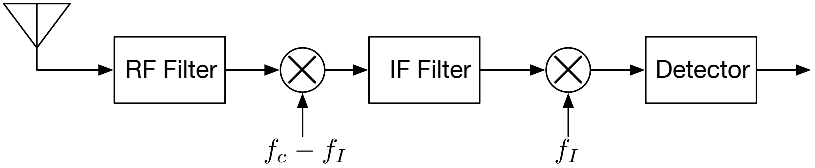
Todays Topics

- Typical Radio Receivers
- Commercial AM
  - Envelope detection
  - AM power
- Single Sideband AM (SSB)
  - SSB idea
  - SSB generation
  - SSD detection

Typical Radio Receiver

Assume we want to listen to a radio signal at f_c. This is a typical receiver.

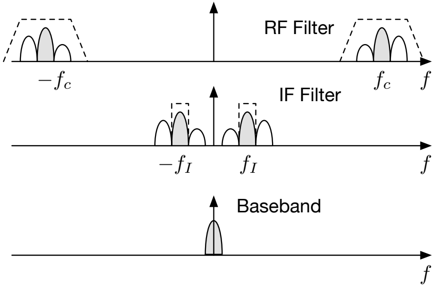


The input RF at f_c is mixed down to a fixed intermediate frequency f_I

- RF filter is not very selective
- First modulation frequency is adjustable
- The IF filter is selective
- Everything from the IF filter onward does not change with tuning

Typical Radio Receiver Spectrum

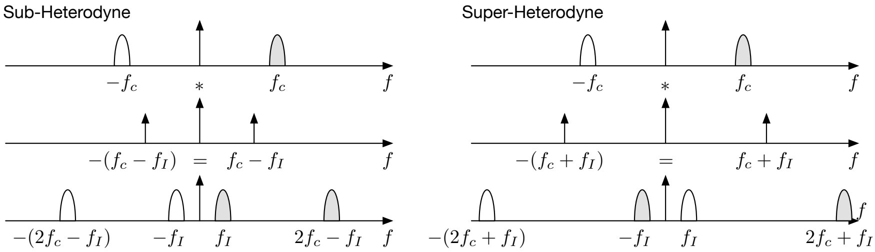
The spectrum of the signals in the receiver look like this:



- RF filter selects part of the band of interest.
- IF filter selects the signal you are interested in.
- With your SDR, you can sample directly instead of using IF filter and do the rest in software.

Frequency Translation

The key to this receiver is being able to translate signals in frequency.



- To help keep track of what is happening, one of the bands has been shaded gray. In fact, both are the same.
- Both produce the same IF signals.

AM Modulation

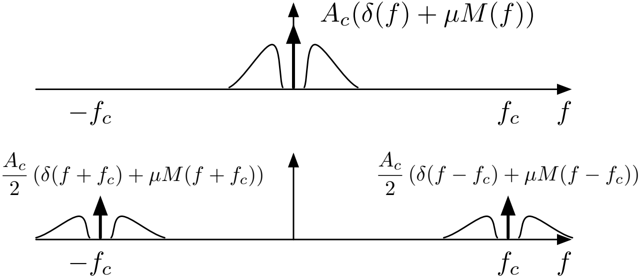
AM modulation is a form of amplitude modulation. For $\mu > 0$,

$$s(t) = (A_c + m(t))\cos(2\pi f_c t) = A_c(1 + \mu m(t))\cos(2\pi f_c t)$$

We need bandwidth of $m(t) \ll f_c$ and modulation index $\mu < 1$.

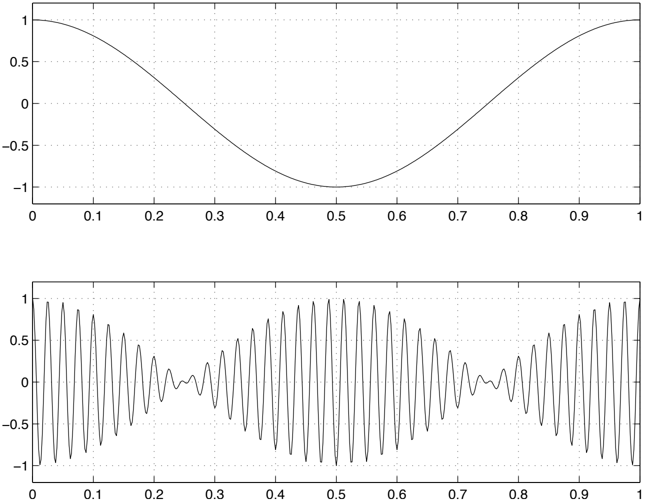
Spectrum of modulated signal:

$$S(f) = \frac{A_c}{2}\left(\delta(f + f_c) + \delta(f - f_c)\right) + \frac{A_c \mu}{2}\left(M(f + f_c) + M(f - f_c)\right)$$



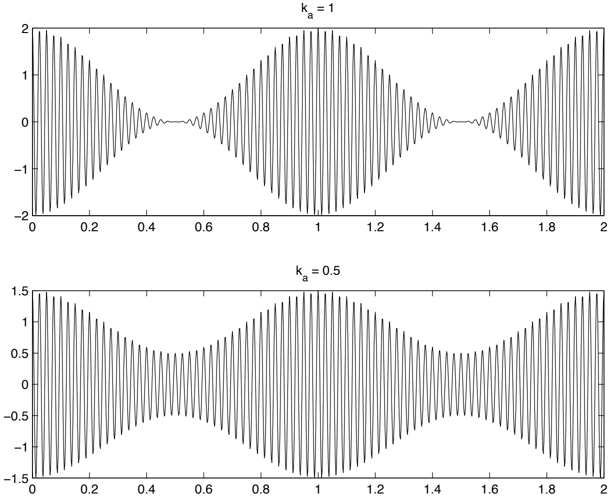
DSB-SC vs. AM

DSB-SC modulated signals undergo phase reversal when $m(t)$ changes sign. It is difficult to extract carrier from received signal.

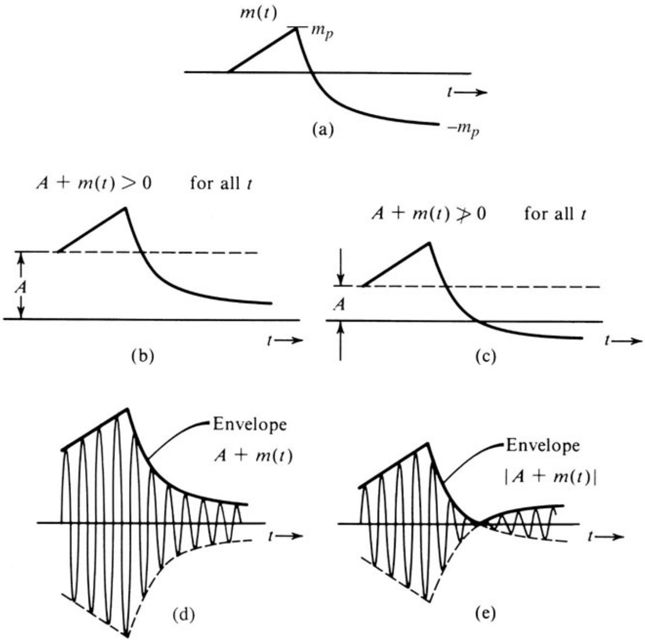


DSB-SC vs. AM (cont.)

In AM, the carrier signal is modulated by $A_c + m(t) = A_c(1 + \mu m(t))$.
Examples: $\mu = 1$ and $\mu = 0.5$.



AM Signal and Its Envelope



Envelope Detection of AM Signals

The term *detection* means extracting signal from received data. In some cases it means demodulation.

Suppose that a signal $x(t)$ can be written as

$$x(t) = E(t)\cos(2\pi f_c t)$$

where $E(t)$ varies slowly compared to the carrier $\cos(2\pi f_c t)$.

Then $|E(t)|$ is called the envelope of $x(t)$.

For envelope detection to work, we need

- $f_c \gg$ bandwidth of $m(t)$
  Otherwise positive and negative spectral components overlap.
- $A + m(t) \geq 0$
  Otherwise phase reversals occur when $A + m(t) < 0$.

Modulation Index

The maximum deviation of $m(t)$ from zero is

$$m_p = \max(|m(t)|)$$

The *modulation index* of the modulated signal is defined by

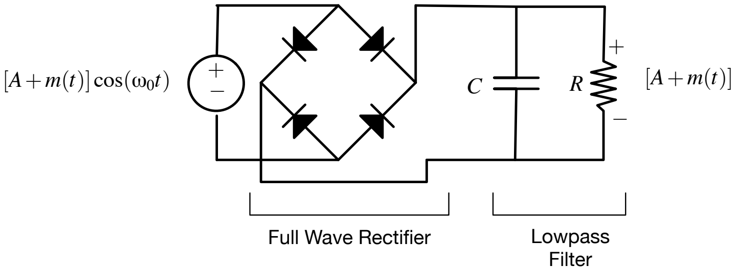
$$\mu = \frac{m_p}{A}$$

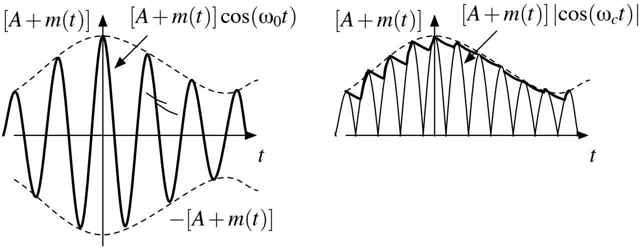
Larger modulation index reduces power but makes demodulation harder.

Broadcast AM stations use modulation index close to 1. Input signals are controlled using automatic gain control (AGC).

Envelope Detector for AM

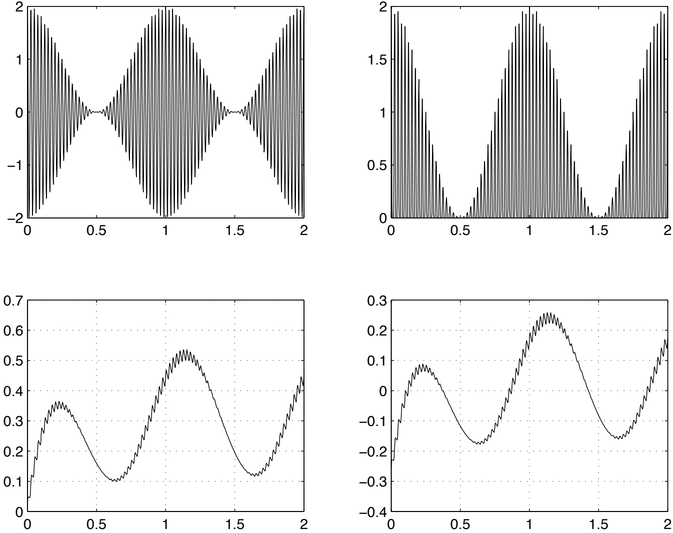
Rectify the RF signal, then lowpass filter:





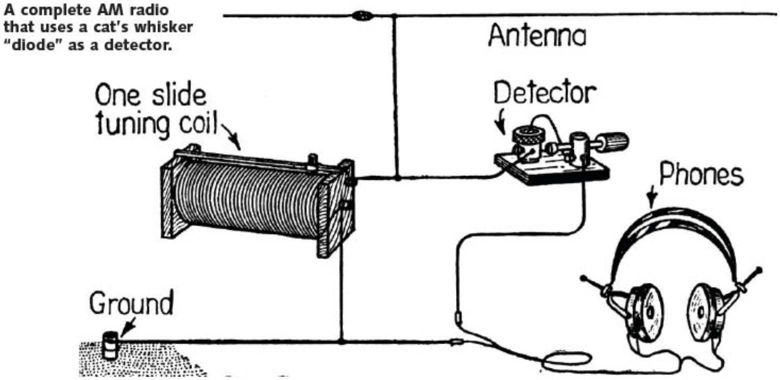
AM Demodulation Experiment

$$m(t) = \cos 2\pi t\,,\ f_c = 10\,,\ h(t) = \frac{1}{RC}e^{-t/RC}u(t)$$



Cat's Whiskers (Crystal) Radio

This radio was powered only by received radio energy.



Wikipedia: Crystal radio wiring pictorial based on Figure 33 in Gernsback's 1922 book *Radio For All* (copyright expired) with "Aerial" changed to Antenna by J.A. Davidson.

The point-contact semiconductor detector was subsequently resurrected around World War II because of the military requirement for microwave radar detectors.

Power of AM Signals

The power of an AM signal is the sum of the power of two components.

$$\varphi_{\mathrm{AM}}(t) = (A + m(t))\cos(2\pi f_c t) = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t)\cos(2\pi f_c t)}_{\text{sidebands}}$$

Carrier and sideband signals are orthogonal, so powers add. Carrier power:

$$P_c = A^2 \int_0^T \cos^2(2\pi f_c t)\, dt = \frac{1}{2}A^2$$

Signal power after modulation is 1/2 the original message power:

$$P_s = \tfrac{1}{2} P_m \,,$$

where message power is average power as T gets large,

$$P_m = \overline{m^2(t)} = \frac{1}{T}\int_{t_0}^{t_0+T} m^2(t)\, dt$$

E.g., the power of a tone $\cos(2\pi f_m t)$ is $\frac{1}{2}$.

Power of AM Signals (cont.)

The carrier tone simplifies demodulation but carries no information.

The power efficiency is defined by

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s} = \frac{\overline{m^2(t)}}{A^2 + \overline{m^2(t)}}$$

Examples: tone modulation $m(t) = \mu A \cos \omega_c t$ where $0 < \mu \leq 1$.

$$\eta = \frac{\frac{1}{2}(\mu A)^2/2}{A^2/2 + \frac{1}{2}(\mu A)^2/2} = \frac{\mu^2}{2 + \mu^2}$$

The efficiency increases with μ. The maximum value is $1/3$.

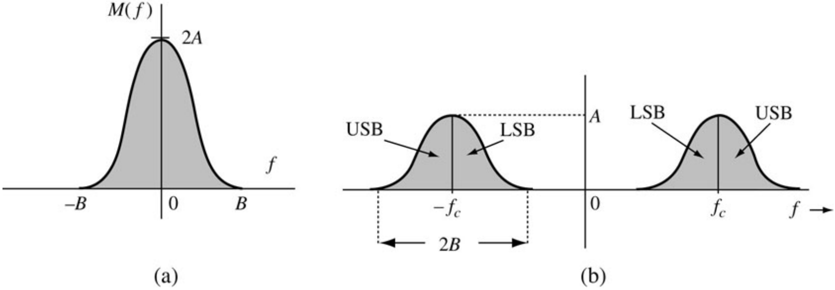
Efficiency falls off rapidly as μ decreases. For $\mu = 0.5$,

$$\eta = \frac{(0.5)^2}{2 + (0.5)^2} = \frac{1}{9}$$

AM is inefficient in both power and bandwidth.

Single Sideband (SSB)

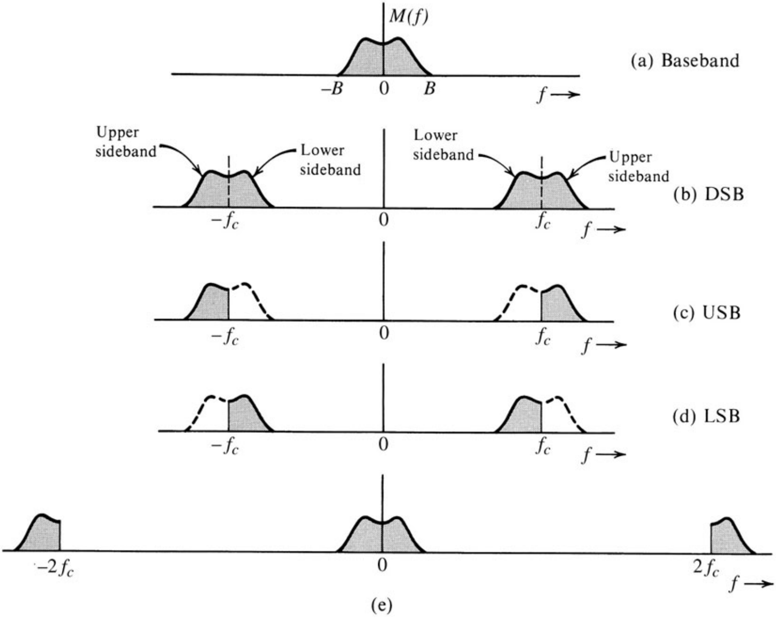
DSB-SC is spectrally inefficient. It uses twice the bandwidth of the message.



The signal can be reconstructed from either the upper sideband (USB) or lower sideband.

SSB transmits a bandpass filtered version of the modulated signal.

Single Sideband (cont.)



Single Sideband Modulation and Demodulation

- SSB can be transmitted using a DSB-SC modulator with a narrower bandpass filter.

  For USB, center frequency is

  $$\tilde{f}_c = f_c + \tfrac{1}{2}B$$

  and cutoff frequency is $B/2$.

  The bandfilter must roll off quickly to eliminate unwanted contributions from the other sideband.

  Message frequencies near 0 will be affected by the nonideal filter.

- SSB demodulation can use a DSB-SC demodulator with no change.

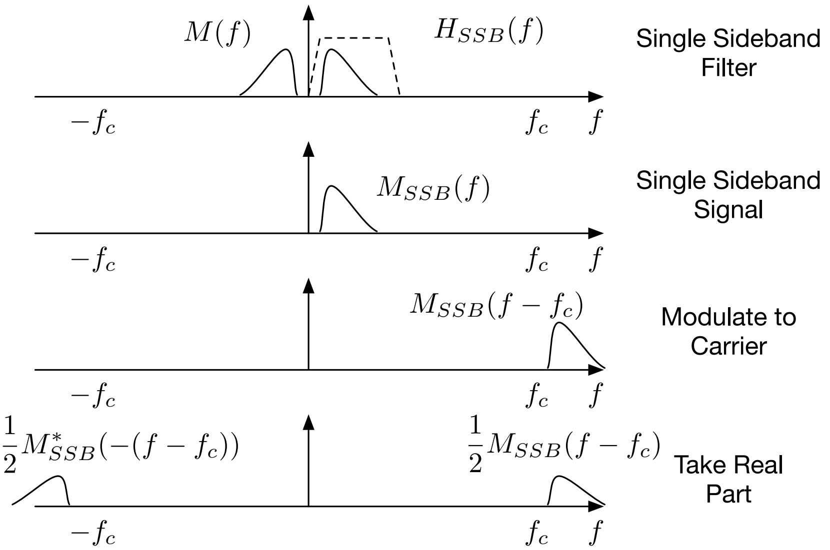
  The input to the lowpass filter is different from that of DSB-SC.

Which SSB Sideband?

- Transmitter and receiver must agree on use of LSB vs. USB.
- SSB is common for amateur radio.
  - Below 10 MHz: LSB
  - Above 10 MHz: USB
  - Exception for 5 MHz: USB
  - Exception for digital modes: USB
- SSB is also common for shortwave.
  - 120m (2300-2495 kHz): LSB
  - 90m (3200-3400 kHz): LSB
  - 75m (3900-4000 kHz): USB
  - 60m (4750-5060 kHz): LSB
  - 49m (5900-6200 kHz): USB
  - 41m (7200-7450 kHz): USB

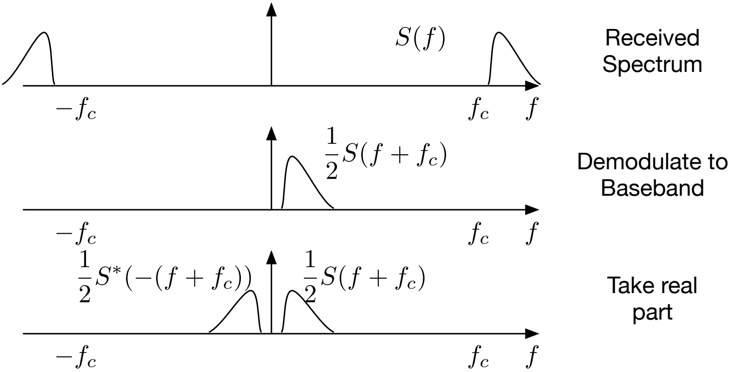
  Your radio knows!

SSB Modulation



SSB Demodulation

To decode the SSB signal, we just reverse the operations



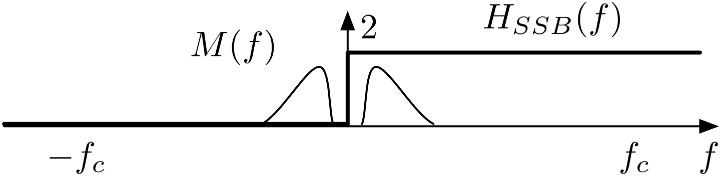
- Ideally we want a synchronous demodulator
- In practice, f_c is estimated by the sound of the signal
- An error of 50 Hz is quite noticeable

SSB in Time Domain

The upper sideband is output of filtering modulated signal $m(t)\cos\omega_c t$ with an ideal bandpass filter:

$$H_{\mathrm{SSB}}(f) = 2u(f) = \begin{cases} 2 & f > 0 \\ 0 & f < 0 \end{cases}$$

This looks like



The impulse response of this filter is

$$h_{SSB}(t) = \mathcal{F}^{-1}\left\{2u(f)\right\}$$

Hilbert Transform

We know

$$u(t) \rightleftharpoons \tfrac{1}{2}\delta(f) + \frac{1}{j2\pi f}$$

By duality, and multiplying by 2

$$\delta(t) + \frac{j}{\pi t} \rightleftharpoons 2u(f)$$

The impulse response of the filter is

$$h_{SSB}(t) = \delta(t) + \frac{j}{\pi t}$$

If $m(t)$ is the input signal, the single sideband signal is

$$m(t) * h_{SSB}(t) = m(t) * \left(\delta(t) + \frac{j}{\pi t}\right) = m(t) + j\left(m(t) * \frac{1}{\pi t}\right)$$

The last term is the *Hilbert transform* of $m(t)$:

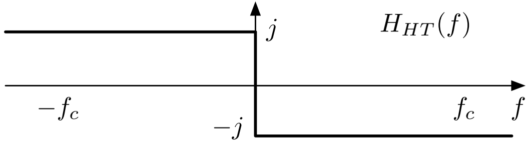
$$m_h(t) = m(t) * \frac{1}{\pi t}$$

Hilbert Transform (cont.)

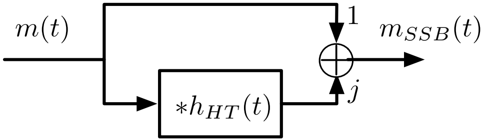
The transfer function of the Hilbert transform is

$$H_{\mathrm{HT}}(f) = -j\,\mathrm{sgn}(f) = \begin{cases} -j & f > 0 \\ j & f < 0 \end{cases}$$

which looks like



The block diagram is



What happens if $m(t) = \cos(2\pi f t)$ or $m(t) = \sin(2\pi f t)$?