Angle Modulation, III

Lecture topics

- FM Modulation (review)
- FM Demodulation
- Spectral pre-emphasis/de-emphasis to improve SNR

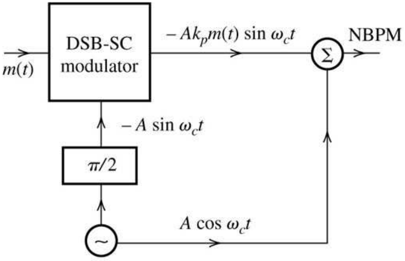
NBFM Modulation

For narrowband signals, $|k_f a(t)| \ll 1$ and $|k_p m(t)| \ll 1$,

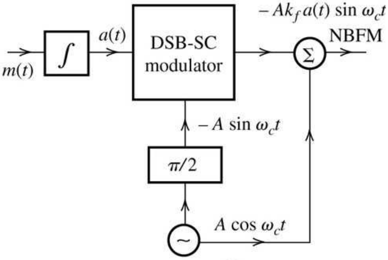
$$\hat{\varphi}_{\text{NBFM}} \approx A(\cos\omega_c - k_f a(t)\sin\omega_c t)$$

$$\hat{\varphi}_{\text{NBPM}} \approx A(\cos\omega_c - k_f m(t)\sin\omega_c t)$$

We can use a DSB-SC modulator with a phase shifter.



(a)



(b)

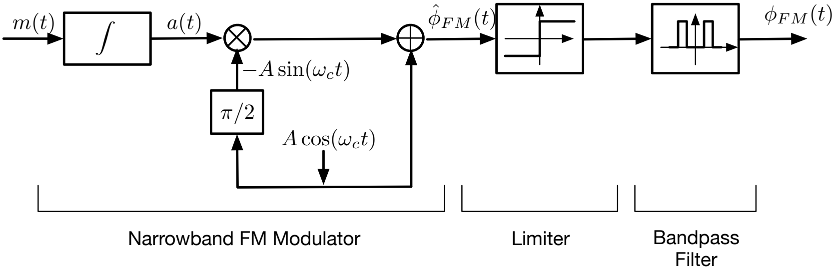
Phase modulation

Frequency modulation

NBFM: Bandpass Limiter

In practice, this modulation will not be perfect, and there will be some amplitude modulation remaining.

To fix this, follow with a limiter and a bandpass filter. For FM,



For PM, the message $m(t)$ does not need to be integrated.

Armstrong's indirect method

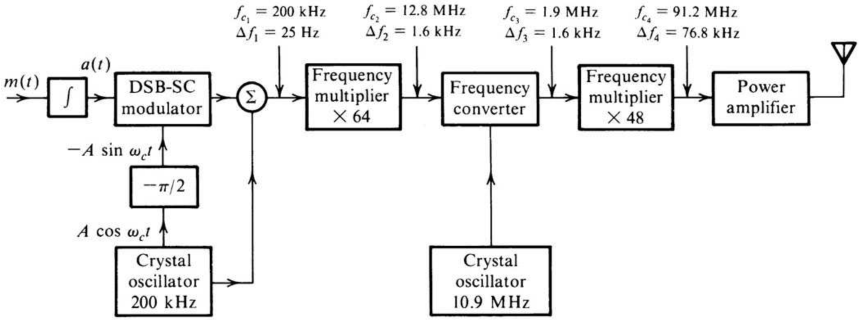
Armstrong wanted to generate WBFM using NBFM and frequency multipliers. Suppose we have a nonlinear device with the response

$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) + \cdots + a_n x^n(t)$$

Let $a(t) = \int_{-\infty}^{t} m(u)\,du$ and $x(t) = A\cos(\omega_c t + k_f a(t))$. Then

$$y(t) \quad = \quad a_0 + a_1 \cos(\omega_c t + k_f a(t)) + \cdots + a_n (\cos(\omega_c t + k_f a(t)))^n$$

A bandpass filter isolate the FM signal with carrier frequency $n\omega_c$.



FM Demodulation

- Frequency-selective filter
  - RC high-pass filter:

$$H(f) = \frac{j2\pi RC}{1 + j2\pi RC} \approx j2\pi RC \quad (2\pi RC \ll 1)$$

  - RLC circuit with carrier frequency $\omega_c < \omega_0 = 1/\sqrt{LC}$
- Differentiator
- Slope detection
- Zero-crossing detectors
- Phase-locked loop (not discussed today)

Derivative Theorem for Fourier Transform

If $G(f)$ is the Fourier transform of $g(t)$, then

$$\frac{dg(t)}{dt} \rightleftharpoons j2\pi f\, G(f)$$

and

$$\int_{-\infty}^{t} g(\tau)\, d\tau \rightleftharpoons \frac{G(f)}{j2\pi f} + \tfrac{1}{2}G(0)\delta(f)$$

"Proof":

$$\begin{aligned}\frac{dg(t)}{dt} &= \frac{d}{dt}\int_{-\infty}^{\infty} G(f)e^{j2\pi ft}\, dt \\ &= \int_{-\infty}^{\infty} \frac{d}{dt}G(f)e^{j2\pi ft}\, dt = \int_{-\infty}^{\infty} j2\pi f\, G(f)\, e^{j2\pi ft}\, dt\end{aligned}$$

By the Fourier inversion theorem, $j2\pi f\, G(f)$ is transform of $g'(t)$.

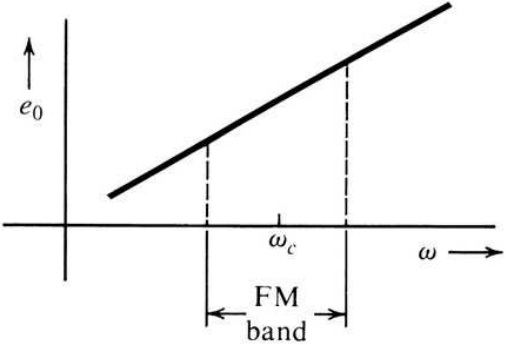
Slope-Detecting Filter

Information in an FM signal is contained in the instantaneous frequency

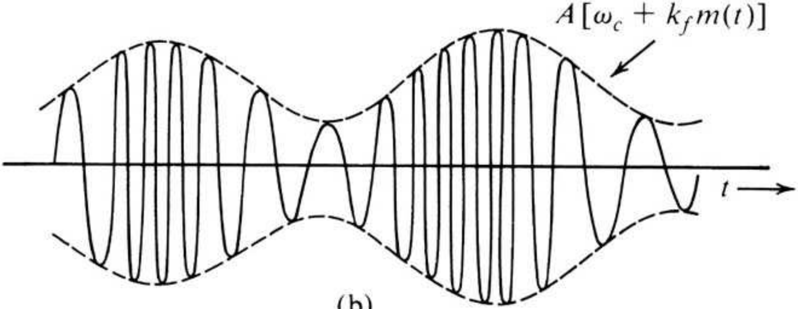
$$\omega_i(t) = \omega_c + k_f m(t)$$

We can extract ω_i using a *slope-detecting filter*, where

$$|H(f)| = a2\pi f + b$$



FM Demodulator and Differentiator



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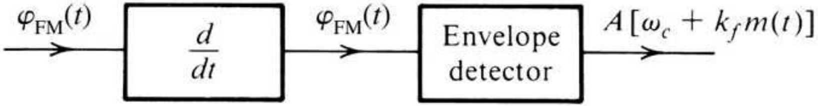
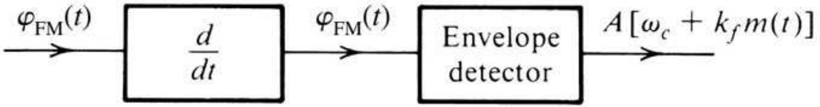


(c)

Envelope Detection using Ideal Differentiator



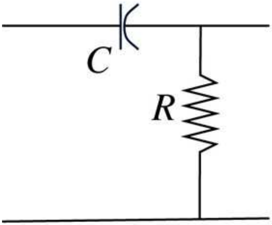
If $\Delta\omega = k_f m_p < \omega_c$ we can use envelope detection.

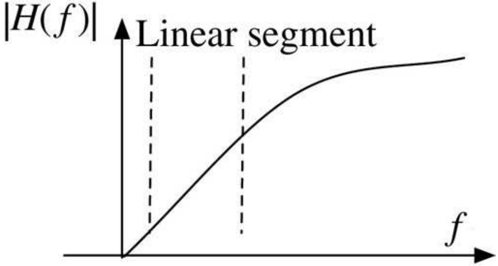
$$\begin{aligned}
\dot{\varphi}_{\text{FM}}(t) &= \frac{d}{dt}\left(A\cos\left(\omega_c t + k_f \int_{-\infty}^{t} m(u)\,du\right)\right) \\
&= -A\,\sin\left(\omega_c t + k_f \int_{-\infty}^{t} m(u)\,du\right)(\omega_c + k_f m(t)) \\
&= A(\omega_c + k_f m(t))\,\sin\left(\omega_c t + k_f \int_{-\infty}^{t} m(u)\,du - \pi\right)
\end{aligned}$$

Envelope of $\dot{\varphi}_{\text{FM}}(t)$ is $A(\omega_c + k_f m(t))$. Important that A is constant.

FM Detection Circuits

RC high-pass filter.





The transfer function is

$$H(f) = \frac{j2\pi RCf}{1 + j2\pi RCf} \approx j2\pi RCf \quad (2\pi RC \ll 1)$$

The impulse response is

$$h(t) = \delta(t) - e^{t/RC}u(t)$$

Advantages of FM

FM is less susceptible to amplifier nonlinearities. If input is

$$x(t) = A\cos(\omega_c t + \psi(t))$$

and the output is

$$\begin{aligned} y(t) &= a_0 + a_1 x(t) + a_2 x^2(t) + \cdots \\ &= c_0 + c_1 \cos(\omega_c t + \psi(t)) + c_2 \cos(2\omega_c t + 2\psi(t)) + \cdots \end{aligned}$$

The extra terms have spectrum outside the carrier signal band.
They will be blocked by bandpass filter.

Nonlinearities in AM cause signal distortion. For $y(t) = ax(t) + bx^3(t)$,

$$\begin{aligned} y(t) &= am(t)\cos\omega_c t + bm^3(t)\cos^3\omega_c t \\ &= (am(t) + \tfrac{3}{4}bm^3(t))\cos\omega_c t + \tfrac{1}{4}b\cos 3\omega_c t \end{aligned}$$

FM is preferred for high-power applications, such as microwave relay towers.

Advantages of FM (cont.)

FM can adjust to rapid fading (change of amplitude) using automatic gain control (AGC).

FM is less vulnerable to signal interference from adjacent channels. Suppose interference is $I\cos((\omega_c + \omega)t)$. Then received signal is

$$
\begin{aligned}
r(t) &= A\cos(\omega_c t) + I\cos((\omega_c + \omega)t) \\
&= (A + I\cos\omega t)\cos\omega_c t - I\sin\omega t\sin\omega_c t \\
&= E_r(t)\cos(\omega_c t + \psi(t))
\end{aligned}
$$

where

$$\psi(t) = \tan^{-1}\left(\frac{I\sin\omega t}{A + I\cos\omega t}\right) \approx \frac{I}{A}\sin\omega t \quad (I \ll A)$$

The output of an ideal frequency modulator is $\dot{\psi}(t)$ for FM is

$$y_d(t) = \frac{I\omega}{A}\cos\omega t\,,$$

which is inversely proportional to amplitude A.

Noise and FM

Suppose that the power spectrum of noise is flat over an FM channel. E.g., *white noise* has constant power spectrum

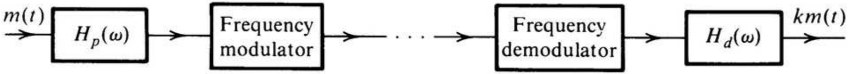
$$H_Z(f) = \frac{N_0}{2}$$

The power of the noise in a frequency band of width $2B$ is

$$2\int_{f_c-B}^{f_c+B} \frac{N_0}{2}\, df = 2BN_0$$

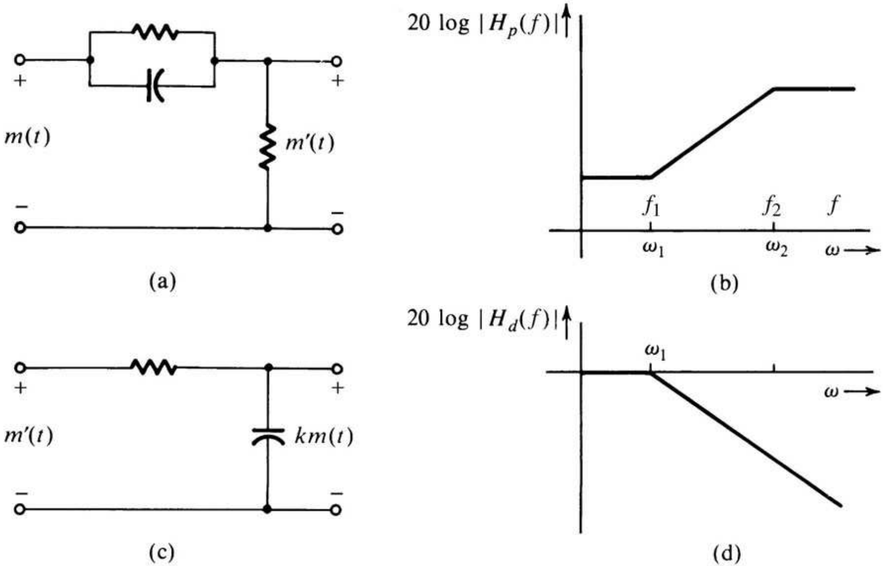
The transfer function for FM demodulator satisfies $|H(f)| = af + b$. This filter increases noise at higher frequencies.

We can reduce high frequency noise by using pre-emphasis/de-emphasis.



FM Pre-emphasis and De-emphasis

Pre-emphasis: RLC high pass filter. De-emphasis: RC low pass filter.



FM Pre-emphasis and De-emphasis (cont.)

The linear pre-emphasis range is $f_1 = 2.1$ kHz to $f_2 = 30$ kHz. Pre-emphasis filter has transfer function

$$H_p(f) = \frac{f_2}{f_1} \frac{f_1 + j2\pi f}{f_2 + j2\pi f}$$

If $f \ll f_1$ then $H_p(f) \approx 1$.

If $f_1 \ll f \ll f_2$ then

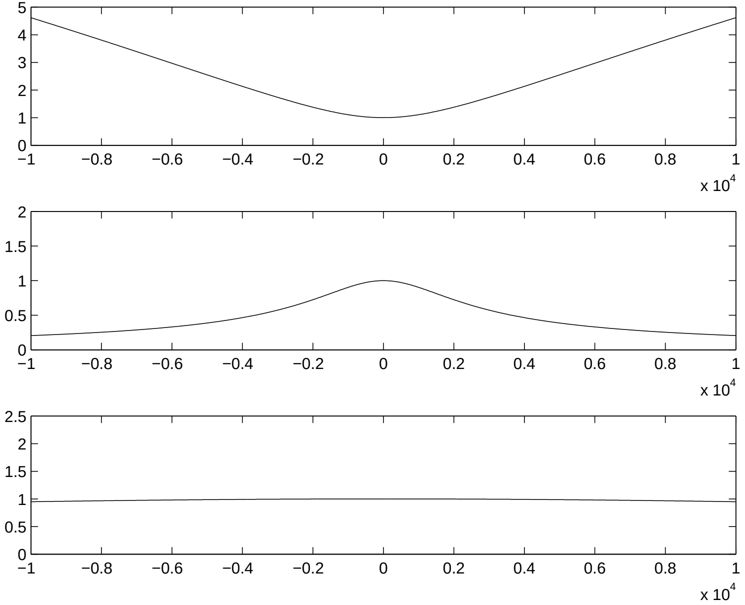
$$H_p(f) \approx \frac{j2\pi f}{f_1}$$

which is a differentiator!

The corresponding de-emphasis filter has transfer function.

$$H_d(f) = \frac{f_1}{j2\pi f + f_1} \approx \frac{1}{H_p(f)}$$

FM Pre-emphasis and De-emphasis Filters



FM Pre-emphasis and De-emphasis Filters (cont.)