Definitions of Regime for Large Antennae

Consider:

\[ \bar{A} = \frac{1}{4\pi} \int_{\text{vol}} \frac{\mathbf{J}(x',y',z')}{R} e^{-jkr} \, dv', \]

\[ \theta \]

\[ \mathbf{R} \]

\[ \mathbf{R} - \mathbf{R}' \]
\[ R^2 = (x-x')^2 + (y-y')^2 + (z-z')^2 \]
\[ = |\mathbf{r} - \mathbf{r}'|^2 = r^2 + r'^2 - 2rr' \cos \xi \]

\[ R = r \left( 1 + \frac{r'^2}{r^2} - 2 \frac{r'}{r} \cos \xi \right)^{1/2} \]

\[ R = \delta - r' \cos \xi + \frac{r'^2 \sin^2 \xi}{r} + \frac{1}{r^2} \left( \frac{r'^3}{2} \cos \xi \sin^2 \xi \right) + \ldots \]

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1st  2nd  3rd

Binomial expansion: order of terms in \( r' \)
What is the Simplest Expansion for $R$?

**First Order**

$$R = r - r' \cos \xi$$

First neglected term is

$$\frac{1}{8} \frac{r'^2 \sin^2 \xi}{2} \quad \xrightarrow{\text{max}} \quad \frac{r'^2}{2 \lambda}$$

Which corresponds to a maximum phase error

$$\Delta \phi = k \Delta R = \frac{2\pi}{\lambda} \Delta R = \frac{2\pi}{\lambda} \frac{r'^2}{2 \lambda}$$
In the expression for $A$ (the Radieeic Ant.)

Let $D$ be the maximum dimension of the region for which $|J| \neq 0$

Then $r' \leq \frac{D}{2}$ (for some choice of origin)

With this picture in mind and some standard value for the maximum tolerable $\Delta \phi$
We are led to

$$\Delta \phi \leq \Delta \phi_{\text{standard}} = \frac{2\pi}{\lambda} \frac{r'}{2r}$$

$$\Rightarrow r > \frac{\pi}{\lambda} \frac{r'}{\Delta \phi_{\text{standard}}}$$

or

$$r > \frac{2D^2}{\lambda}$$

for $\Delta \phi = \frac{2\pi}{16} = \frac{\pi}{8}$ rad.

$$= 22\frac{1}{2} \, ^\circ \text{ phase}$$

Which is the standard definition for the "Far-Field" of an antenna!
Far Field Relationships \( r > \frac{2D^2}{\lambda} \)

\[ R = r - r' \cos \gamma = \bar{r} - \bar{r}' \cdot \bar{u}_r \]

\[ \frac{2D^2}{\lambda} = \frac{2(46)^2}{1} = 4.2 \text{ km} \Rightarrow 42 \text{ km} \]

\begin{align*}
D &= 46 \text{ m} \\
\lambda &= 1 \\
f &= 3 \times 10^8 \\
&\Rightarrow 3 \times 10^{-9} \\
d &= 0.1
\end{align*}
Far Field (cont)

\[ k \bar{R} = R \cdot \bar{R} = \bar{k} \cdot (\bar{r} - \bar{r}') ; \quad \bar{R} \parallel \bar{r} \]

\[ A = \frac{-j \bar{k} \cdot \bar{r}}{4\pi r} \int \frac{\bar{J}(\bar{r}') e^{j \bar{k} \cdot \bar{r}'}}{r'} \, d\bar{r}' \]

Greens function -

\[ = k \bar{r} \]

Far Field General Case: Also called Fraunhofer Region -
In the above note that the Green's function factor depends only on \( r \), and \( \vec{k} \cdot \vec{F} \). But
\[
\vec{k} \cdot \vec{F} = \frac{2\pi}{A} r
\]
depends only on \( r \), also.

On the other hand, the integrand depends on \( \vec{F}' \) and \( \vec{k} \cdot \vec{F}' = \vec{u}_R \cdot \vec{v}' \cdot \frac{2\pi}{A} \),
where
\[
\vec{u}_R = \frac{\vec{F} - \vec{F}'}{|\vec{F} - \vec{F}'|}
\]
That is, the integral
is a function of direction \( \vec{u}_R \). Seen another way
\[
\vec{k} \cdot \vec{F}' = \frac{2\pi}{A} r' \cos \theta.
\]
Far Field Region or Fraunhofer Region

Characterized by retention of linear phase term in expansions of $kR$.

$R$ must be precise on scale of $2\pi/a$ in all phase calculations. (ehn Rad. dat.)

$R$ must be precise only on scale of $D$ for amplitude calculations. (ehn Rad. dat.)
What if \( r < 2D^2/\lambda \), then what?

\[ R = r - r' \cos \xi + \frac{1}{2} r'' \sin^2 \xi \left( \frac{r^2}{2} \right) + \ldots \]

First neglected term is \( \frac{1}{r^2} \left( \frac{r^3 \cos \xi \sin^2 \xi}{2} \right) \)

\( \frac{\partial}{\partial \xi} = 0 = -\sin^3 \xi + 2 \cos^2 \xi \sin \xi = \sin \xi \left( -\sin^2 \xi + 2 \cos^2 \xi \right) \)

\( \xi = 0 \Rightarrow \min \text{ error} \); \( \tan^2 \xi = 2 \), \( \xi = 0.96 \text{ rad} \)

⇒ max error.

...
\[ \frac{r^{13}}{r^2} \left( \cos \frac{\xi}{2} \sin^2 \frac{\xi}{2} \right) \left|_{\xi = 0.96 \text{ radians}} \right. = \frac{r'^3}{r^2} (0.19) = \Delta R \]

\[ \text{Error from neglect of cubic term} \]

Applying the same criterion, that \( \frac{2\pi}{\lambda} \cdot \Delta R < \frac{\pi}{8} \),

\[ \frac{2\pi}{\lambda} \cdot \frac{r^{13}}{r^2} 0.19 < \frac{\pi}{8} ; \quad r^2 > \left[ \frac{(16)(0.19)}{\lambda} \right] r'^3 \]

\[ r > 1.74 \sqrt{\frac{r'^3}{\lambda}} ; \quad r > 0.62 \left[ \frac{D^3}{\lambda} \right]^{1/2} \]
Radiating Near-Field Region

\[ 0.62 \left( \frac{D^3}{\lambda} \right)^{1/2} < r < \frac{2D^2}{\lambda} \]

1. Function with \( \bar{U} \), ie now a function of distance.

\[
\int_{\text{Vol}} -j \bar{k} \bar{r}' \begin{array}{c} \vdots \\ \int \end{array} J(\cdot) e^{j k \bar{r}'} \text{ d}r' \text{ now depends on } |\bar{r}'| \quad \text{(see below)}
\]

2. Fields have a significant curvature —

Hence Fresnel integrals appear.
Radiating Near-field (cont.)

3. Region called "Radiating" because radiating
   power density is much greater than the reactive
   power density

4. Also called the Irregular Region
\[ \vec{A} = \mu \int \frac{\vec{f}(\vec{r}')}{4\pi R} e^{-j k R} \, d\vec{r}' \]

Consider only the integral

"Far out" \( \vec{F} \parallel \vec{R} \), so the integral depends only on the direction of \( \vec{F} \), and \( \vec{A} \propto \frac{1}{r} \)

Close in \( \vec{F} \perp \vec{R} \), so \( \vec{R} \), and hence the integral depend on \( \vec{r}' \) and \( \vec{r}'' \). In this case \( \vec{A} \) does not vary as \( \frac{1}{r} \), but as some higher power of \( \frac{1}{r} \).
Free head Region or Radiating Near-Field Region, keep second-order term as well as $r$-dependence.

Far Field or Frontofar Region - only need this:

$$
\mathbf{A} = \frac{e^{-jkr}}{4\pi r} \int_{\text{vol}} \bar{F} \bar{f} e^{jkr} \, dv'
$$

Denominator easily approximated by $r \gg R$ for all of this!
Far-field and Near-field definitions are confusing when applied to small and large antennas alike.

\[ \frac{2D^2}{\lambda} \] makes no sense for an "ideal dipole".

\[ \frac{10^{-2} \lambda}{1} \]

\[ \frac{2D^2}{\lambda} = \frac{2 \times 10^{-4} \lambda^2}{\lambda} = 2 \times 10^{-4} \lambda \] (!)

\[ \lambda = 1 \text{ m} \Rightarrow 2 \times 10^{-4} \text{ m} - \text{ridiculous} ! \]
Fraunhofer and Fresnel Region concepts apply at distances beyond which the effect of individual current elements can be considered to vary as $1/r$, i.e., in the "local" far-field of the individual current elements. These in terms of superposition of waves from individual radiators.

Reactivi Region and Transition Region concepts are based on the behavior of the fields — they apply in the vicinity of the active current distribution...
Which concept you use will depend on the situation. Away from the immediate structure of large antennas the Fraunhofer/Fresnel approach is appropriate; close to the vicinity of wire antennas or waveguides, field concepts are needed.

Some people use the term "near-field" to refer to reactive and transition regions.
Summary of Antenna Field Regions

Four Regions:

Reactive (sometimes "near field" or "fringing-field")

Transition (sometimes "intermediate")

Radiating Near-Field (sometimes just "near field")

Far-Field (always far-field)
Summary

Transition Region

F Source

Reactive Region

Determined by source effects

Near-field

0.62 \( \left[ \frac{D^3}{\lambda} \right]^{1/2} \)

\( \frac{2D^2}{\lambda} \)

Determined by propagation effects