Line Codes and Pulse Shaping

- Review
  - Line codes
  - Pulse width and polarity
  - Power spectral density
- Intersymbol interference (ISI)
- Pulse shaping to reduce ISI
- Embracing ISI

Line Code Examples (review)

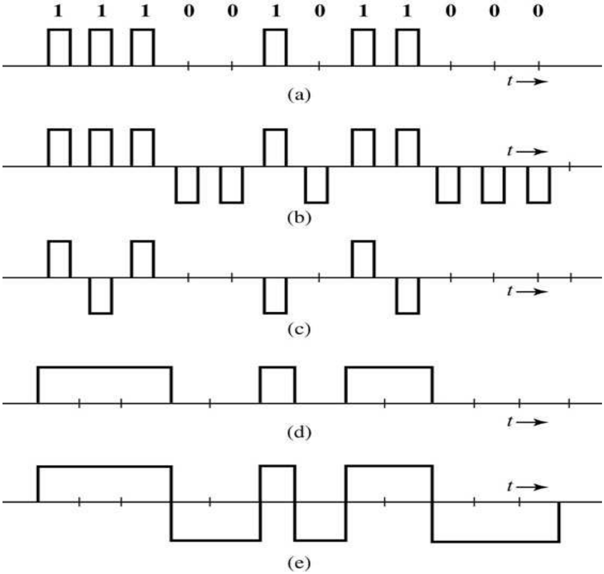
on-off (RZ)

polar (RZ)

bipolar/AMI (RZ)

on-off (NRZ)

polar (NRZ)



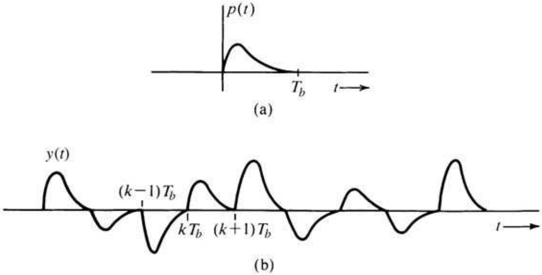
Line Code Desiderata (review)

Features we want:

- Minimum bandwidth (NRZ)
- Easy clock recovery (RZ)
- Frequently, no DC value (bipolar pulses)

PSD of Line Codes

The PSD of a line code depends on the shapes of the pulses that correspond to digital values. Assume PAM.



The transmitted signal is the sum of weighted, shifted pulses.

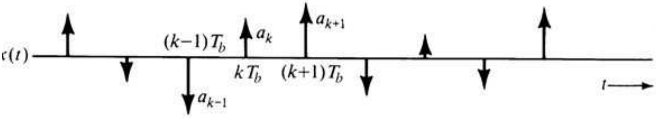
$$y(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT_b)$$

where T_b is spacing between pulses. (Pulse may be wider than T_b.)

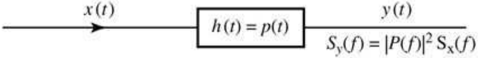
PSD of Line Codes (cont.)

PSD depends on pulse shape, rate, and digital values $\{a_k\}$.

We can simplify analysis by representing $\{a_k\}$ as impulse train.



(c)



(d)

PSD of $y(t)$ is $S_y(f) = |P(f)|^2 S_x(f)$.

- $P(f)$ depends only on the pulse, independent of digital values or rate.
- $S_x(f)$ depends only on the bit sequence, and can be altered by the signaling scheme.

Power Spectral Density of Line Codes (review)

- In general, the PSD of a line code is

$$S_y(f) = |P(f)|^2 \, S_x(f)$$

  where

$$S_x(f) = \mathcal{F}\left\{R_x(t)\right\} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-jn2\pi f T_b}$$

  so

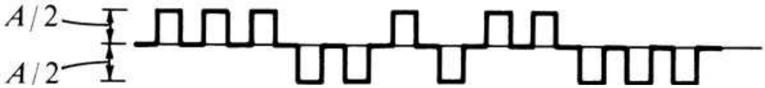
$$S_y(f) = |P(f)|^2 \left(\frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-jn2\pi f T_b} \right)$$

  In many cases, only $R_0 \neq 0$, or just a few terms are not equal to zero.

- We wantto limit the bandwidth of the transmitted signal, so we can
  - modify $P(f)$, or
  - modify $x(t)$, or
  - both

Modifying $x(t)$

Polar signaling. Transmit 1 as $+p(t)$ and 0 as $-p(t)$,



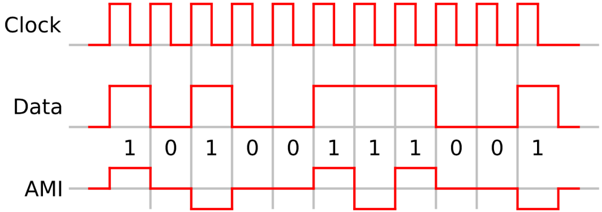
Only $R_0 \neq 0$, and

$$S_y(f) = \frac{|P(f)|^2}{T_b} R_0 = \frac{|P(f)|^2}{T_b}$$

PSD of polar signaling depends only on spectrum of $p(t)$.

Modifying $x(t)$ (cont.)

Bipolar signaling: transmit 1's as alternating $\pm p(t)$, and 0's as 0.



Bipolar signaling for full-width (NRZ) pulses.

$$S_y(f) = \frac{|P(f)|^2}{2T_b}(1 - \cos 2\pi T_b f) = \frac{|P(f)|^2}{T_b}\sin^2(\pi T_b f)$$

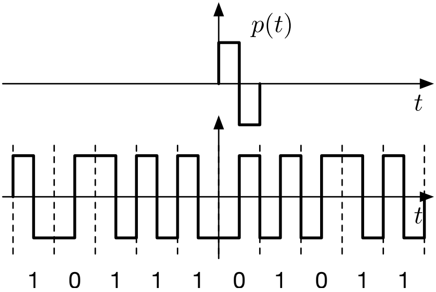
For half-width (RZ) pulses:

$$S_y(f) = \frac{|P(f)|^2 T_b}{4}\operatorname{sinc}^2\left(\frac{\pi f T_n b}{2}\right)\sin^2(\pi T_b f)$$

PSD depends on $P(f)$, but decays more rapidly due to $\sin^2$ terms.

Modifying $p(t)$

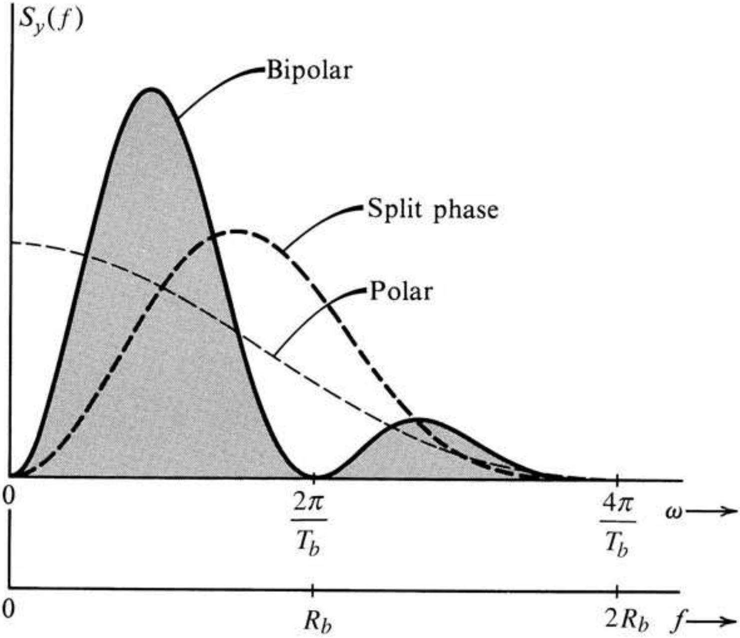
Split phase or Manchester encoding:



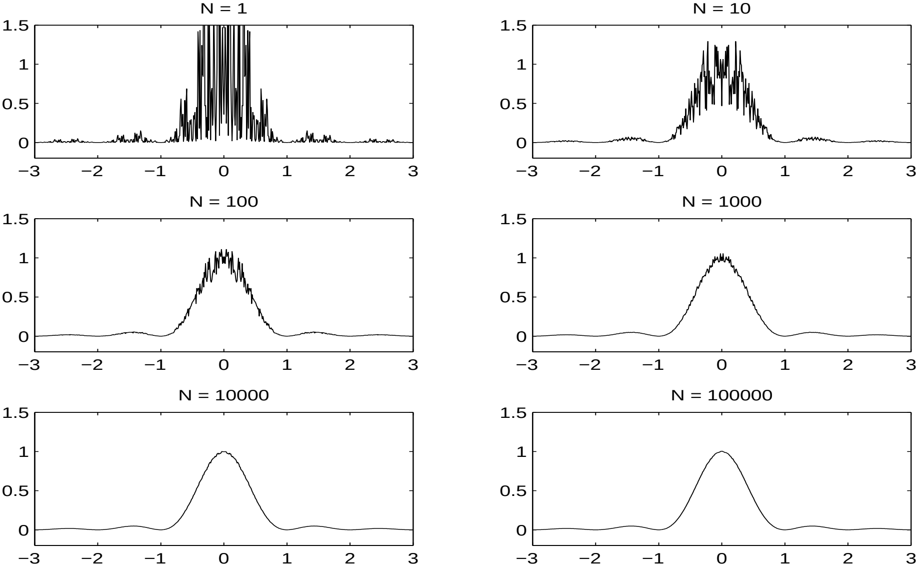
PSD is

$$
\begin{aligned}
S_y(f) &= |P(f)|^2 \left(\frac{1}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-jn2\pi f T_b} \right) \\
&= |P(f)|^2 \frac{1}{T_b}
\end{aligned}
$$

Comparison of PSD's



PSD of Polar Signaling (Matlab Experiment)



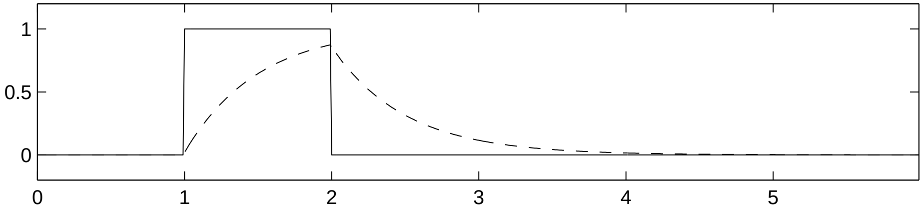
Intersymbol Interference (ISI)

Pulses transmitted over a physical channel (linear time-invariant system) are distorted—smoothed and stretched.

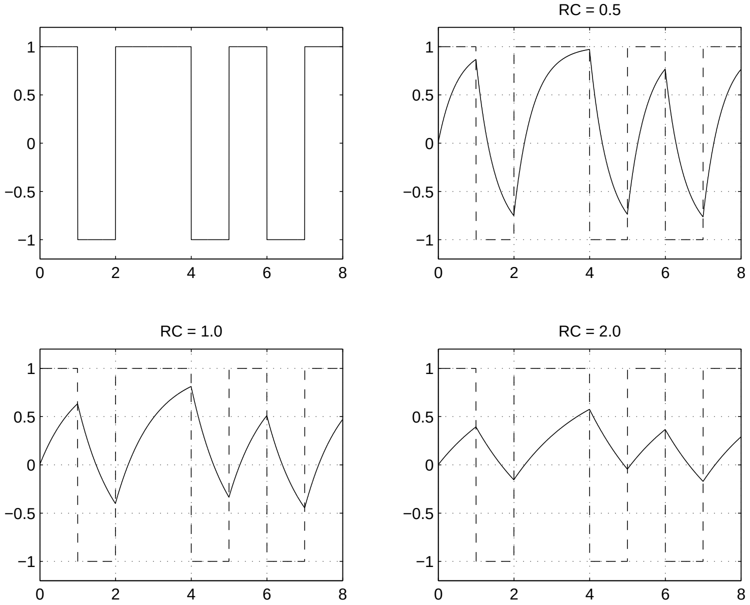
Example: impulse response of an RC circuit is $h(t) = e^{-t/RC}u(t)$.

For $RC = 0.5$, the *pulse* response is

$$h(t) * \Pi(t - \tfrac{1}{2}) = \begin{cases} 0 & t < 0 \\ 1 - e^{-2t} & 0 < t < 1 \\ (1 - e^{-2})e^{-2t} & t > 1 \end{cases}$$



ISI Example: $h(t) = \frac{1}{RC}e^{-t/RC}$

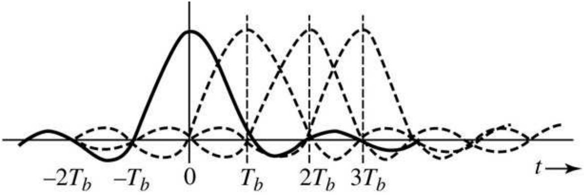


Reducing ISI: Pulse Shaping

- A time-limited pulse cannot be bandlimited
- Linear channel distortion results in spread out, overlapping pulses
- Nyquist introduced three criteria for dealing with ISI.

  The first criterion was that each pulse is zero at the sampling time of other pulses.

$$p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm kT_b\,,\ k = \pm 1, \pm 2, \ldots \end{cases}$$



Harry Nyquist, "Certain topics in telegraph transmission theory", Trans. AIEE, Apr. 1928

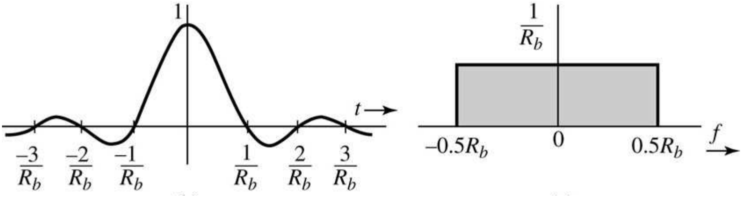
Pulse Shaping: sinc Pulse

- Let $R_b = 1/T_b$. The sinc pulse $\mathrm{sinc}(\pi R_b t)$ satisfies Nyquist's first crierion for zero ISI:

$$\mathrm{sinc}(\pi R_b t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm k T_b\,,\ k = \pm 1, \pm 2, \ldots \end{cases}$$

- This pulse is bandlimited. Its Fourier transform is

$$P(f) = \frac{1}{R_b}\Pi\Big(\frac{f}{R_b}\Big)$$

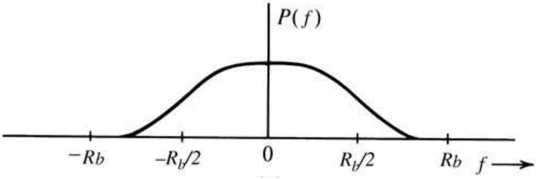


- Unfortunately, this pulse has infinite width and decays slowly.

Nyquist Pulse

Nyquist increased the width of the spectrum in order to make the pulse fall off more rapidly.

The Nyquist pulse has spectrum width $\frac{1}{2}(1+r)R_b$, where $0 < r < 1$.



If we sample the pulse $p(t)$ at rate $R_b = 1/T_b$, then

$$\overline{p}(t) = p(t)\,\mathrm{III}_{T_b}(t) = p(t)\delta(t) = \delta(t)\,.$$

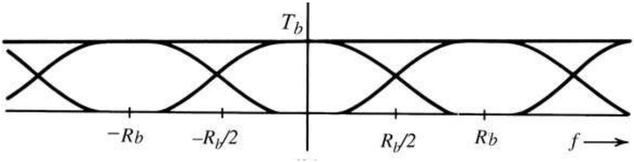
The Fourier transform of the sampled signal is

$$\overline{P}(f) = 1 = P(f) * R_b\,\mathrm{III}_{R_b}(f) = \sum_{k=-\infty}^{\infty} P(f - kR_b)$$

Nyquist Pulse (cont.)

Since we are sampling below the Nyquist rate $2R_b$, the shifted transforms overlap.

Nyquist's criterion requires pulses whose overlaps add to 1 for all f.



For parameter r with $0 < r < 1$, the resulting pulse has bandwidth

$$B_r = \frac{1}{2}(R_b + rR_b)$$

The parameter r is called *roll-off factor* and controls how sharply the pulse spectrum declines above $\frac{1}{2}R_b$.

Nyquist Pulse (cont.)

Many pulse spectra satisfy this condition. e.g., trapezoid:

$$P(f) = \begin{cases} 1 & |f| < \frac{1}{2}(1-r)R_b \\ 1 - \frac{|f|-(1-r)R_b}{2R_b} & \frac{1}{2}(1-r)R_b < |f| < \frac{1}{2}(1+r)R_b \\ 0 & |f| > \frac{1}{2}(1-r)R_b \end{cases}$$

A trapezoid is the difference of two triangles. Thus the pulse with trapezoidal Fourier transform is the difference of two sinc^2 pulses.

Example: for $r = \frac{1}{2}$,

$$P(f) = \frac{3}{2}\Lambda\left(\frac{f}{\frac{3}{2}R_b}\right) - \frac{1}{2}\Lambda\left(\frac{f}{\frac{1}{2}R_b}\right)$$

so the pulse is

$$p(t) = \frac{9}{4}\,\mathrm{sinc}^2(\tfrac{3}{2}R_b t) - \frac{1}{4}\,\mathrm{sinc}^2(\tfrac{1}{2}R_b t)$$

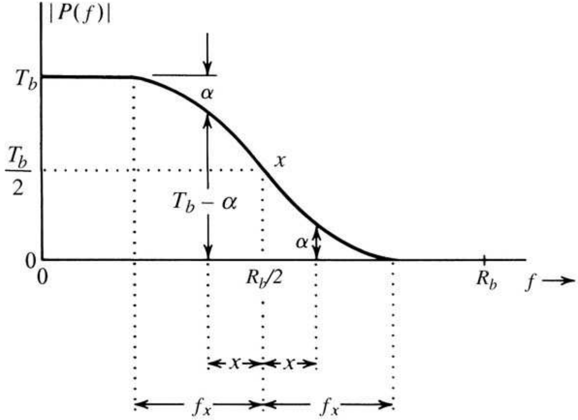
This pulse falls off as $1/t^2$.

Nyquist Pulse (cont.)

Nyquist chose a pulse with a “vestigial” raised cosine transform.

This transform is smoother than trapezoid, so pulse decays more rapidly.

The Nyquist pulse is parametrized by r. Let $f_x = rR_b/2$.

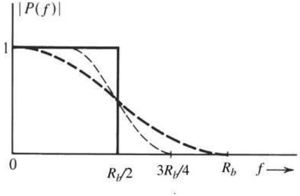


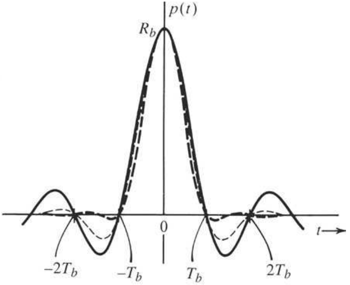
Nyquist Pulse (cont.)

Nyquist pulse spectrum is raised cosine pulse with flat *porch*.

$$P(f) = \begin{cases} 1 & |f| < \frac{1}{2}R_b - f_x \\ \frac{1}{2}\left(1 - \sin\pi\left(\frac{f - \frac{1}{2}R_b}{2f_x}\right)\right) & \left||f| - \frac{1}{2}R_b\right| < f_x \\ 0 & |f| > \frac{1}{2}R_b + f_x \end{cases}$$

The transform $P(f)$ is differentiable, so the pulse decays as $1/t^2$.





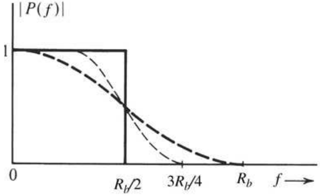
Nyquist Pulse (cont.)

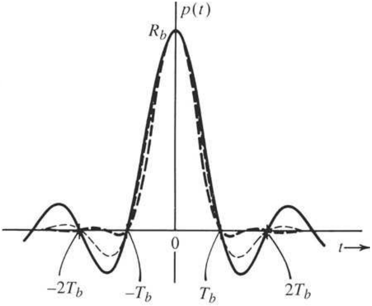
Special case of Nyquist pulse is $r = 1$: *full-cosine roll-off*.

$$P(f) = \tfrac{1}{2}(1 + \cos \pi T_b f)\Pi(f/R_b)$$
$$= \cos^2(\tfrac{1}{2}\pi T_b f)\,\Pi(\tfrac{1}{2}T_b f)$$

This transform $P(f)$ has a second derivative so the pulse decays as $1/t^3$.

$$p(t) = R_b \frac{\cos \pi R_b t}{1 - 4R_b^2 t^2}\,\mathrm{sinc}(\pi R_b t) = \frac{\sin(2\pi R_b t)}{2\pi t(1 - 4R_b^2 t^2)}$$

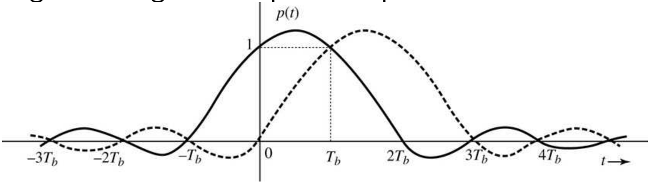




Controlled ISI (Partial Response Signaling)

The second Nyquist approach embraced ISI: if we know the adjacent bits then interference is known!

We can achieve lower bandwidth by using an even wider pulse. ISI can be canceled using knowledge of the pulse shape.



In this case $p(t)$ is equal to 1 for $t = 0$ and $t = T_b$.

The signal at time T_b depends on both $x(0)$ and $x(T_b)$.

- If both are 1, the output $y(t) = 2$
- if one is 1, and the other is -1, the output is 0
- if both are -1, the output is -2.

Given a starting value for a_k, where $x(t) = \sum_k a_k \delta(t - kT_b)$, we can subtract the ISI term by term and recover the bit sequence $\{a_k\}$.

Partial Response Signaling (cont.)

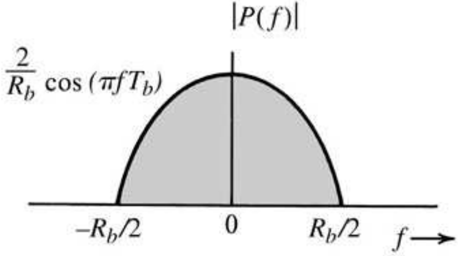
The ideal duobinary pulse is

$$p(t) = \frac{\sin \pi R_b t}{\pi R_b t(1 - R_b t)}$$

The Fourier transform of $p(t)$ is

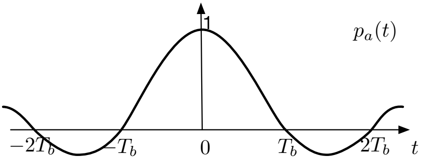
$$P(f) = \frac{2}{R_b} \cos\left(\frac{\pi f}{R_b}\right) \Pi\left(\frac{f}{R_b}\right) e^{-j\pi f/R_b}$$

The spectrum is confined to the theoretical minimum of $R_b/2$.



Zero-ISI, Duobinary, Modified Duobinary Pulses

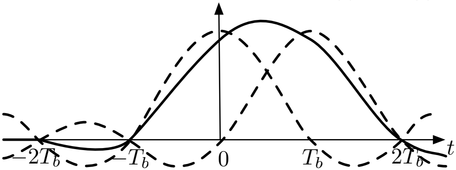
Suppose $p_a(t)$ satisfies Nyquist's first criterion (zero ISI).



Then

$$p_b(t) = p_a(t) + p_a(t - T_b)$$

is a duobinary pulse with controlled ISI.



Zero-ISI, Duobinary, Modified Duobinary Pulses (cont.)

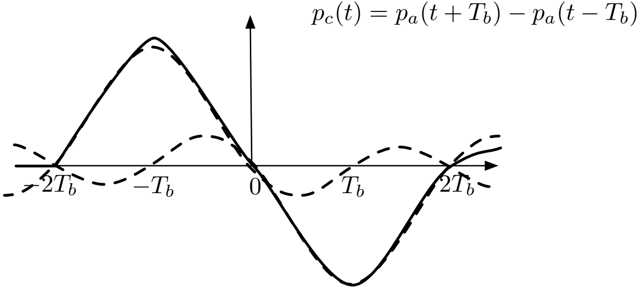
By shift theorem, $P_b(f) = P_a(1 + e^{-j2\pi T_b f})$

Since $P_b(R_b/2) = 0$, most (or all) of the pulse energy is below $R_b/2$.

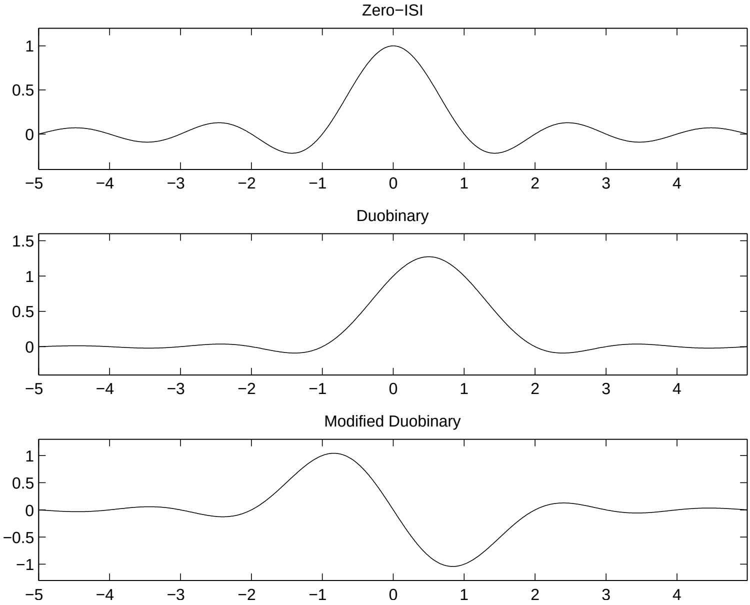
We can eliminate unwanted DC component using modified duobinary, where $p_c(-T_b) = 1$, $p_c(T_b) = -1$, and $p_c(nT_B) = 0$ for other integers n.

$$p_c(t) = p_a(t + T_b) - p_a(t - T_b) \implies P_c(f) = 2jP_a(f)\sin 2\pi T_b f$$

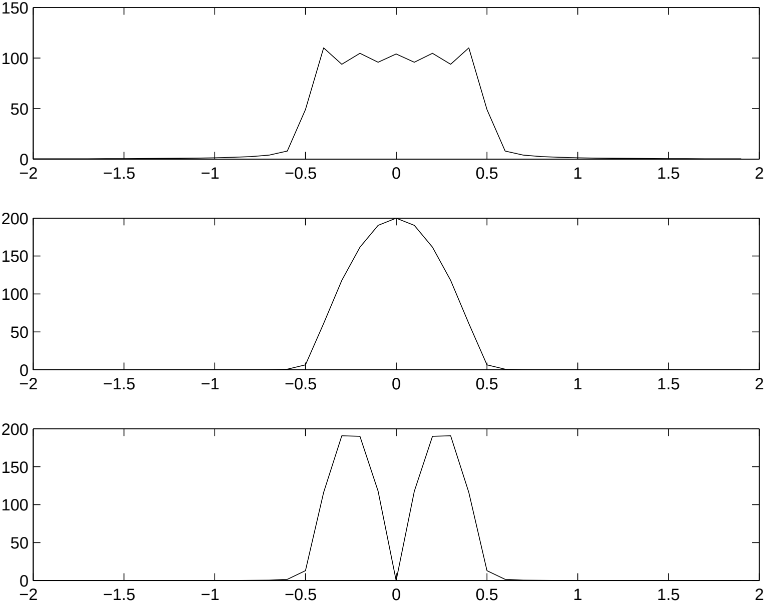
The transform of $p_c(t)$ has nulls at 0 and $\pm R_b/2$.



Zero-ISI, Duobinary, Modified Duobinary Pulses (cont.)



Zero-ISI, Duobinary, Modified Duobinary Pulses (cont.)



Partial Response Signaling Detection

Suppose that sequence 0010110 is transmitted (first bit is startup digit).

Digit x_k	0	0	1	0	1	1	0
Bipolar amplitude	-1	-1	1	-1	1	1	-1
Combined amplitude		-2	0	0	0	2	0
Decode sequence		0	1	0	1	1	0

Partial response signaling is susceptible to *error propagation*.

If a nonzero value is misdetected, zeros will be misdetected until the next nonzero value.

This can be eliminated by differential encoding, where the input stream is preprocessed, so that a receiver can directly read out the data bits.