Albrecht Dürer, “Underweysung der Messung mit dem Zirckel und Richtscheyt”, 1525
Lecture Overview

- what is computer graphics?
- the graphics pipeline
- primitives: vertices, edges, triangles!
- model transforms: translations, rotations, scaling
- view transform
- perspective transform
- window transform
Modeling 3D Geometry

Courtesy of H.G. Animations
https://www.youtube.com/watch?v=fewbFvA5oGk
What is Computer Graphics?

• at the most basic level: conversion from 3D scene description to 2D image

• what do you need to describe a static scene?
  • 3D geometry and transformations
  • lights
  • material properties

• most common geometry primitives in graphics:
  • vertices (3D points) and normals (unit-length vector associated with vertex)
  • triangles (set of 3 vertices, high-resolution 3D models have M or B of triangles)
The Graphics Pipeline

- geometry + transformations
- cameras and viewing
- lighting and shading
- rasterization
- texturing
Some History

- Stanford startup in 1981
- computer graphics goes hardware
- based on Jim Clark’s geometry engine
Figure 3: Geometry System; each block is a Geometry Engine.

The subsystems are:

- **Matrix Subsystem** - A stack of 4x4 floating-point matrices for completely general, 2D or 3D floating point coordinate transformation of graphical data.

- **Clipping Subsystem** - A windowing, or clipping, capability for clipping 2D or 3D graphical data to a window into the user's virtual drawing space. In 3D, this window is a volume of the user's virtual floating-point space, corresponding to a truncated viewing pyramid with "near" and "far" clipping.

- **Scaling Subsystem** - Scaling of 2D and 3D coordinates to the coordinate system of the particular output device of the user. In 3D, this scaling phase also includes either orthographic or perspective projection onto the viewer's virtual window. Stereo coordinates are computed and optionally supplied as the output of the system.
The Graphics Pipeline

• monolithic graphics workstations of the 80s have been replaced by modular GPUs (graphics processing units); major companies: NVIDIA, AMD, Intel

• early versions of these GPUs implemented *fixed-function* rendering pipeline in hardware

• GPUs have become programmable starting in the late 90s
  • e.g. in 2001 Nvidia GeForce 3 = first programmable shaders

• now: GPUs = programmable (e.g. OpenGL, CUDA, OpenCL) processors
The Graphics Pipeline

GPU = massively parallel processor
• OpenGL is our interface to the GPU!

• right: “old-school” OpenGL state machine

• today’s lecture: vertex transforms

I had this poster hanging on my dorm wall during undergrad
WebGL

- JavaScript application programmer interface (API) for 2D and 3D graphics
- OpenGL ES 2.0 running in the browser, implemented by all modern browsers
- overview, tutorials, documentation: see lab 1
three.js

- cross-browser JavaScript library/API

- higher-level library that provides a lot of useful helper functions, tools, and abstractions around WebGL – easy and convenient to use

- https://threejs.org/
- simple examples: https://threejs.org/examples/
- great introduction (in WebGL):
  http://davidscottlyons.com/threejs/presentations/frontporch14/
3D Graphics Rendering Pipeline: Output of one stage is fed as input of the next stage. A vertex has attributes such as \((x, y, z)\) position, color (RGB or RGBA), vertex-normal \((n_x, n_y, n_z)\), and texture. A primitive is made up of one or more vertices. The rasterizer raster-scans each primitive to produce a set of grid-aligned fragments, by interpolating the vertices.

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
1. **Vertex Processing**: Process and transform individual vertices & normals.
2. **Rasterization**: Convert each primitive (connected vertices) into a set of fragments. A fragment can be interpreted as a pixel with attributes such as position, color, normal and texture.
3. **Fragment Processing**: Process individual fragments.
4. **Output Merging**: Combine the fragments of all primitives (in 3D space) into 2D color-pixel for the display.
The Graphics Pipeline

- **Vertex Shader**
  - transforms & (per-vertex) lighting

- **Fragment Shader**
  - texturing
  - (per-fragment) lighting

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Coordinate Systems

• right hand coordinate system

• a few different coordinate systems:
  • object coordinates
  • world coordinates
  • viewing coordinates
  • also clip, normalized device, and window coordinates
Primitives

• vertex = 3D point v(x,y,z)

• triangle = 3 vertices

• normal = 3D vector per vertex describing surface orientation \( \mathbf{n}=(n_x,n_y,n_z) \)
Pixels v Fragments

- fragments have rasterized 2D coordinates on screen but a lot of other attributes too (texture coordinates, depth value, alpha value, …)
- pixels appear on screen
- won’t discuss in more detail today

A *primitive* is formed by one or more *vertices*. Vertices are not grid-aligned.

Grid-aligned *fragments* are interpolated from vertices.

All primitives are merged to produce 2D *pixels* on the display.

**Vertex, Primitives, Fragment and Pixel**

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Vertex Transforms

Raw Vertices & Primitives → Transformed Vertices & Primitives → Rasterizer → Fragments → Fragment Processor (Programmable) → Processed Fragments → Output Merging → Pixels → Display

3D

2D array of color-values

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Vertex Transforms

Vertex/normal transforms

Model Space
- Model Transform
- World Space
- View Transform
- Camera Space
- Projection Transform
- Clipping-Volume Space
- Viewport Transform
- Screen Space

Coordinates Transform Pipeline

Raw Vertices & Primitives
- Transformed Vertices & Primitives
- Fragments
- Processed Fragments
- Pixels

Display

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Vertex Transforms

1. Arrange the objects (or models, or avatar) in the world (Model Transform).
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Model Transform

- transform each vertex $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ from object coordinates to world coordinates

Objects are typically created in their local spaces. We need to bring them into the common world space, via a series of affine transforms (translation, rotation and scaling).

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Model Transform - Scaling

- transform each vertex \( v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \) from object coordinates to world coordinates

1. scaling as 3x3 matrix

\[
S(s_x, s_y, s_z) = \begin{pmatrix}
  s_x & 0 & 0 \\
  0 & s_y & 0 \\
  0 & 0 & s_z \\
\end{pmatrix}
\]

scaled vertex = matrix-vector product:

\[
sv = \begin{pmatrix}
  s_x & 0 & 0 \\
  0 & s_y & 0 \\
  0 & 0 & s_z \\
\end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} s_x x \\ s_y y \\ s_z z \end{pmatrix}
\]
Model Transform - Rotation

- transform each vertex \( \mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \) from object coordinates to world coordinates

2. rotation as 3x3 matrix

\[
R_z(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix} \quad R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta \end{bmatrix} \quad R_y(\theta) = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta \end{bmatrix}
\]

rotated vertex = matrix-vector product, e.g.

\[
R_z \mathbf{v} = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \cos \theta - y \sin \theta \\
x \sin \theta + y \cos \theta \\ z \end{bmatrix}
\]
Model Transform - Translation

- transform each vertex $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ from object coordinates to world coordinates

3. translation cannot be represented as 3x3 matrix!

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} = \begin{pmatrix} x + d_x \\ y + d_y \\ z + d_z \end{pmatrix}$$

that’s unfortunate 😐
Model Transform - Translation

- solution: use homogeneous coordinates, vertex is \( v = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \)

3. translation is 4x4 matrix

\[
T(d) = \begin{pmatrix}
1 & 0 & 0 & d_x \\
0 & 1 & 0 & d_y \\
0 & 0 & 1 & d_z \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
Tv = \begin{pmatrix}
1 & 0 & 0 & d_x \\
0 & 1 & 0 & d_y \\
0 & 0 & 1 & d_z \\
0 & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + d_x \\ y + d_y \\ z + d_z \\ 1 \end{pmatrix}
\]
Summary of Homogeneous Matrix Transforms

- **translation** \( T(d) = \begin{pmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

- **scale** \( S(s) = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

- **rotation** \( R_z(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

\( R_x = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

\( R_y = \begin{pmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

Read more: https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Summary of Homogeneous Matrix Transforms

- **translation** \( T(d) = \begin{pmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \)
  
  inverse translation \( T^{-1}(d) = T(-d) = \begin{pmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

- **scale** \( S(s) = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)
  
  inverse scale \( S^{-1}(s) = S\left(\frac{1}{s}\right) = \begin{pmatrix} 1/s_x & 0 & 0 & 0 \\ 0 & 1/s_y & 0 & 0 \\ 0 & 0 & 1/s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

- **rotation** \( R_z(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)
  
  inverse rotation \( R_z^{-1}(\theta) = R_z(-\theta) = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) & 0 & 0 \\ \sin(-\theta) & \cos(-\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \)

Read more: https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Summary of Homogeneous Matrix Transforms

• successive transforms: \( v' = T \cdot S \cdot R_z \cdot R_x \cdot T \cdot v \)

• inverse successive transforms: 
  \[
  v = \left( T \cdot S \cdot R_z \cdot R_x \cdot T \right)^{-1} \cdot v' \\
  = T^{-1} \cdot R_x^{-1} \cdot R_z^{-1} \cdot S^{-1} \cdot T^{-1} \cdot v'
  \]

Read more: https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Vector and Normal Transforms

- homogeneous representation of a vector $t$, i.e. pointing from $v_1$ to $v_2$:

$$
\begin{bmatrix}
(v_2-v_1)_x \\
(v_2-v_1)_y \\
(v_2-v_1)_z \\
(1-1)
\end{bmatrix}
\begin{bmatrix}
t_x \\
t_y \\
t_z \\
0
\end{bmatrix}
= 
\begin{bmatrix}
(v_2-v_1) \\
M \cdot v_2 - M \cdot v_1
\end{bmatrix}
$$

- successive transforms: $t' = M \cdot t = M \cdot (v_2 - v_1) = M \cdot v_2 - M \cdot v_1$

- this works!
Vector and **Normal** Transforms

- homogeneous representation of a normal (unit length, perpendicular to surface)

- successive transforms ??? \( n' = M \cdot n \)

- this does **NOT** work! (non-uniform scaling is a problem)

![Diagram](image-url)
Vector and **Normal** Transforms

- homogeneous representation of a normal (unit length, perpendicular to surface)

- need to use normal matrix = transpose of inverse for transformation!

\[
\begin{pmatrix}
    n_x \\
    n_y \\
    n_z \\
    0
\end{pmatrix} = \left( M^{-1} \right)^T \cdot n
\]
Vector and **Normal** Transforms

- homogeneous representation of a normal (unit length, perpendicular to surface)

\[
n = \begin{pmatrix} n_x \\ n_y \\ n_z \\ 0 \end{pmatrix}
\]

- need to use normal matrix = transpose of inverse for transformation!

\[
n' = \left(M^{-1}\right)^T \cdot n
\]

- fine print: only use upper left 3x3 part of modelview matrix for inverse transpose (no homogeneous normal representation) OR drop \(w\) component from \(n'\) after multiplying 4x4 inverse transpose (i.e. don’t use \(w\) for normalization of \(n'\))
Attention!

- rotations and translations (or transforms in general) are not commutative!
- make sure you get the correct order!
so far we discussed model transforms, e.g. going from object or model space to world space

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
View Transform

- so far we discussed model transforms, e.g. going from object or model space to world space
- one simple 4x4 transform matrix is sufficient to go from world space to camera or view space!
View Transform

specify camera by

- eye position  \( \text{eye} = \begin{pmatrix} \text{eye}_x \\ \text{eye}_y \\ \text{eye}_z \end{pmatrix} \)

- reference position  \( \text{center} = \begin{pmatrix} \text{center}_x \\ \text{center}_y \\ \text{center}_z \end{pmatrix} \)

- up vector  \( \text{up} = \begin{pmatrix} \text{up}_x \\ \text{up}_y \\ \text{up}_z \end{pmatrix} \)
View Transform

specify camera by

- eye position  \( \text{eye} = \begin{pmatrix} \text{eye}_x \\ \text{eye}_y \\ \text{eye}_z \end{pmatrix} \)

- reference position  \( \text{center} = \begin{pmatrix} \text{center}_x \\ \text{center}_y \\ \text{center}_z \end{pmatrix} \)

- up vector  \( \text{up} = \begin{pmatrix} \text{up}_x \\ \text{up}_y \\ \text{up}_z \end{pmatrix} \)

compute 3 vectors:

\[
\begin{align*}
z^c &= \frac{\text{eye} - \text{center}}{||\text{eye} - \text{center}||} \\
x^c &= \frac{\text{up} \times z^c}{||\text{up} \times z^c||} \\
y^c &= z^c \times x^c
\end{align*}
\]
View Transform

view transform \( M \) is translation into eye position, followed by rotation

compute 3 vectors:

\[
\begin{align*}
z^c &= \frac{\text{eye} - \text{center}}{||\text{eye} - \text{center}||} \\
x^c &= \frac{\text{up} \times z^c}{||\text{up} \times z^c||} \\
y^c &= z^c \times x^c
\end{align*}
\]
view transform $M$ is translation into eye position, followed by rotation

$$M = R \cdot T(-e) = \begin{pmatrix} x^c_x & x^c_y & x^c_z & 0 \\ y^c_x & y^c_y & y^c_z & 0 \\ z^c_x & z^c_y & z^c_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -\text{eye}_x \\ 0 & 1 & 0 & -\text{eye}_y \\ 0 & 0 & 1 & -\text{eye}_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

compute 3 vectors:

$$z^c = \frac{\text{eye} - \text{center}}{||\text{eye} - \text{center}||}$$

$$x^c = \frac{\text{up} \times z^c}{||\text{up} \times z^c||}$$

$$y^c = z^c \times x^c$$
View Transform

The view transform $M$ is translation into eye position, followed by rotation.

$$M = R \cdot T(-e) = \begin{pmatrix}
x^c_x & x^c_y & x^c_z & -\left(x^c_{\text{eye}_x} + x^c_{\text{eye}_y} + x^c_{\text{eye}_z}\right) \\
y^c_x & y^c_y & y^c_z & -\left(y^c_{\text{eye}_x} + y^c_{\text{eye}_y} + y^c_{\text{eye}_z}\right) \\
z^c_x & z^c_y & z^c_z & -\left(z^c_{\text{eye}_x} + z^c_{\text{eye}_y} + z^c_{\text{eye}_z}\right) \\
0 & 0 & 0 & 1
\end{pmatrix}$$
many graphics APIs have a function called lookat that automatically computes the view matrix for you

Three.js also has such a function, but that only computes the rotation, not the translation, of the view matrix. So best implement the view matrix yourself!
View Transform

- in camera/view space, the camera is at the origin, looking into negative z
- **modelview matrix** is combined model (rotations, translations, scaling) and view matrix!
View Transform

- in camera/view space, the camera is at the origin, looking into negative z
Projection Transform

- similar to choosing lens and sensor of camera – specify field of view and aspect

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Projection Transform - Perspective Projection

- have symmetric view frustum
- fovy: vertical angle in degrees
- aspect: ratio of width/height
- zNear: near clipping plane (relative from cam)
- zFar: far clipping plane (relative from cam)

\[ f = \cot\left(\text{fovy} / 2\right) \]

\[
M_{\text{proj}} = \begin{pmatrix}
\frac{f}{\text{aspect}} & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & \frac{z\text{Far} + z\text{Near}}{z\text{Far} - z\text{Near}} & -\frac{2 \cdot z\text{Far} \cdot z\text{Near}}{z\text{Far} - z\text{Near}} \\
0 & 0 & -1 & 0 \\
\end{pmatrix}
\]

projection matrix
(symmetric frustum)
Projection Transform - Perspective Projection

more general: a perspective “frustum” (truncated, possibly sheared pyramid)

- left (l), right (r), bottom (b), top (t): corner coordinates on near clipping plane (at zNear)

\[
M_{proj} = \begin{pmatrix}
\frac{2 \cdot z_{Near}}{r - l} & 0 & \frac{r + l}{r - l} & 0 \\
0 & \frac{2 \cdot z_{Near}}{t - b} & \frac{t + b}{t - b} & 0 \\
0 & 0 & -\frac{z_{Far} + z_{Near}}{z_{Far} - z_{Near}} & -\frac{2 \cdot z_{Far} \cdot z_{Near}}{z_{Far} - z_{Near}} \\
0 & 0 & \frac{z_{Far} - z_{Near}}{z_{Far} - z_{Near}} & 0 \\
\end{pmatrix}
\]

projection matrix
(asymmetric frustum)
Projection Transform - Orthographic Projection

more general: a “box frustum” (no perspective, objects don’t get smaller when farther away)

- left (l), right (r), bottom (b), top (t): corner coordinates on near clipping plane

\[
M_{proj} = \begin{pmatrix}
\frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\
0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\
0 & 0 & \frac{-2}{f-n} & -\frac{f+n}{f-n} \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

projection matrix (orthographic)
Projection Transform

- possible source of confusion for zNear and zFar:
  - Marschner & Shirley define it as absolute z coordinates, thus zNear > zFar and both values are always negative
  - OpenGL and we define it as positive values, i.e. the distances of the near and far clipping plane from the camera (zFar > zNear)
Modelview Projection Matrix

• put it all together with 4x4 matrix multiplications!

\[ \mathbf{v}_{clip} = \mathbf{M}_{proj} \cdot \mathbf{M}_{view} \cdot \mathbf{M}_{model} \cdot \mathbf{v} = \mathbf{M}_{proj} \cdot \mathbf{M}_{mv} \cdot \mathbf{v} \]

vertex in clip space  projection matrix  modelview matrix
Clip Space

Clip Volume Space

Clipping Volume (2x2x1 Cuboid)

Coordinates Transform Pipeline

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Normalized Device Coordinates (NDC)

- not shown in previous illustration
- get to NDC by perspective division

\[ v_{clip} = \begin{pmatrix} x_{clip} \\ y_{clip} \\ z_{clip} \\ w_{clip} \end{pmatrix} \quad \xrightarrow{\text{perspective division}} \quad v_{NDC} = \begin{pmatrix} x_{clip} / w_{clip} \\ y_{clip} / w_{clip} \\ z_{clip} / w_{clip} \\ 1 \end{pmatrix} \in (-1,1) \]

vertex in clip space \quad \text{vertex in NDC}
Viewport Transform

define (sub)window as viewport(x,y,width,height),

- x,y lower left corner of viewport rectangle (default is (0,0))
- width, height size of viewport rectangle in pixels

\[
\begin{align*}
x_{\text{window}} &= \frac{\text{width}}{2}(x_{\text{NDC}} + 1) + x \\
y_{\text{window}} &= \frac{\text{height}}{2}(y_{\text{NDC}} + 1) + y \\
z_{\text{window}} &= \frac{1}{2}z_{\text{NDC}} + \frac{1}{2}
\end{align*}
\]

vertex in NDC

\[
\begin{pmatrix}
x_{\text{clip}} / w_{\text{clip}} \\
y_{\text{clip}} / w_{\text{clip}} \\
z_{\text{clip}} / w_{\text{clip}}
\end{pmatrix}
\]

vertex in window coords

\[
\begin{pmatrix}
x_{\text{window}} \\
y_{\text{window}} \\
z_{\text{window}}
\end{pmatrix} \in (0, \text{width}) \\
\in (0, \text{height}) \\
\in (0,1)
\]
The Graphics Pipeline – Another Illustration

1. Object Space → Modeling Transform → World Space → View Transform → Eye Space
2. Eye Space → Projection Transform → Clip Space → Perspective Divide → Normalized Device Space
3. Normalized Device Space → Viewport and Depth Range Transform → Window Space → Window Space
The Graphics Pipeline

Raw Vertices & Primitives → Transformed Vertices & Primitives → Rasterizer → Fragments → Fragment Processor (Programmable) → Processed Fragments → Output Merging → Pixels → Display

all vertex transforms from today!
assign fixed color (e.g. red) to each vertex in window coordinates (fragment)
interpolate (i.e. rasterize) lines between vertices (as defined by user)
… and we can almost do this …

Nintendo Virtual Boy Game “Red Alarm”
Summary

- Graphics pipeline is a series of operations that takes 3D vertices/normals/triangles as input and generates fragments and pixels.

- Today, we only discussed a part of it: vertex and normal transforms.

- Transforms include: rotation, scale, translation, perspective projection, perspective division, and viewport transform.

- Most transforms are represented as 4x4 matrices in homogeneous coordinates.

→ Know your matrices & be able to create, manipulate, invert them!
Next Lecture: Lighting and Shading, Fragment Processing

- **vertex shader**
  - transforms & (per-vertex) lighting

- **fragment shader**
  - texturing
  - (per-fragment) lighting

https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html
Further Reading

• course notes on transforms (see course website)

• good overview of OpenGL (deprecated version) and graphics pipeline (missing a few things):
  https://www.ntu.edu.sg/home/ehchua/programming/opengl/CG_BasicsTheory.html


• WebGL / three.js tutorials: https://threejs.org/