Digital Carrier Modulation

Lecture topics

- Eye diagrams
- Pulse amplitude modulation (PAM)
- Binary digital modulation
  - Amplitude shift keying (ASK)
  - Frequency shift keying (FSK)
  - Phase shift keying (PSK)
- Quadrature PSK and QAM

Based on lecture notes from John Gill
Polar Signaling with Raised Cosine Transform \((r = 0.5)\)

\[
P(f) = \begin{cases} 
1 & |f| < \frac{1}{4} R_b \\
\frac{1}{2} \left(1 - \sin \pi \left(\frac{f - \frac{1}{2} R_b}{R_b}\right)\right) & \left|\frac{f}{2} - \frac{1}{2} R_b\right| < \frac{1}{2} R_b \\
0 & |f| > \frac{3}{4} R_b 
\end{cases}
\]
Polar Signaling with Raised Cosine Transform ($r = 0.5$)

The pulse corresponding to $P(f)$ is

$$p(t) = \text{sinc}(\pi R_b t) \frac{\cos(\pi r R_b t)}{1 - 4r^2 R_b^2 t^2}$$
Eye Diagram Measurements

- Maximum opening affects noise margin
- Slope of signal determines sensitivity to timing jitter
- Level crossing timing jitter affects clock extraction
- Area of opening is also related to noise margin
PAM: $M$-ary Baseband Signaling

We can generalize polar signaling to

$$y(t) = \sum_{k} a_k p(t - kT_b)$$

where $a_k$ is chosen from a set of more than two values (i.e., not just $\pm 1$).

Example: one widely used encoding of two bits into four levels is

$$a_k = \begin{cases} 
-3 & \text{message bits 00} \\
-1 & \text{message bits 01} \\
+1 & \text{message bits 11} \\
+3 & \text{message bits 10} 
\end{cases}$$

This is used in ISDN.
PAM: $M$-ary Baseband Signaling (cont.)

Power of 4-ary signaling:

$$R_0 = \frac{1}{4}((-3)^2 + (-1)^2 + 1^2 + 3^2) = \frac{1}{4} \cdot 20 = 5.$$ 

If digital values are independent, $R_n = 0$ for $n \neq 0$. Thus PSD is

$$S_y(f) = \frac{5}{T_s}|P_x(f)|^2,$$

The PSD is the same as binary signaling. More bits use more power.
On-Off Keying (OOK) = Amplitude Shift Keying (ASK)

Modulated signal is $m(t) \cos 2\pi f_c t$.

Baseband signal may use shaped pulses, so cosine amplitude varies.
OOK Example

Digital input: 1 0 0 1 1 0 1 0 0. Square wave and shaped pulses.
**PSK and FSK**

Binary PSK is the same as polar ASK.

Phase shift keying can use more than two phases (4 and 8 are common). (FSK often uses only two frequencies, but more are not unusual.)
PSD of binary ASK, PSK, FSK

ASK

PSK (same as ASK)

FSK
Demodulation of ASK and PSK

- ASK demodulation
  - envelope detector (signal vs. not signal)
  - coherent detector (requires synchronous detection)
- Binary PSK is equivalent to binary PAM with
  \[ y(t) = \pm A \cos \omega_c t \]

Constant amplitude means envelope detection is not possible.

Coherent binary PSK detector is similar to DSB-SC demodulator.
Demodulation of FSK

FSK can also use envelope or coherent detector. In both cases, these are parallel ASK detectors.

Example: Bell 103 modem (V.21, 300 bps) uses 1270 Hz and 1070 Hz for originating station, only 3 or 4 cycles per bit.
FSK Example: $f_0 = 8$, $f_1 = 12$
Differential PSK (DPSK)

Encode 1 by change of phase, \(0 \rightarrow \pi\) or \(\pi \rightarrow 0\)

- Advantage: local carrier not needed
- Disadvantage: less noise immunity than PSK, more bandwidth, errors occur in pairs
- More than two different phases can be used.
\( M \)-ary Digital Carrier Modulation

- **\( M \)-ary ASK**

\[ \varphi(t) = 0, \ A \cos \omega_c t, \ 2A \cos \omega_c t, \ldots, \ (M - 1)A \cos \omega_c t \]

One symbol contains \( \log_2 M \) bits of information.

Example: \( M = 3, \ \log_2 M = 1.584 \): 2 trits (ternits) > 3 bits

- **\( M \)-ary FSK**

\[ \varphi(t) = A \cos \omega_1 t, \ A \cos \omega_2 t, \ldots, \ A \cos \omega_M t \]

Ideally, the possible signals are orthogonal over a bit period. Then

\[ \omega_m = \omega_1 + (m - 1)\delta f \]

where smallest \( \delta f \) is \( 1/2T_b \). Bandwidth is (Carson’s rule)

\[ 2(\Delta f + B) = \frac{M + 3}{2T_b} \]

Not bandwidth efficient.
\(M\)-ary PSK

- In general,

\[
\varphi_{\text{PSK}}(t) = a_m \sqrt{\frac{2}{T_b}} \cos \omega_c t + b_m \sqrt{\frac{2}{T_b}} \sin \omega_c t
\]

Binary PSK: \(a_m = A \cos \theta_m, \ b_m = -A \cos \theta_m. \) (Ideally, \(\theta_m = 0.\))

- In orthogonal signal space, we use more values \(a_m, b_m\) where \(a^2 + b^2 = A^2.\)

- Bell 212A (1200 bps) uses 4-PSK = 4-QAM
**M-ary QAM**

QAM, like $M$-PSK, uses linear combination of orthogonal sinusoids:

$$
\varphi_{QAM}(t) = a_m \sqrt{\frac{2}{T_b}} \cos \omega_c t + b_m \sqrt{\frac{2}{T_b}} \sin \omega_c t
$$

However, amplitude $A = \sqrt{a^2 + b^2}$ can have more than one value.

V.22bis (2400-bps) uses 16-QAM (3 amplitudes, 12 phases)
QAM (cont.)

- Modulation and demodulation are combination of PSK and AM.

- V.32 9600 bps uses 32-QAM with trellis coding.

- All modern digital electronic communication uses QAM.
Constellation Examples

- baud = symbol per second

- baud “rate” is proportional to bandwidth
Next Couple of Classes

- Monday: other rtl_sdr hacks
- Wednesday: Questions about the final project.
- Final Project: due at the end of finals