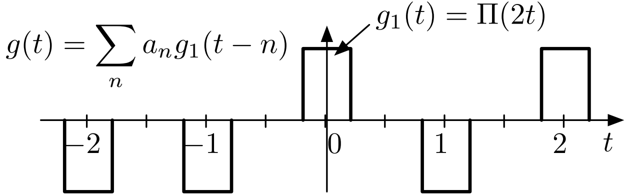
$$\mathcal{F}\left\{\mathcal{R}_g(t)\right\} = \lim_{T\to\infty} \frac{1}{T} \Psi_{g,T}(f) = \mathcal{S}_g(f)$$

$\mathcal{S}_g(f)$ is the power spectral density, PSD.

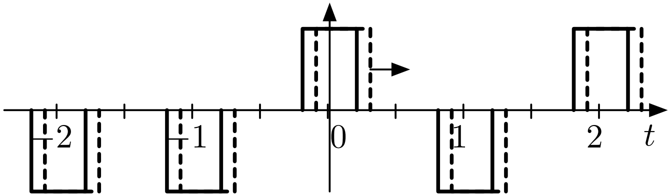
Again, this shows the frequency distribution of the power of the signal.

Power Spectral Density Example

Let $g(t)$ be a random binary sequence of rectangle pulses $g_1(t) = \Pi(2t)$

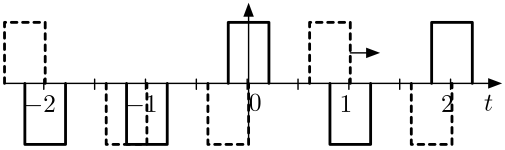


For small displacements, the autocorrelation looks like

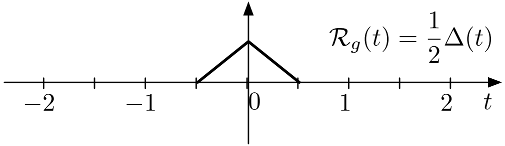


After normalizing by the interval T, this is the same a for a single pulse.

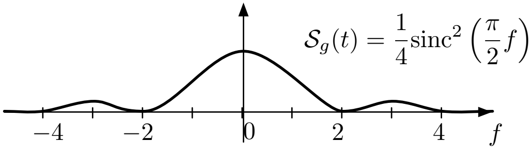
For large displacements, overlaps are just as likely to be ± 1, and will cancel.



This will go to zero as T gets large. The autocorrelation is then



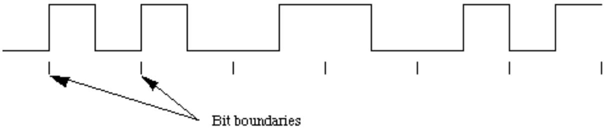
The power spectral density is then



Baseband Communication

The *baseband* is the frequency band of the original signal.

- Telephones: 300–3700 Hz
- High-fidelity audio: 0–20 KHz
- Television (NTSC) video: 0–4.3 MHz
- Ethernet (10 Mbs): 0–20 MHz



Baseband communication usually requires wire (single, twisted pair, coax).

Multiple baseband signals cannot share a channel without time division multiplexing (TDM).

Carrier Communication

Carrier communication uses modulation to shift spectrum of signal.

- Wireless communication requires frequencies higher than baseband
- Multiple signals can be sent at same time using different frequencies: frequency division multiplexing (FDM)

In carrier communication, the signal modulates a sinusoidal carrier.
The signal modifies the amplitude, frequency, or phase of carrier.

$$s(t) = A(t)\cos\big(2\pi f_c(t)t + \phi(t)\big)$$

- amplitude modulation: $A(t)$ is proportional to $m(t)$
- frequency modulation: $f_c(t)$ is proportional to $m(t)$
- phase modulation: $\phi(t)$ is proportional to $m(t)$

Frequency and phase modulation are called *angle modulation*.

Double-Sideband Amplitude Modulation

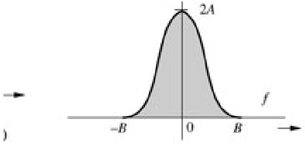
The simplest modulation method is multiplication by sinusoid:

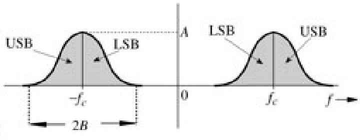
$$x(t) = m(t)\cos(2\pi f_c t + \phi)$$

We usually set phase ϕ to 0 to simplify mathematical discussion.

The Fourier transform of the modulated signal is

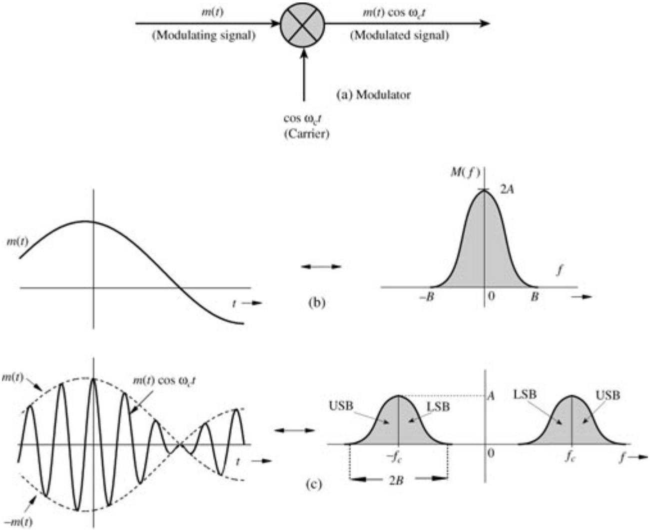
$$X(f) = \frac{1}{2}(M(f + f_c) + M(f - f_c))$$





Double-Sideband Amplitude Modulation (cont.)

This scheme is called *double-sideband, suppressed-carrier (DSB-SC)*.



Signal Bandwidth vs. Carrier Frequency

Transmitters can radiate only a narrow band without distortion. Thus we choose the carrier frequency such that

$$\frac{f_c}{B} \gg 1 \iff \frac{B}{f_c} \ll 1$$

Examples:

- AM radio: $B = 5$ KHz, $550 \leq f_c \leq 1600$ KHz

$$\Rightarrow\ 100 < f_c/B < 320$$

- FM: $B = 200$ KHz, $87.7 \leq f_c \leq 108.0$ MHz

$$\Rightarrow\ 43 < f_c/B < 54$$

- US television: $B = 6$ MHz, $54 \leq f_c \leq 862$ MHz

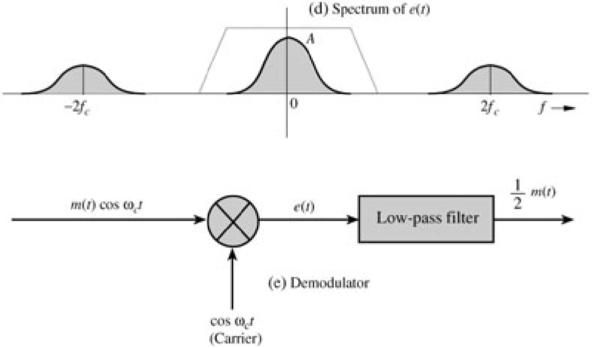
$$\Rightarrow\ 9 \leq f_c/B \leq 142$$

  Digital TV uses the same frequency bands as analog TV.

Demodulation of DSB-SC Signals

Demodulation uses a multiplier and a low-pass filter.

$$e(t) = x(t)\cos(2\pi f_c t) = m(t)\cos^2(2\pi f_c t) = \tfrac{1}{2}m(t) + \tfrac{1}{2}\cos(4\pi f_c t)$$



The low pass filter does not have to be very sharp. But it should be flat over the signal baseband.

DSB-SC Example

Modulating a sinusoid is an important way to test the system. Let

$$m(t) = \cos(2\pi f_m t)$$

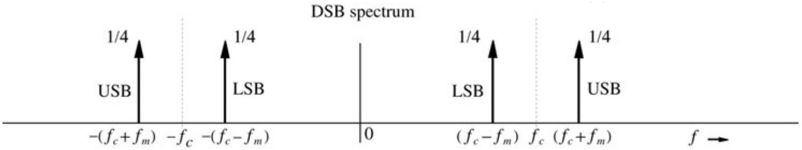
Then

$$M(f) = \tfrac{1}{2}\delta(f + f_m) + \tfrac{1}{2}\delta(f - f_m)$$

and

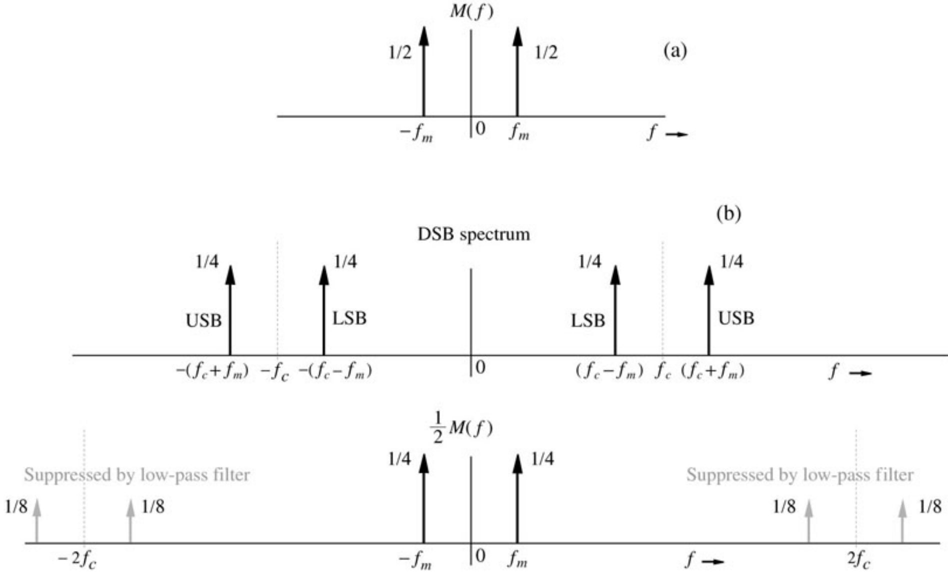
$$\begin{aligned}\varphi_{\text{DSB-SC}}(t) &= m(t)\cos(2\pi f_c t) = \cos(2\pi f_m t)\cos(2\pi f_c t)\\ &= \tfrac{1}{2}\big(\cos((f_c + f_m)t) + \cos((f_c - f_m)t)\big)\end{aligned}$$

The transform of the modulated signal contains two impulse pairs separated by $2f_c$.

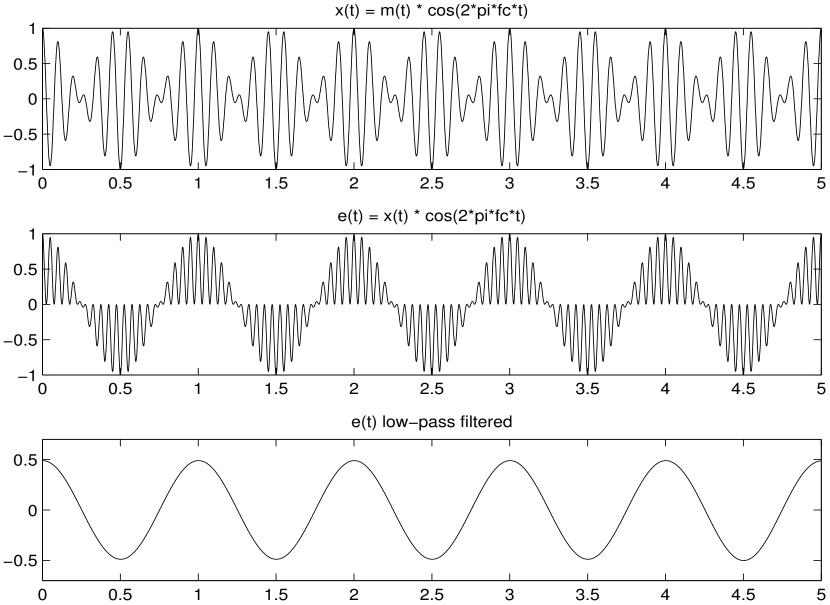


DSB-SC Example: Frequency Domain

Modulation and demodulation of cosine.



DSB-SC Example: Time Domain



Types of Modulators

- Multiplier modulators using variable gain amplifiers.
- Nonlinear modulator. Suppose the input-output characteristic is
$$y(t) = ax(t) + bx^2(t)$$
Let
$$x_1(t) = \cos(2\pi f_c t) + m(t)$$
$$x_2(t) = \cos(2\pi f_c t) - m(t)$$
It we apply $x_1(t)$ and $x_2(t)$ to the nonlinear modulator and look at the difference
$$\begin{aligned} y_1(t) - y_2(t) &= a(\cos(2\pi f_c t) + m(t)) + b(\cos(2\pi f_c t) + m(t))^2 \\ &\quad - a(\cos(2\pi f_c t) - m(t)) - b(\cos(2\pi f_c t) - m(t))^2 \\ &= 2a\, m(t) + 4b\, m(t) \cos(2\pi f_c t) \end{aligned}$$
Convince yourself this is true!

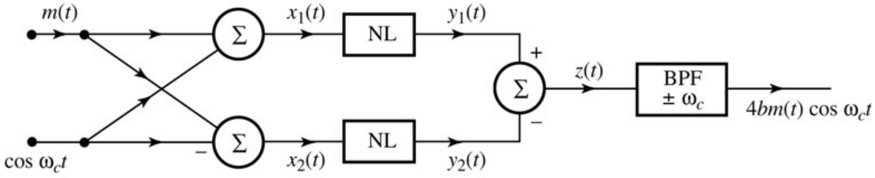
Types of Modulators (cont.)

From the previous page

$$y_1(t) - y_2(t) = 2a\, m(t) + 4b\, m(t) \cos(2\pi f_c t)$$

This has the term we want at $\omega_c = 2\pi f_c$, plus another copy of the message at baseband.

The unwanted baseband component is blocked by bandpass filter. This could be the antenna or the amplifier.



Or we can just forget about the baseband signal, it won't propagate!

Types of Modulators (cont.)

Switching modulator: multiply message by a simple periodic function.

Suppose $w(t)$ is periodic with a fundament frequency f_c:

$$w(t) = \sum_{n=-\infty}^{\infty} D_n e^{j2\pi f_c n t}$$

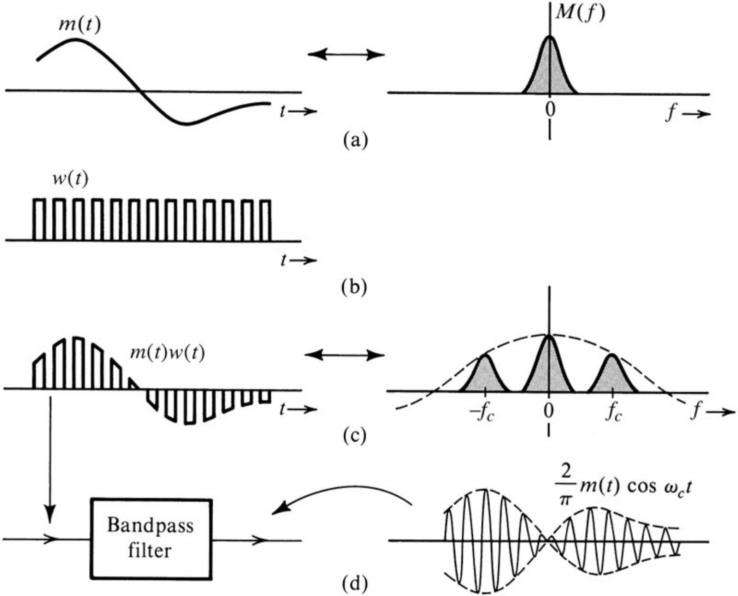
This weighted sum of complex exponentials that are impulses at all multiples of f_c. Then

$$m(t)w(t) = \sum_{n=-\infty}^{\infty} D_n m(t) e^{j2\pi f_c n t}$$

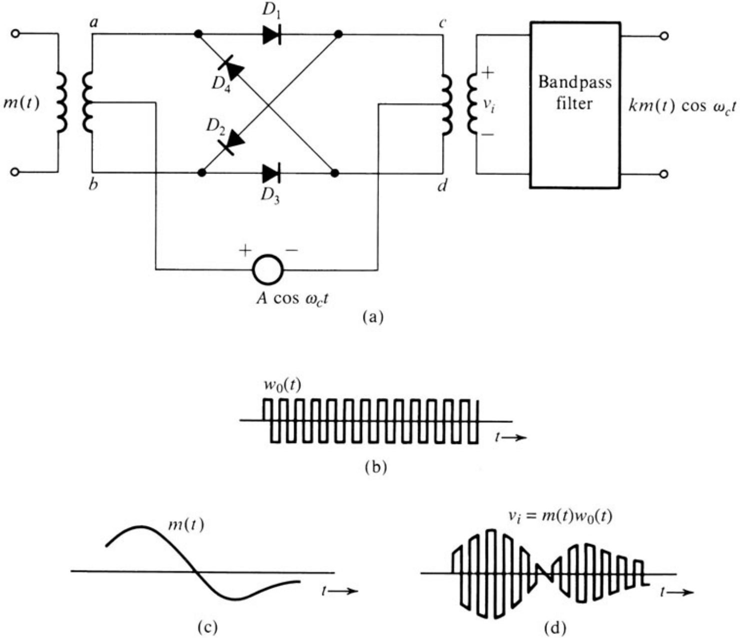
By the convolution theorem, the spectrum of $m(t)w(t)$ consists of $M(f)$ shifted to $\pm f_c, \pm 2f_c, \pm 3f_c, \ldots$ Suppose $w(t)$ is a square wave centered at $t = 0$. Then from Lecture 3,

$$w(t) = \frac{1}{2} + \frac{1}{\pi} \sum_{n=-\infty}^{\infty} \frac{1}{n} e^{j2\pi f_c n t}, \; n \text{ odd}$$

Switching Modulator

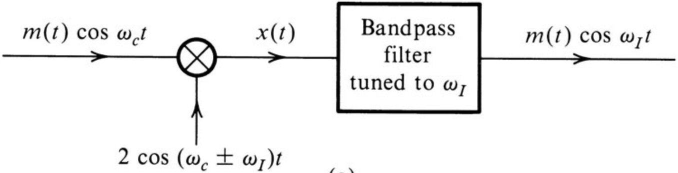


Ring Modulator

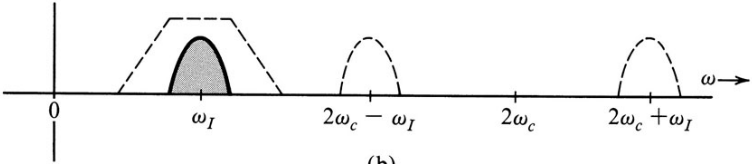


Frequency Converter

Multiplying a modulated signal by a sinusoidal moves the frequency band to sum and difference frequencies.



(a)



(b)

Super-heterodyning: $\omega_{\text{mix}} = \omega_c + \omega_I$.

Sub-heterodyning: $\omega_{\text{mix}} = \omega_c - \omega_I$.

Demodulation of DSB-SC Signals

Both modulator and demodulator use a multiplier by carrier signal.

- Modulator uses bandpass filter
- Demodulator uses lowpass filter

The carrier used by the demodulator must be in phase with the transmitter carrier (taking into account transmission delay).

Such a receiver is called *synchronous*, *coherent*, *homodyne*.

The receiver has a local oscillator that must be adjusted to stay in phase with the received signal.

A voltage-controlled oscillator (VCO) that is controlled by a phase-locked loop (PLL) is commonly used.

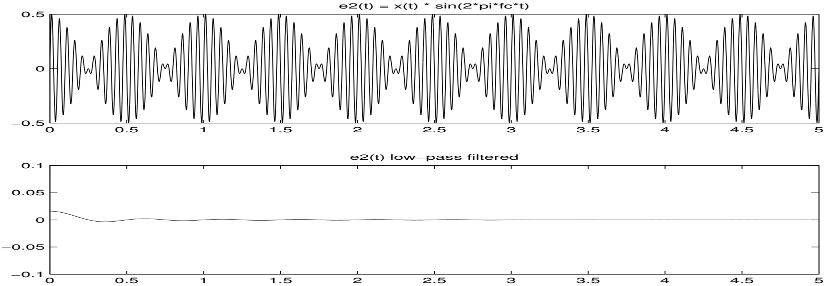
The phase of the carrier in the received signal must be extracted.

Demodulation of DSB-SC Signals (cont.)

Suppose that the signal is not ideal,

$$\begin{aligned} r(t) &= A_c m(t - t_0) \cos\big(2\pi f_c (t - t_0)\big) \\ &= A_c m(t - t_0) \cos\big(2\pi f_c t - \theta_d\big) \end{aligned}$$

where $\theta_d = 2\pi f_c t_0$.



We can end up transmitting with a cosine, and receiving with a sine. These are orthogonal, and we get nothing!

Commercial AM

- If the goal is cheap receivers, then we can eliminate the PLL by transmitting the carrier signal along with the modulated message.

$$\begin{aligned}\varphi_{\mathsf{AM}}(t) &= A\cos(2\pi f_c t) + m(t)\cos(2\pi f_c t) \\ &= (A + m(t))\cos(2\pi f_c t)\end{aligned}$$

- The tone $A\cos(2\pi f_c t)$ contains the desired carrier in correct phase.
- As long as A is larger than $|m(t)|$, then we can recover $m(t)$ from $\varphi_{\mathsf{AM}}(t)$, as we will show next time.

Next time

- Commercial AM, and power
- Single Sideband AM (SSB)
- Vestigial Sideband AM (VSB)
- Quadrature Amplitude Modulation (QAM)