Problem Set #1

Due Date: October 7, 2005. Submit in class, or outside Packard Room 331 before 4:30 PM.

Reading Assignment:

“Reader” Chapters 1–3 Suggested Reading:

E&H Sections 1.1–1.5 (1.6–1.11 opt.)
Sections 5.1–5.3 (5.4 opt.)

Problems:

1. Operator notation [10 points]
   (a) Verify that \( \phi(x,t) \) is a solution to by the wave equation by direct substitution. The one dimensional wave equation is given by \( \phi_{xx} = \frac{1}{c^2} \ddot{\phi} \) and \( \phi(x,t) \) is of the form
   \[ \phi(x,t) = f_1(x-ct) + f_2(x+ct) \]. [5 points]

   (b) By use of the “chain rule” of differential calculus, establish the operator formulae
   \[ 2 \frac{\partial}{\partial u} = \frac{\partial}{\partial x} - \frac{1}{c} \frac{\partial}{\partial t} \]
   \[ 2 \frac{\partial}{\partial w} = \frac{\partial}{\partial x} + \frac{1}{c} \frac{\partial}{\partial t} \]
   where \( u = x-ct \) and \( w = x+ct \). [5 points]

2. Wave Equation [20 points]
   (a) Use the separation of variables method to find the general solutions for a three-dimensional wave equation (see below). Use \( k^2 \) for the separation constant, where \( k^2 = k_x^2 + k_y^2 + k_z^2 \). [15 points]
   \[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} \]

   (b) Interpret the physical meaning of your solution. Consider all possible values for \( k_x^2 \), \( k_y^2 \), and \( k_z^2 \). [5 points]

3. Initial Value Problem [30 points]
(a) A long string, for which the transverse velocity is $c$, is given a displacement specified by some function $\eta = f(x)$ that is localized near the center of the string. The string is released at $t = 0$ with zero initial velocity. Find the equations for the resulting traveling waves. Sketch the waves at several instants of time for $t > 0$ (Assume a simple shape for $f(x)$ to make your job easier). Hint: You can solve this readily by finding two oppositely traveling waves that together satisfy the boundary condition at $t = 0$. [15 points]

(b) Now consider the situation where the string has not only an initial displacement but an initial velocity $\frac{\partial \eta}{\partial t} = g(x)$ at the time of release. Find the more general form of the resulting waves. [15 points]

4. Direction of Waves [20 points] Which of the following disturbances represents a travelling wave in one dimension (Give reason)? If its a travelling wave, what is the speed of the wave, and in which direction is it travelling? (Ignore the fact that some of these are unrealistic.)

(a) $\eta = (3x - 4t)^2$
(b) $\eta = x^2t^2$
(c) $\eta = e^{-\alpha x}e^{i\omega t}$
(d) $\eta = \exp \left[-\alpha (2x - t)^2\right]$
(e) $\eta = \sin (4x + 3t) + \sin (4x - 3t)$

5. Longitudinal waves [20 points]

(a) What are longitudinal waves? [5 points]
(b) Give three examples of longitudinal waves. [5 points]
(c) A plane acoustic wave has incremental pressure

$$p(x, y, t) = 1.5\sin\left[2\pi (1.2x - \beta y + 100t + 2.41)\right]$$

The propagation speed of the wave is 330 m/s and Air density is 1.29 Kg/m$^3$, and for a fixed “x” the wave appears to move in the positive “y” direction. Find $\beta$, the direction of propagation, wave number, frequency, angular frequency, period, peak pressure and average power carried by the wave.

6. System of springs and masses [10 points] With regard to the discussion of the system of springs and masses in Chapter 1, Fig. 2 of the "Course Reader":

(a) How does the spring constant “K” depend on the length of the spring? Please be quantitative and state your reasoning. [5 points]
(b) For the distributed system of uniform springs and masses, what are the units of $\mu \ast \kappa$? Does your answer balance the units of Eq. (15), p. 1-5 of the "Course Reader"? [5 points]