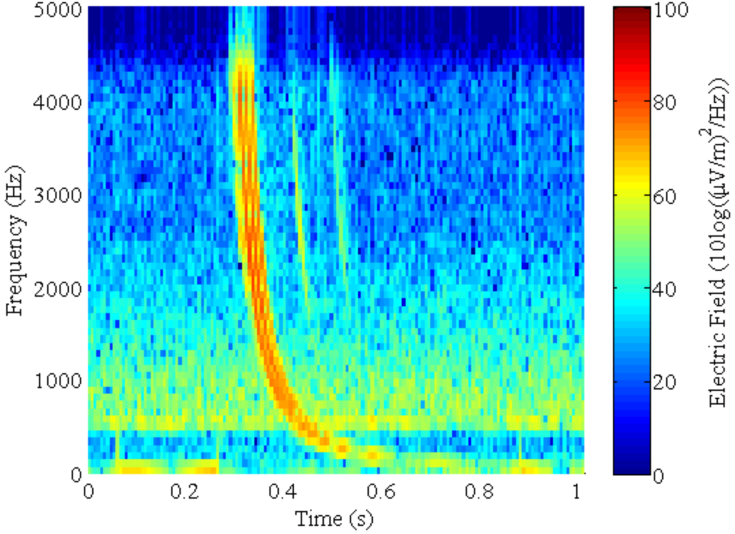
Angle Modulation

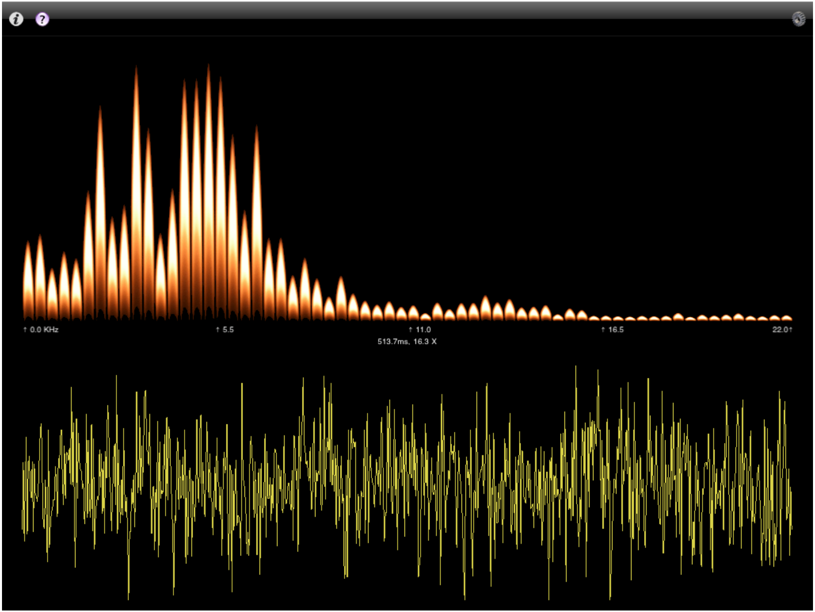
- Time-varying frequency
- Introduction to angle modulation
- Relationship between FM and PM
- FM bandwidth

Spectrogram

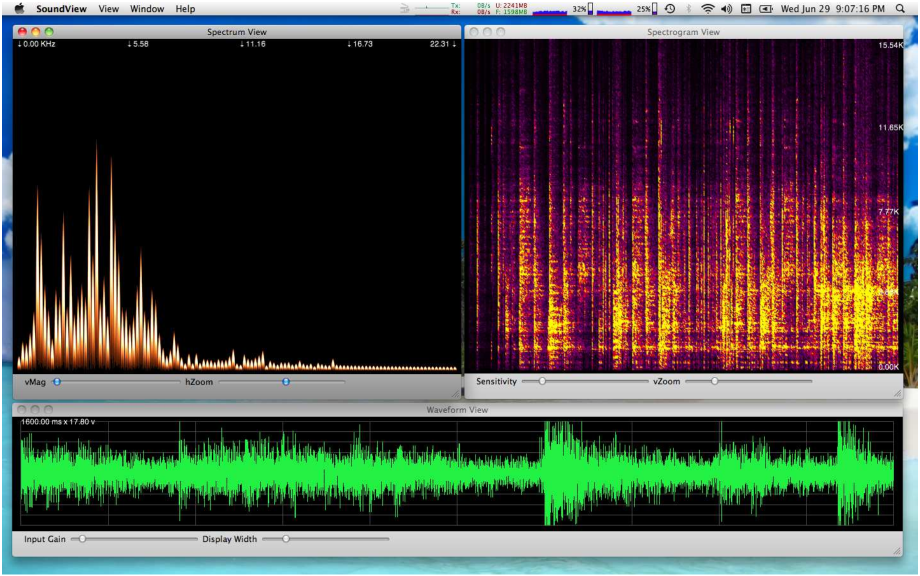
This spectrogram shows a *whistler* in the VLF band.



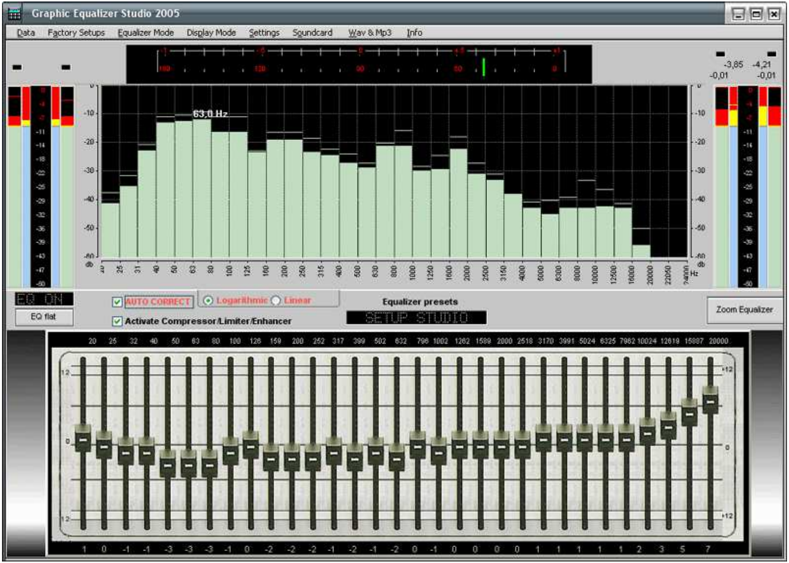
Spectrum Analyzer



Spectrum Analyzer (cont.)



Graphic Equalizer



Instantaneous Frequency

- In principle, frequency of a signal at an instant in time depends on entire signal (Hilbert transform)
- For generalized sinusoids, we can use a simpler approach. Suppose

$$\varphi(t) = A\cos\theta(t)\,.$$

  Then $\theta(t)$ is the *generalized angle*. For a true sinusoid,

$$\theta(t) = \omega_c t + \theta_0\,.$$

  This is linear with slope ω_c and offset θ_0
- The generalized angle is *not* limited to $[0, 2\pi]$. Wrapping introduces discontinuities.
- MATLAB has `unwrap` function to remove discontinuities.

Instantaneous Frequency (cont.)

- Instananeous frequency is derivative of generalized angle:

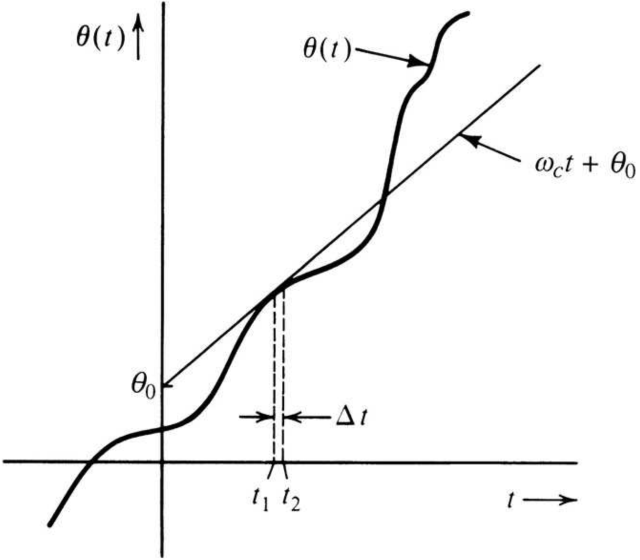
$$\omega_i(t) = \frac{d\theta}{dt} = \theta'(t)$$

- By Fundamental Theorem of Calculus,

$$\theta(t) = \int_{-\infty}^{t} \omega_i(u)\, du = \theta(0) + \int_{0}^{t} \omega_i(u)\, du$$

- We can modulate a generalized sinusoid by using a signal $m(t)$ to vary either $\theta(t)$ or $\omega_i(t)$.
- In either case, the frequency of the modulated signal changes as a function of $m(t)$.

Instantaneous Frequency (cont.)



Phase Modulation (PM) & Frequency Modulation (FM)

- In PM, *phase* is varied *linearly* with $m(t)$:

$$\theta(t) = \omega_c t + k_p m(t) \;\Rightarrow\; \varphi_{\mathrm{PM}}(t) = \cos\big(\omega_c t + k_p m(t)\big)$$

The instantaneous frequency is

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t)$$

If $m(t)$ varies rapidly, then the frequency deviations are larger.

- In FM, *frequency deviation* is *linear* in $m(t)$:

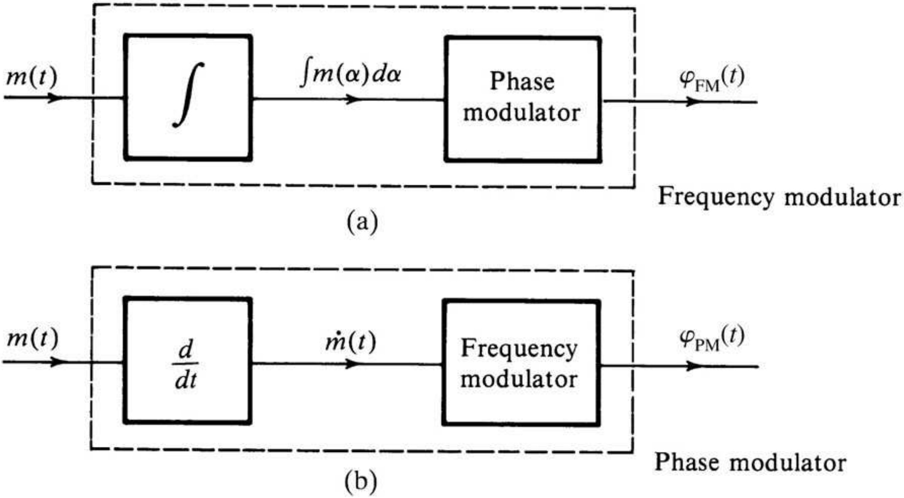
$$\omega_i(t) = \omega_c + k_f m(t)$$

The angle is

$$\theta(t) = \int_{-\infty}^{t} \big(\omega_c + k_f m(t)\big)\, du = \omega_c t + k_f \int_{-\infty}^{t} m(u)\, du$$

Relationship Between FM and PM

- Phase modulation of $m(t)$ = frequency modulation of $\dot{m}(t)$.
- Frequency modulation of $m(t)$ = phase modulation of $\int m(u)\,du$.



Generalized Angle Modulation

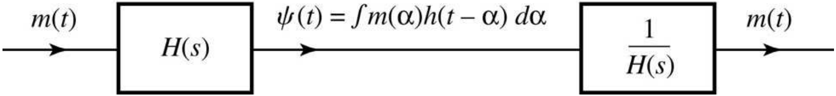
- We can vary phase using a linear transform of message signal:

$$\begin{aligned}\varphi_{\mathrm{EM}}(t) &= A\cos\left(\omega_c t + h(t) * m(t)\right)\\ &= A\cos\left(\omega_c t + \int_{-\infty}^{t} m(u)h(t-u)\,du\right)\end{aligned}$$

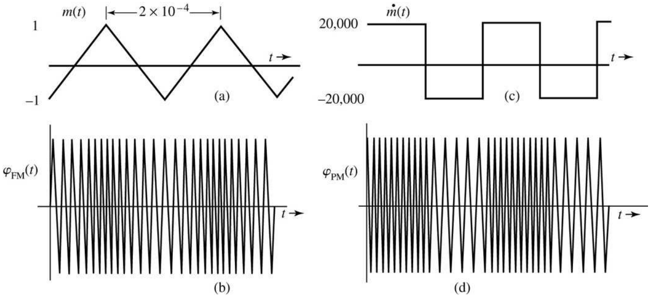
  We assume that impulse response $h(t)$ is causal.

- PM ($h(t) = k_p\delta(t)$) and FM ($h(t) = k_f u(t)$) are special cases.
- We recover $m(t)$ from phase by using inverse filter $H^{-1}(s)$.

  E.g., for FM the inverse of integration is differention.



FM Example: $f_c = 100$ MHz, $k_f = 2\pi \times 10^5$



Instantaneous frequency:

$$f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$$

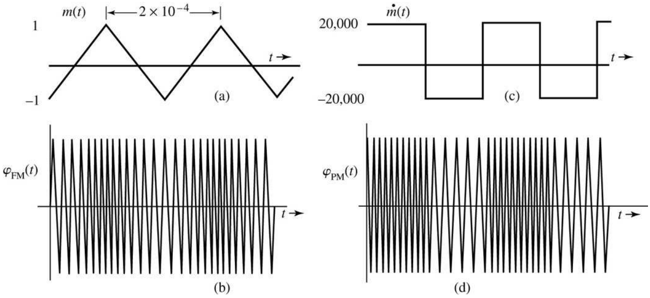
$$f_{i,\min} = 99.9 \text{ MHz}$$

$$f_{i,\max} = 100.1 \text{ MHz}$$

Is the modulated signal confined to the frequency band $99.9^- - 10.1^+$?

No! The bandwidth is approximately 300 KHz.

PM Example: $f_c = 100$ MHz, $k_p = 10\pi$



Instantaneous frequency:

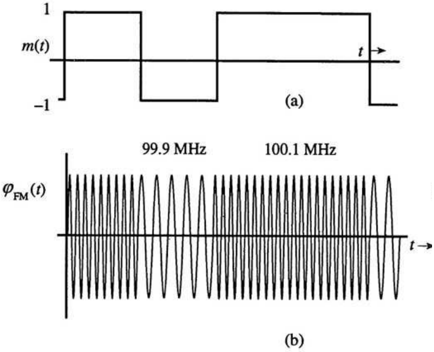
$$f_i = f_c + \frac{k_p}{2\pi}\dot{m}(t) = 10^8 + 5\dot{m}(t)$$

$$f_{i,\min} = 99.9 \text{ MHz}$$

$$f_{i,\max} = 100.1 \text{ MHz}$$

FM modulation by $\dot{m}(t)$ is same as PM modulation by $m(t)$.

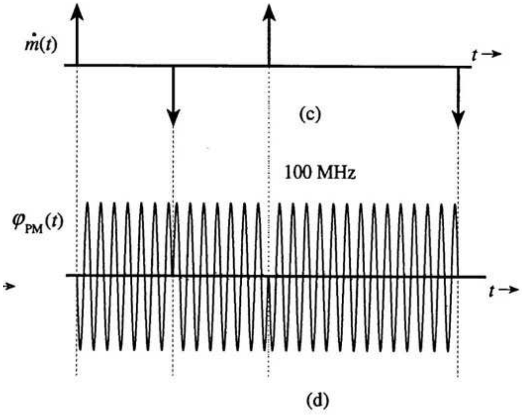
Frequency Shift Keying (FSK)



The binary message can be detected using two filters tuned for the two frequencies.

FSK is a historically important digital modulation scheme. The Bell 103 modem used frequencies 1070 and 1270 for originating station.

Phase Shift Keying (PSK)



The binary message can be detected by correlating with sinusoids of a fixed frequency but different phases.

The sinusoids are 180° out of phase but synchronized with the carrier.

PSK: Direct Modulation

If $k_p = \pi/2$ and $m(t)$ takes on only values $+1$ and -1, then

$$\begin{aligned}\varphi_{\mathrm{FM}}(t) &= A\cos\big(\omega_c t + k_p m(t)\big)\\ &= A\cos\Big(\omega_c t + \frac{\pi}{2} m(t)\Big)\\ &= \begin{cases} A\sin\omega_c t & \text{when } m(t) = -1\\ -A\sin\omega_c t & \text{when } m(t) = 1\end{cases}\end{aligned}$$

The figure on the previous slide corresponds to bit duration $7/f_c$, where f_c is the carrier frequency.

Phase changes of $\pm\pi$ are possible at the end of each bit period.
The transmitter must low pass filter the PSK signal.