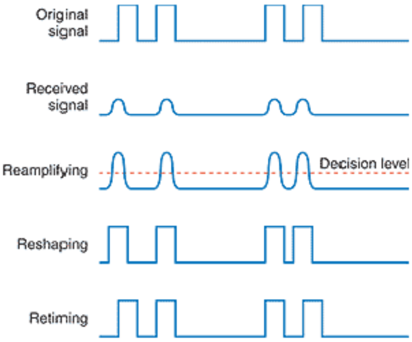
Digital Communications

- Analog vs Digital Communication
- Pulse Code Modulation (PCM)
- Quantization
  - Uniform Quantization
  - Non-Uniform Quantization
- Quantization Error
- PCM Bandwidth
- PCM SNR

Analog vs. Digital Communication

- Analog communication (baseband and modulated) is subject to noise.
- Pulse modulations (PAM, PWM, PPM) represent analog signals by analog variations in pulses and are also sunbject to noise.
- Long distance communication requires repeaters, which amplify signal and noise. Each link adds noise.
- Digital communication suppresses noise by regenerating signal.

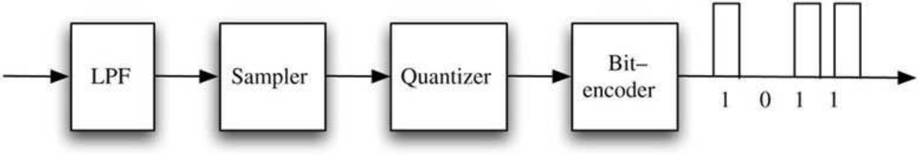


Pulse Code Modulation (PCM)

In PCM, a signal value is represented by a sequence of pulses (digits).

Width and spacing of pulses is constant. Value of pulse is chosen from a small number of values.

Usually PCM uses only two pulse values, which represent 0 and 1.



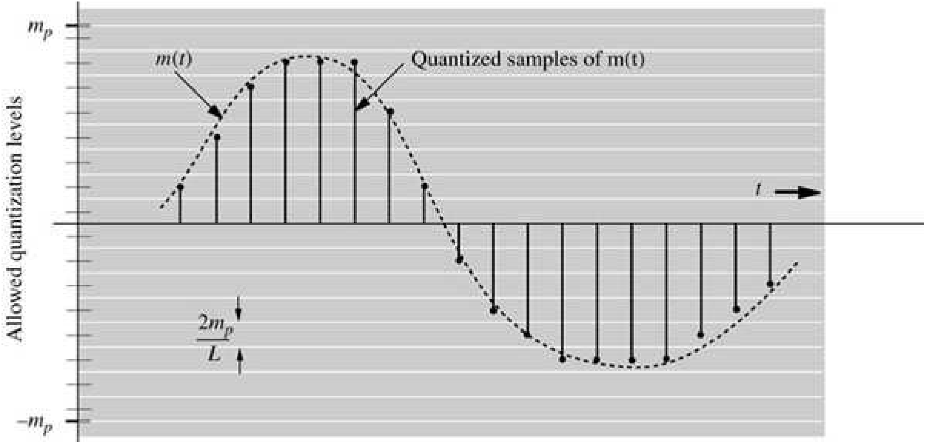
If m bits are used, then 2^m signal values can be represented.

- unsigned linear: $0, \nu, \ldots, \Delta\nu(2^m - 1)$
- two's complement: $-\Delta\nu 2^{m-1}, \ldots, \Delta\nu(2^{m-1} - 1)$
- A-law and μ-law: approximately logarithmic (more dynamic range)

PCM and Quantization

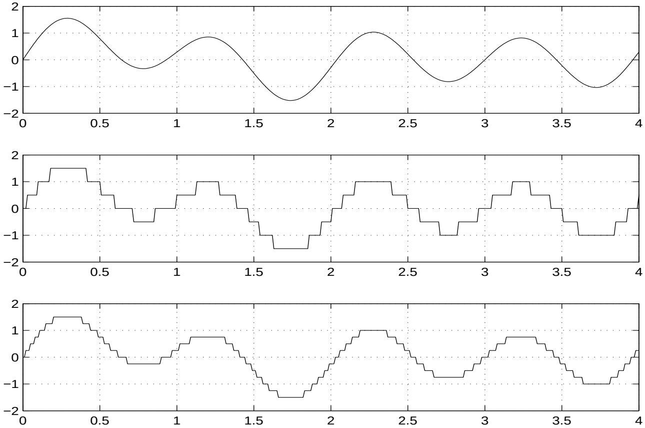
Quantization of a signal produces the closest representable value.

For fixed number of values, spacing between values increases with range.



PCM Tradeoffs

- Signal bandwidth determines minimum sample rate
- Desired signal fidelity determines precision of reproduced signal
- Signals can be quantized using digital-to-analog converter (DAC)

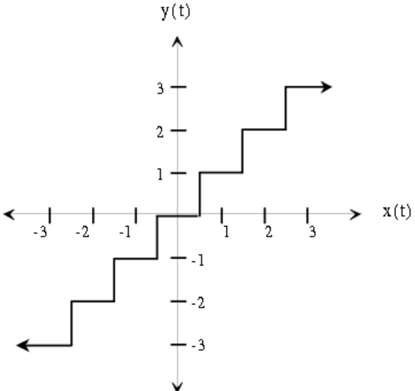


Uniform Quantization

An ideal uniform quantizer is a nonlinear time invariant system:

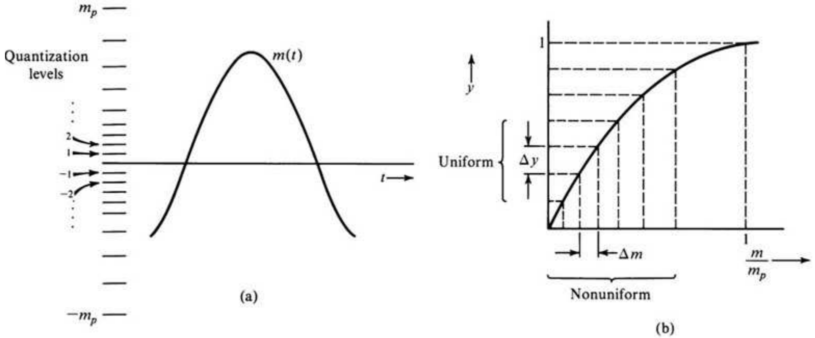
$$\widetilde{g}(t) = \begin{cases} -N_l\Delta\nu & g(t) < -N_l\Delta\nu \\ n\Delta\nu & (n-\frac{1}{2})\Delta\nu < g(t) < (n+\frac{1}{2})\Delta\nu,\ -N_l < n < N_h \\ N_h\Delta\nu & g(t) > N_h\Delta\nu \end{cases}$$

$\Delta\nu$ is quantization interval. $N_l + N_h$ is number of levels.

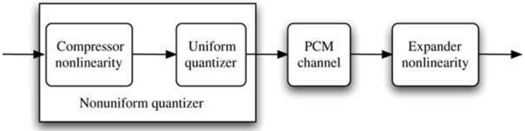


Nonuniform Quantization

Nonuniform quantizers increase quantization intervals as magnitude of value. Interval proportional to value implies logarithmic curve.



An analog compressor (semiconductor diode) can be used.



Nonuniform Quantization (cont.)

Logarithmic compression can be approximated by floating point, A-law, and μ-law representations.

- Binary floating point corresponds to scientific notation:

  $$\pm f \times 2^e$$

  where f is *significand* or *fraction* and e is exponent. Both f and e are represented in unsigned binary.

  Example: 8-bit code with one bit for sign, 4 bits for f, 3 bits for e,

  $$f = 0, 1, \ldots, 15\,, \quad e = 0, 1, \ldots, 7$$

  The representable values range from $-15 \cdot 2^7 = -1920$ to 1920. Quantization spacing range from 1 for $|y| < 16$ to 128 for $|y| > 960$.

  Disadvantage of floating point is that many values have multiple representations. E.g., this scheme represents only 72 values.

Some writers use mantissa instead of significand.

Nonuniform Quantization (cont.)

Telephone systems use ITU standardized compression formula.

- μ-law: North America and Japan. For $\mu = 255$ (for 8-bit codes),

$$y = \operatorname{sgn}(x)\frac{1}{\ln(1+\mu)}\ln\left(1+\mu|x|\right), \quad (0 < x < 1)$$

- A-law: Europe, rest of world.

$$y = \begin{cases} \operatorname{sgn}(x)\dfrac{A|x|}{1+\ln(A)} & |x| < \dfrac{1}{A} \\ \operatorname{sgn}(x)\dfrac{1+\ln(A|x|)}{1+\ln(A)} & \dfrac{1}{A} < |x| < 1 \end{cases}$$

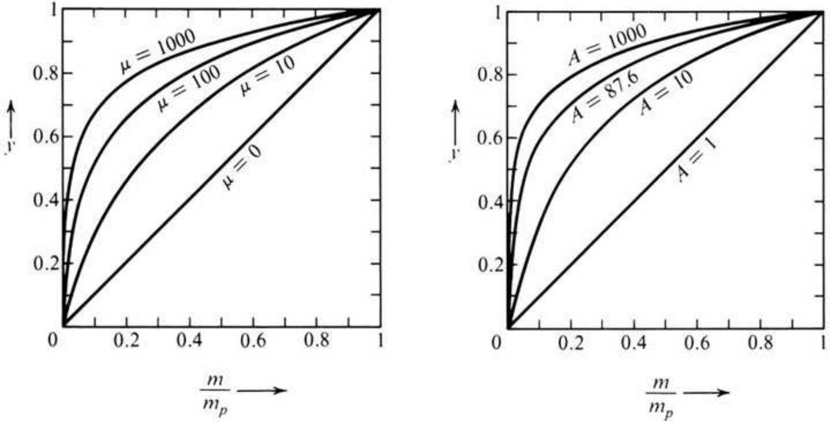
  The standard value is $A = 87.7$.

For both laws, the input to the compressor is

$$x = \frac{m(t)}{m_p}$$

where $-m_p \leq m(t) \leq m_p$.

Comparison of μ-Law and A-Law



- μ-law provides slightly larger dynamic range than A-law.
- A-law has smaller proportional distortion for small signals.
- A-law is used for international connections if at least one country uses it.

μ-Law Implementation

Both μ-law and A-law expanders are piecewise linear.

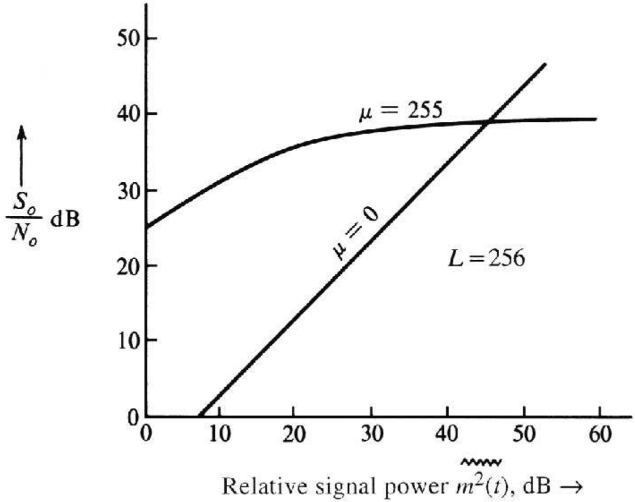
This table shows how 7 bits are expanded.

Numerical Value Range	Number Of Intervals	Interval Size	Total Intervals
0	1	1	1
1 - 16	15	2	30
17 - 32	16	4	64
33 - 48	16	8	128
49 - 64	16	16	256
65 - 80	16	32	512
81 - 96	16	64	1024
97 - 112	16	128	2048
113 - 127	16	256	4096
			8159

In practice this table is used by the μ-law encoder.

μ-Law Signal-to-Noise Ratio

The average power of a compressed signal is closer to the peak power.



Quantization Error

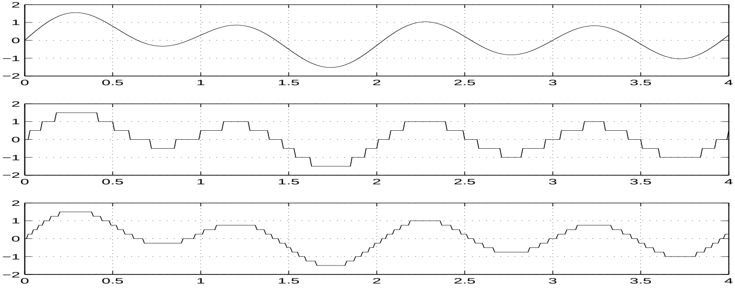
Uniform quantization with L levels of a signal with peak amplitude m_p has maximum quantization error

$$\text{max error} = \frac{m_p}{L},$$

and mean square error

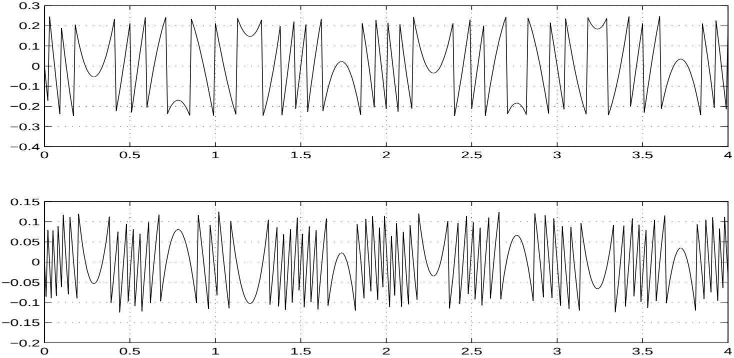
$$\text{average square error} = \frac{m_p^2}{3L^2}$$

Example: signal quantized to 4 and 16 levels.



Quantization Error (cont.)

Quantization error for quantizing to 4 and 16 levels.



Power of quantization error for above example is

$$0.0863 \approx \tfrac{1}{3}\,2^{-2}\ (L=4) \ \text{ and } \ 0.0180 \approx \tfrac{1}{3}\,2^{-4}\ (L=16)$$

Fact: for μ-law, SNR is $\dfrac{3L^2}{(\ln(1+\mu))^2}$ if $\mu \gg m_p/\text{rms}(m(t))$.

Bandwidth vs. Quantization Error

What bandwidth is needed to transmit a PCM encoded signal?

Example: suppose that we want maximum error $0.5\% m_p$ for a 3 kHz signal.

$$\frac{\Delta\nu}{2} = \frac{m_p}{L} = \frac{0.5}{100} m_p \implies L = 200 < 2^8$$

At Nyquist sample rate

$$R_N = 2 \cdot 3000 = 6000 \text{ Hz}$$

we need $6000 \cdot 8 = 48000$ bits/sec.

Fact: a bandlimited signal can convey two symbols per Hz.

For binary PCM, we need $48000/2 = 24000$ Hz.

For practical reasons, we sample faster than the Nyquist rate.
E.g., at rate 4000 Hz, the required bandwidth is 32 kHz.

PCM SNR

The signal-to-noise ratio is

$$SNR = \frac{\text{average signal power}}{\text{average noise power}}$$

For uniform quantization noise,

$$\text{average signal power} \approx am_p^2 \quad (a \approx \tfrac{1}{2})$$

$$\text{quantization error} \approx \tfrac{1}{3}(m_p/L)^2$$

$$\text{SNR} \approx cL^2 = c2^{2m}$$

where m is the number of bits in the PCM sample, so $L = 2^m$. c is a constant.

SNR grows exponentially with the number of bits.

If we measure SNR in dB,

$$SNR_{dB} = 10\log_{10}(c2^{2m}) = 10\log_{10}(c) + 2m\ log_{10}2 = (\alpha + 6m)\text{dB}$$

where $\alpha = 10\log_{10} c$.

Increasing n by one bit improves SNR by 6 dB! One bit quadruples SNR.

PCM SNR

Consider two cases for a 4 kHz bandwidth signal

- $L = 64$, $m = 6$ bits

$$SNR_{dB} = \alpha + 36 \text{ dB}$$

- $L = 256$, and $m = 8$ bits

$$SNR_{dB} = \alpha + 48 \text{ dB}$$

We've gained 12 dB in SNR. However, the PCM bandwidth has increased only from

$$(2 \times 4 \text{ kHz})(6 \text{ bits})/2 = 24 \text{ kbits/sec}$$

to

$$(2 \times 4 \text{ kHz})(8 \text{ bits})/2 = 32 \text{ kbits/sec}$$

We only need 1/3 greater bandwidth for a 12 dB improvements in SNR. The value of α (and c) depend on the quantization method, but are constants given that.

Logarithmic Units

In communications we often measure ratios using logarithms.
The *bel* (B) is the $\log_{10}$ of a ratio. More useful is the *decibel* (dB):

$$\frac{a}{b} \text{ in dB } \text{ is } 10 \log_{10} \frac{a}{b}$$

Examples: $2 \Leftrightarrow 3.01 \approx 3$ dB, $5 \Leftrightarrow 4.77 \approx 5$ dB
Why measure in dB?

- Some sensors (human eyes, ears) respond to logarithm of signal power.
- Many transmission media have attenuation that is exponential in length. Thus the signal loss in dB is proportional to length.
- Calculating how much power is needed in a communications system requires a *link budget*, which is additive in dB.

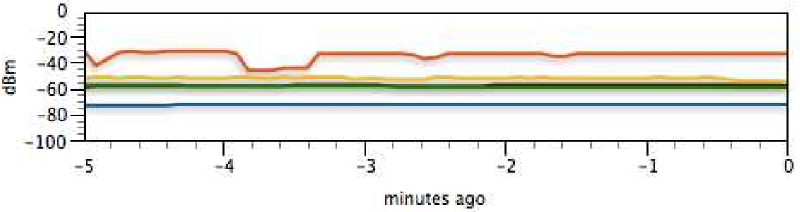
  rcv power (dBm) = xmit power (dBm) + gains (dB) − losses (dB)
- Since dB measures ratio, we must specify a reference value for 0 dB.
  - dbW: 0 dB = 1 W
  - dBm: 0 dB = 1 mW

Audio Volume (Sound Pressure)

- The SI unit of pressure is the pascal (Pa): 1 N/m^2
- Atmospheric pressure at sea level is 14.7 lb/in^2 or 101325 Pa
- Audio reference level: 20 μPa, threshold of human hearing. Another reference level is 1 pW $= 10^{-12}$ W.
- Example audio levels (rms of sound pressure):
  - 20–30 dB: quiet room
  - 60 dB: TV at normal volume
  - 85 dB: hearing damage (long term)
  - 100 dB: jack hammer
  - 140 dB: aircraft carrier deck
  - 120 dB: vuvuzuela, thunderclap, chain saw
  - 175 dB: stun grenade
  - 194 dB: atmospheric pressure (shock wave)

Wireless Receive Power and SNR

Data from John Gill.



Client	Signal	Noise	Rate	Type
D8:30:62:5A:79:1C	-32	-79	104	802.11b/g/n
B8:F6:B1:19:51:63	-54	-79	216	802.11b/g/n
4C:B1:99:AE:C4:89	-58	-79	65	802.11b/g/n
9C:20:7B:92:C3:02	-71	-79	65	802.11b/g/n
78:E4:00:BE:AF:B3	-56	-79	54	802.11b/g

Receive power ranges from -32 to -71 dB $(6.3 \times 10^{-7}$ to 7.9×10^{-11} W$)$.

The most powerful transmitter was 4 feet from the receiver.