Solutions For Homework #6

Problem 1:[30 pts]

(a) The geometry of this problem is shown in Figure 1. The 21 geophones are denoted by $g_1, ...g_{21}$. The spacing between the geophones is $s = 80$ meters. From the geometry, we see that the time τ_i it takes for a pulse of acoustic energy to propagate from the source (assumed to coincide with g_1 - i.e. zero offset to the first geophone), reflect off a layer at depth d and arrive at the i-th geophone is

$$\tau_i = 2\frac{\sqrt{d^2 + (\frac{(i-1)s}{2})^2}}{v} \tag{1}$$

The depth of a layer causing an echo that is received at time t is simply $\frac{vt}{2}$. Thus, the depth corresponding to the first sample is $\frac{vt_0}{2} = 5000$ meters. The depth corresponding to the last sample for the same geophone is $\frac{v\times(t_0+(512-1)\Delta t)}{2} = 6277.5$ meters. MATLAB code: when reading the file *hw6prob1data*, we used the following code

```
fid = fopen('hw6prob1data','rb');
data = fread(fid, inf, 'float');
fclose(fid);
im = reshape(data, [512 21]); im = im';
```

That is, echoes recorded at each geophone are arranged end-to-end in the data array *hw6prob1data*.

(b) The result of simply summing all 21 lines together is shown in Figure 2. We see that direct averaging of the 21 signals yields a noisy return that does not immediately indicate the presence of layers.

(c) Stacking here basically means searching for the appropriate sample in the entire signal recorded at a particular geophone to include in finding the averaged return at a certain time. Specifically, the averaged received intensity

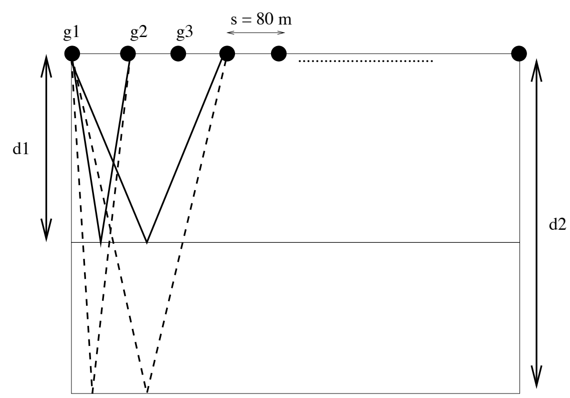

Figure 1:

at a particular time τ is computed as follows

$$g_{avg}(\tau) = \sum_{i=1}^{21} g_i(k) \tag{2}$$

where k refers to the k-th sample in the signal recorded at the i-th geophone. We can calculate k as follows

$$k = \left\lfloor \frac{\frac{\sqrt{(v\tau/2)^2+((i-1)s/2)^2}}{v}}{\Delta t} \right\rfloor \tag{3}$$

where Δt is the time spacing between samples recorded at any geophone. We note that we reference time τ to the first geophone. Of course, if the equation above yields a value of k that is greater than 512, the length of the signal, then that geophone signal is not incorporated in the sum for g_{avg}

above. The MATLAB code for performing the stacking is given at the back. The stacked and averaged return is plotted in Figure Figure 3 shows significant echoes located at **depths 5117.5, 5310 and 5597.5 meters**, as can be seen by the 3 clearly visible peaks.

Problem 2:[30 pts]

This problem involves repeated application of the stacking procedure described in Problem 1 to signals recorded from 512 geophone arrays. The averaged profile shown in Figure 3 gives us the locations of layers (three of them as we saw) at only one horizontal location. In this problem, we are given data recorded by geophone arrays at multiple horizontal locations, thereby allowing us to generate a 2D image whose large values will denote layers. The MATLAB code that performs this is given at the back. The final image is shown in Figure 4 The two-dimensional distribution of intensity clearly indicates the presence of 3 layers, as we had deduced in Problem 1. Moreover, we can see that middle layer, at depth of approximately at 5400 meters, curves thus providing a location where oil might collect. So, a possible drill location might be at position 250.

Problem 3:[30 pts]

We form a sum of sinusoids, all equally weighted, as follows

$$s = \sum_{i=0}^{90} \cos(2\pi(i \times 0.1)) \tag{4}$$

That is, we sum up cosine functions at 91 different frequencies. The resulting signal is shown in Figure 5 What would we expect to see? The sum above is a discrete approximation to the following integral

$$s(t) = 2\int_{1\text{ Hz}}^{10\text{ Hz}} 1\cos(2\pi f t)\, df \tag{5}$$

The integral above can be interpreted as the inverse Fourier Transform of a spectrum that has constant (unity) amplitude in the frequency range $1\text{ Hz} < f < 10\text{ Hz}$. The factor of two arises from the fact that the integrand is symmetric. Mathematically, this spectrum is described as

$$S(f) = \text{rect}\left(\frac{f-5.5}{9}\right) + \text{rect}\left(\frac{f+5.5}{9}\right) \tag{6}$$

The spectrum $S(f)$ above can be written as a convolution of a rect with two delta functions, as follows

$$S(f) = 2\text{rect}\left(\frac{f}{9}\right) * \frac{1}{2}[\delta(f-5.5)+\delta(f+5.5)] \tag{7}$$

By the Convolution Theorem, the time signal should be

$$s(t) = 18\text{sinc}(9t)\cos(2\pi 5.5t); \tag{8}$$

Figure 6 shows the effect of aliasing. The figure on the left shows the spectrum of sum of cosines of frequencies from 1 Hz to 10 Hz when the time spacings between samples are 1 second, 0.1 seconds and 0.01 seconds. When the sample rate is high, we observe two clearly distinguishable passbands, with roughly constant amplitude in the frequency range between 1 Hz and 10 Hz. For low sample rates, in contrast, we see the "smearing" out of the passbands into a spectrum that is centered at DC.

MATLAB code for Problem 1

```
% load in data
fid = fopen('hw6prob1data','rb');
data = fread(fid, inf,'float');fclose(fid);

% arrange data into a matrix with 21 rows and 512 columns
im = reshape(data,[512 21]); im = im';

% parameters
v = 5000;    % meters/second
t0 = 2.0;    % seconds
delta_t = 0.001;    % seconds
s = 80;      % meters

% loop through all samples in first array
ave_array = zeros(1,512);

si = [0:21-1]*s;
[time,ss] = meshgrid([0:512-1]*delta_t,si);
```

```
d = (t0 + time)*v/2;
tau = sqrt(4*d.^2 + ss.^2)./v;
ind = floor((tau-t0)./delta_t) + 1;

for i=1:512
  I = (1:21)'; J = ind(:,i); ii=find(J<=512);
  I = I(ii); J=J(ii);
  ave_array(i) = mean(im(sub2ind(size(im),I,J)));
end

depth = (t0 + [0:512-1]*delta_t)*v/2;
figure(1); plot(depth, sum(im));
h =gca;set(h,'FontSize',20); xlabel('depth (m)');
ylabel('averaged intensity');
title('direct sum of profiles as a function of depth');
grid on;

figure(2); plot(depth, ave_array);
h =gca;set(h,'FontSize',20); xlabel('depth (m)');
ylabel('averaged intensity');
title('stacked profile of echoes as a function of depth');
grid on;
```

MATLAB code for Problem 2

```
% load in data
fid = fopen('hw6prob2data','rb');
data = fread(fid, inf,'uint8');fclose(fid);

% arrange data into a matrix with 21
% rows and 512 columns
im2 = reshape(data,[512*21 512]);
im2 = im2'; im2 = im2 - 128;

% parameters
v = 5000;    % meters/second
t0 = 2.0;    % seconds
```

```
delta_t = 0.001;    % seconds
s = 80;       % meters

% loop through all samples in first array

fin = zeros(512,512);

si = [0:21-1]*s;
for pos = 1:512

  if(mod(pos,10)==0); disp(pos); end;
  dat = im2(pos,:); im = reshape(dat,[512 21]);
  im = im';

  si = [0:21-1]*s;
  [time,ss] = meshgrid([0:512-1]*delta_t,si);
  d = (t0 + time)*v/2;
  tau = sqrt(4*d.^2 + ss.^2)./v;
  ind = floor((tau-t0)./delta_t) + 1;
  ave_array = zeros(1,512);
  for i=1:512
    I = (1:21)'; J = ind(:,i); ii=find(J<=512);
    I = I(ii); J=J(ii);
    ave_array(i) = mean(im(sub2ind(size(im),I,J)));
  end

  fin(:,pos) = ave_array(:);
end

depth = (t0 + [0:512-1]*delta_t)*v/2;
imagesc([1:512],depth,fin);axis square;
h=gca;set(h,'FontSize',20);
ylabel('depth (m)'); xlabel('positions');
colormap gray;
```

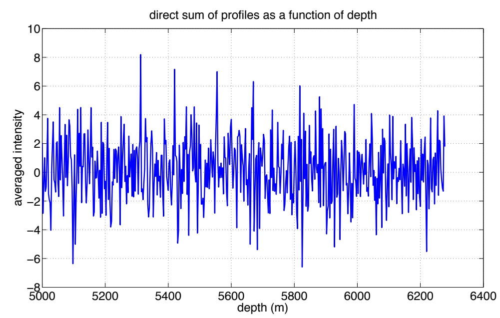

Figure 2:

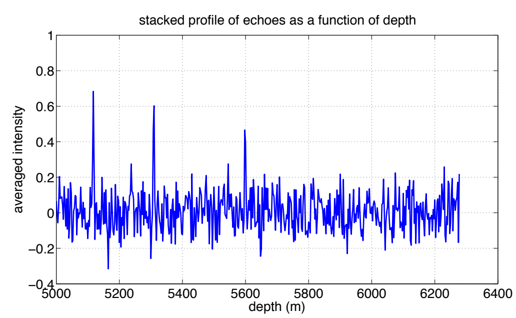

Figure 3:

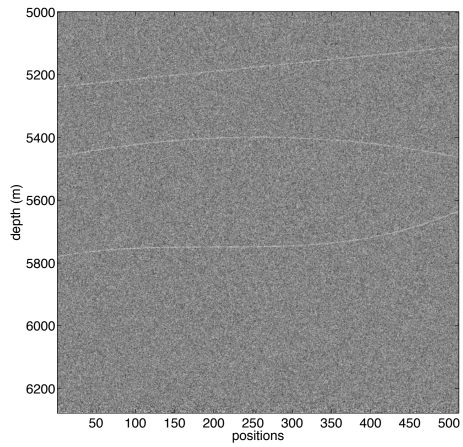

Figure 4:

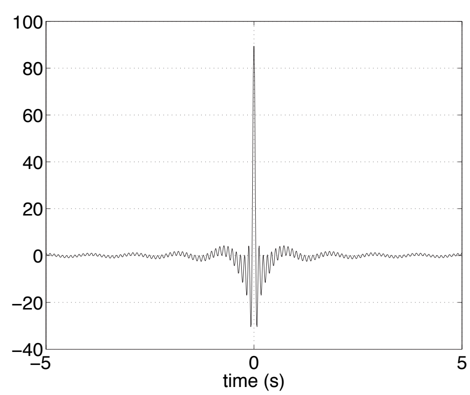

Figure 5:

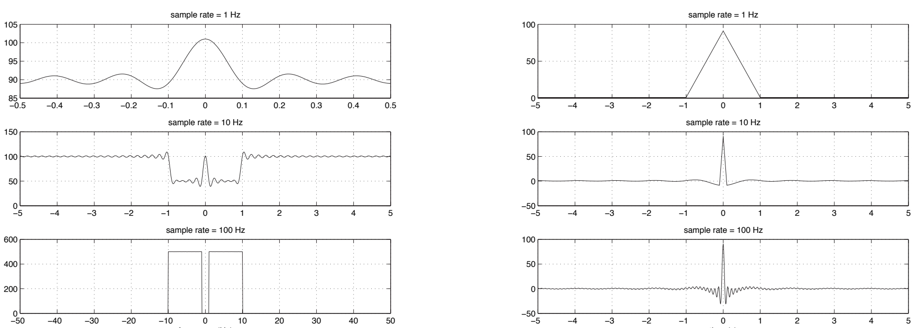
sample rate = 1 Hz
sample rate = 10 Hz
sample rate = 100 Hz
frequency (Hz)
sample rate = 1 Hz
sample rate = 10 Hz
sample rate = 100 Hz
time (s)

Figure 6: