# Head Mounted Display Optics I


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Stanford University

EE 267 Virtual Reality

Lecture 7

stanford.edu/class/ee267/

# Logistics

- HW3 is probably the longest homework, so get started asap if you have not done so already

- hardware kits will be handed out in TA office hours this week

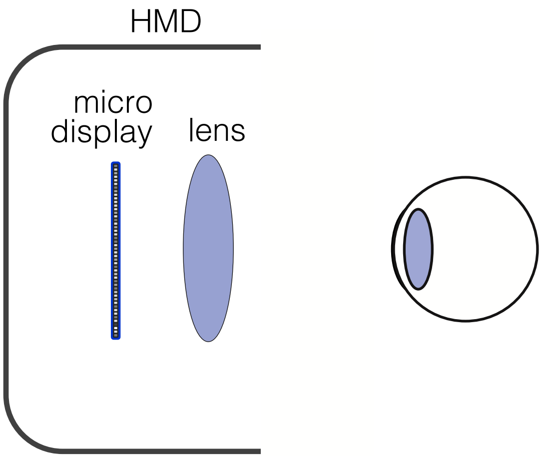
# Lecture Overview

1. stereo rendering for HMDs
2. field of view and visual field
3. lens distortion correction using GLSL
4. overview of microdisplay technology

# Stereo Rendering for HMDs

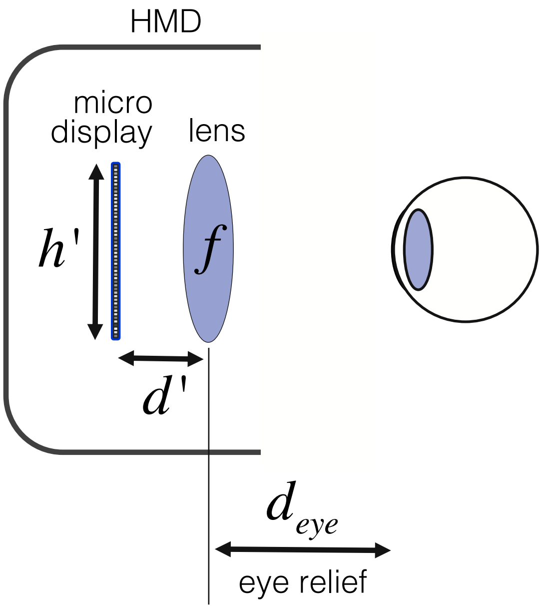
*All Current-generation VR HMDs are "Simple Magnifiers"*

# Image Formation



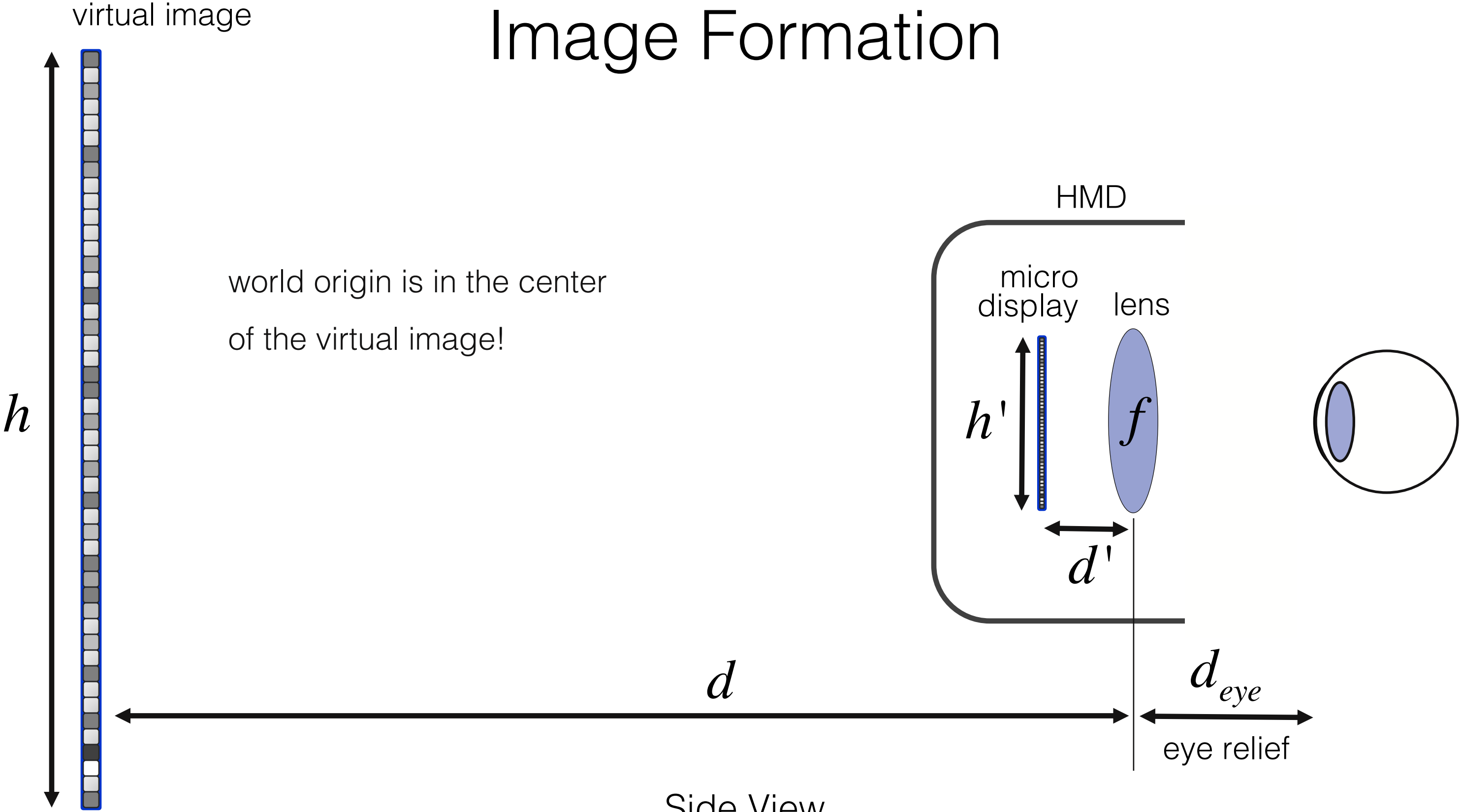

Side View

# Image Formation




Side View

# Image Formation

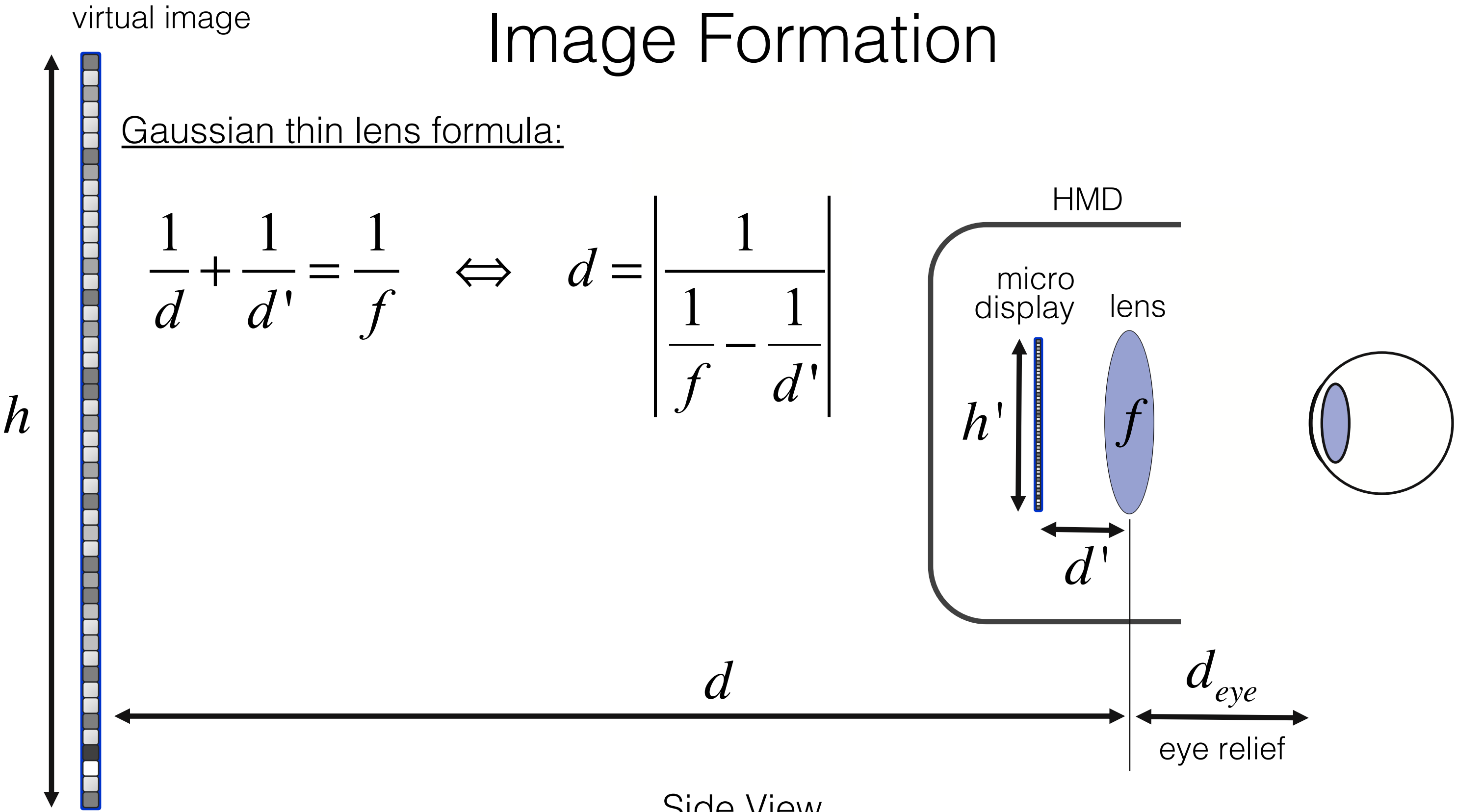



Side View

# Image Formation

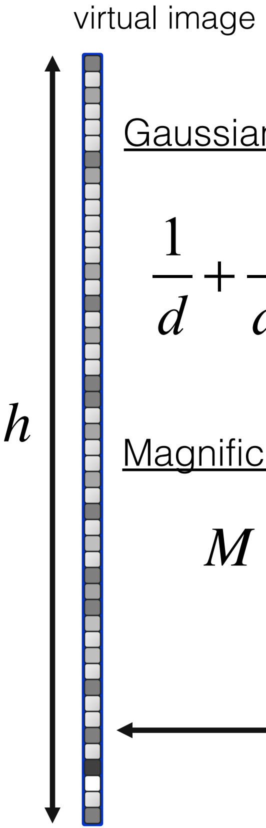
Gaussian thin lens formula:

$$\frac{1}{d} + \frac{1}{d'} = \frac{1}{f} \quad \Leftrightarrow \quad d = \left| \frac{1}{\frac{1}{f} - \frac{1}{d'}} \right|$$




Side View

# Image Formation

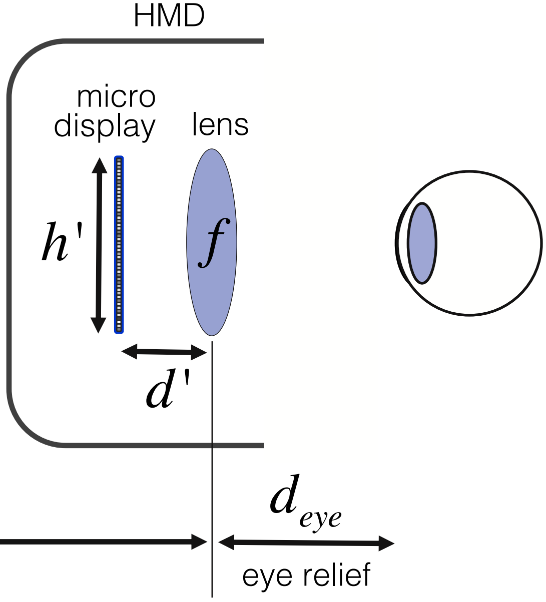



Gaussian thin lens formula:

$$\frac{1}{d}+\frac{1}{d'}=\frac{1}{f} \quad \Leftrightarrow \quad d=\left|\frac{1}{\frac{1}{f}-\frac{1}{d'}}\right|$$

Magnification:

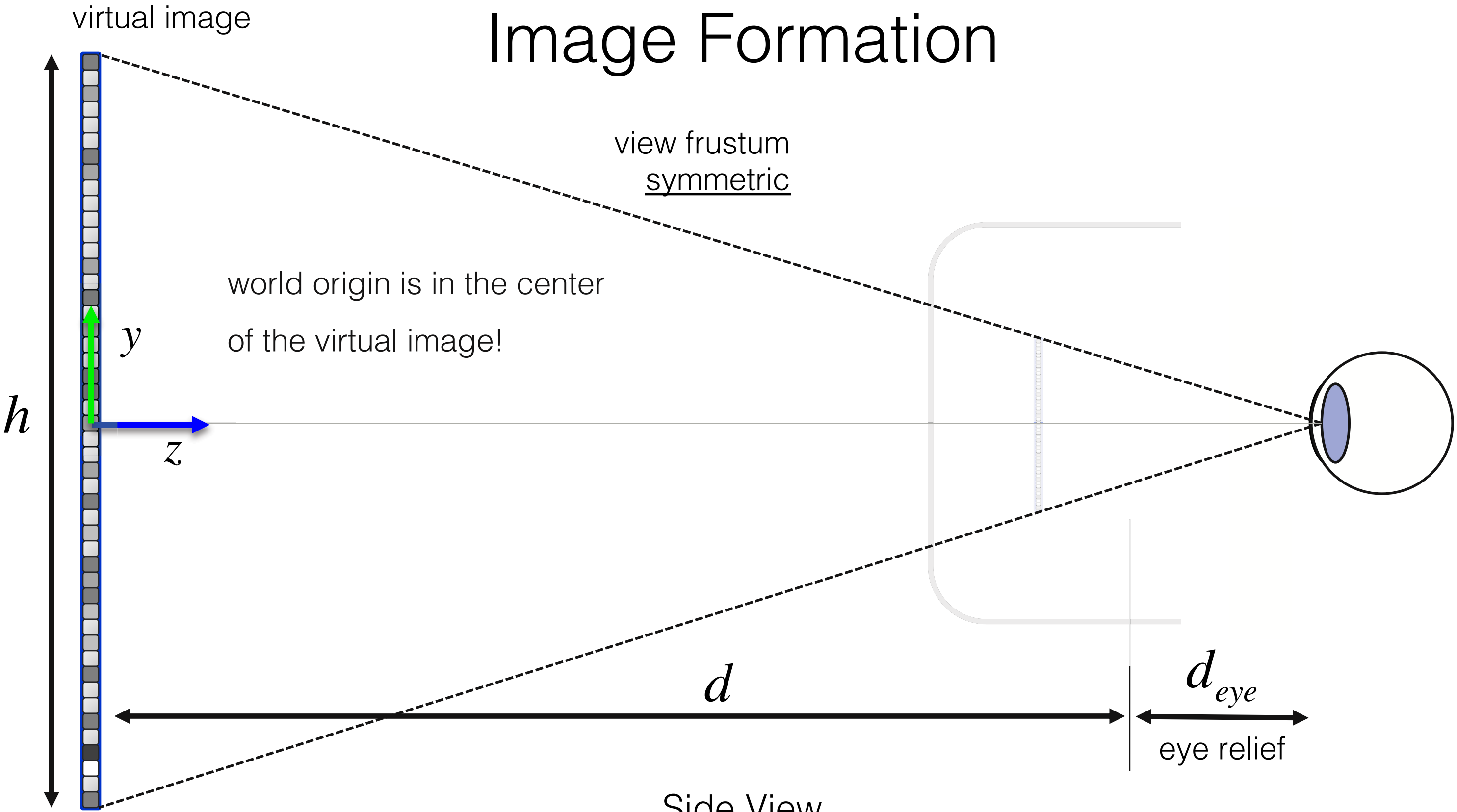
$$M=\frac{f}{f-d'} \quad \Rightarrow \quad h=Mh'$$





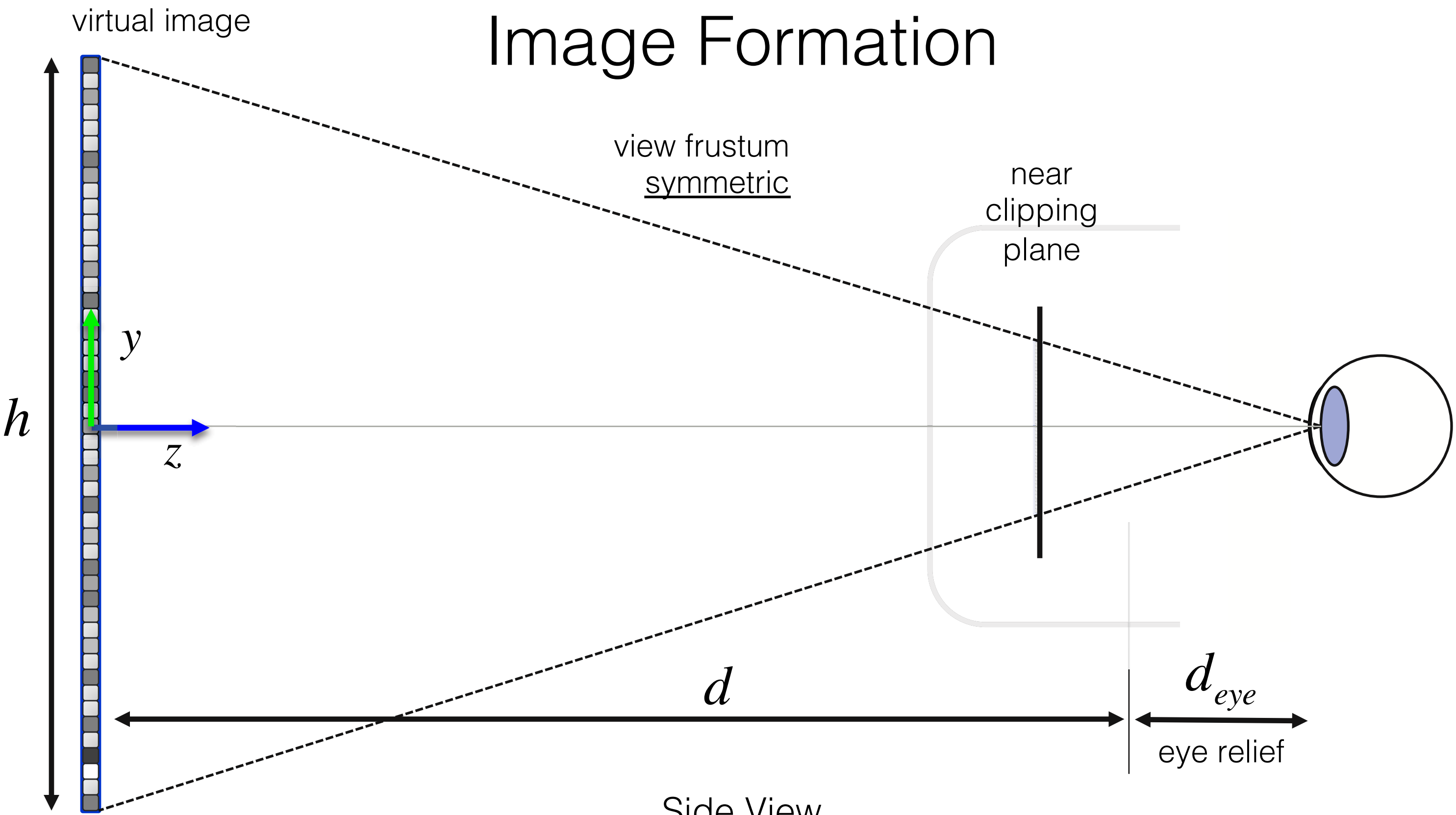
Side View

# Image Formation



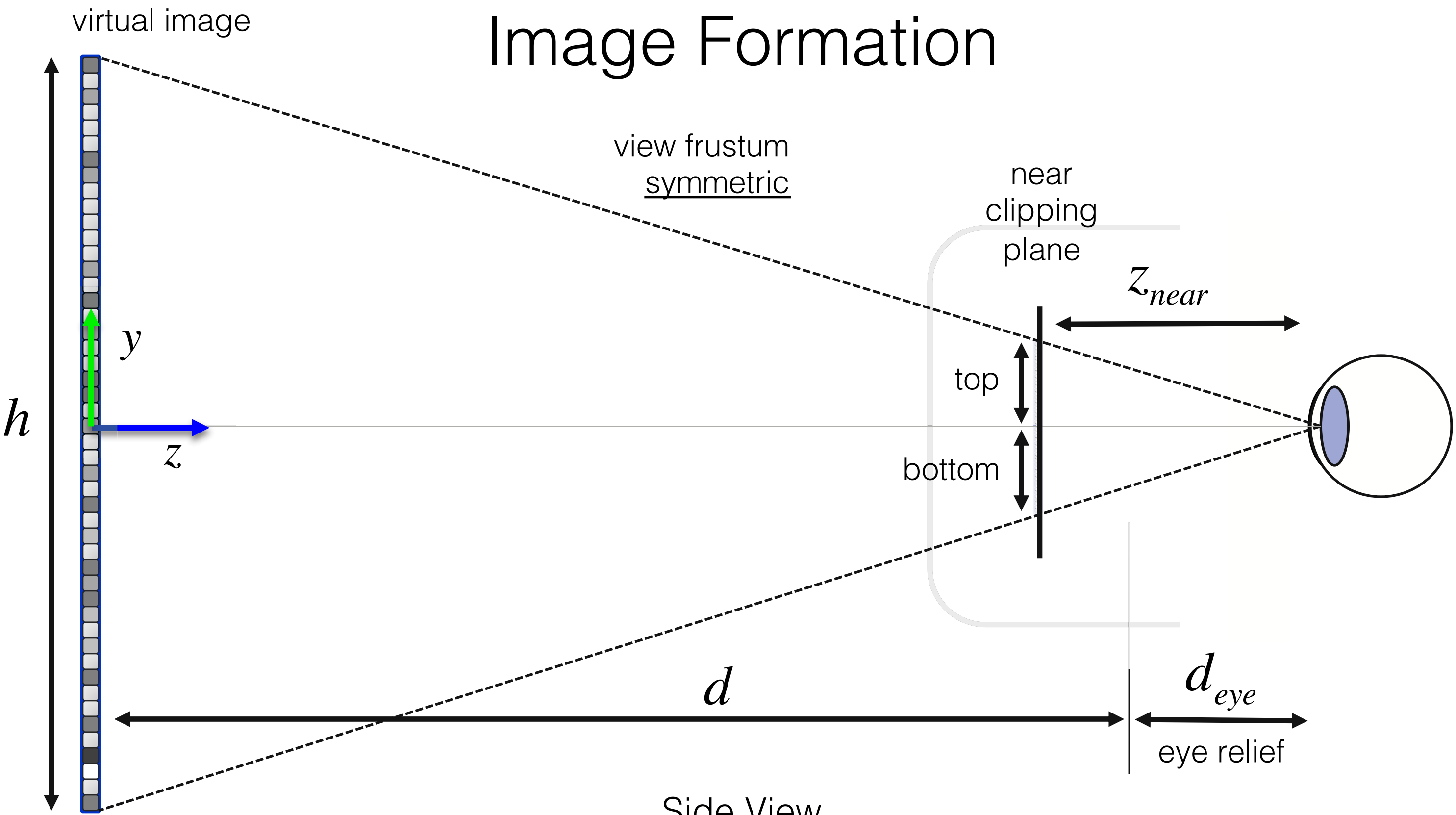

Side View

# Image Formation



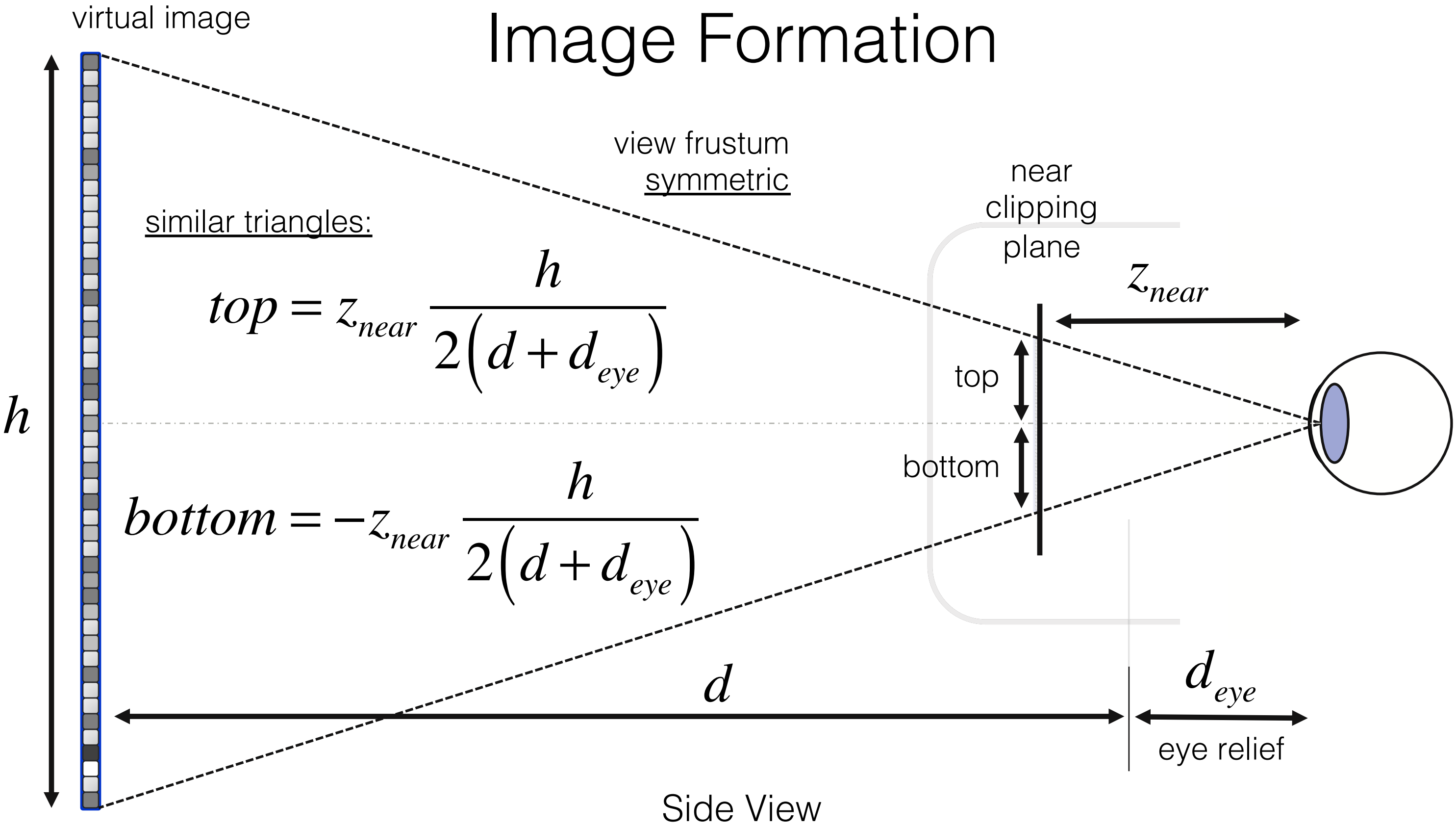

Side View

# Image Formation




Side View

# Image Formation

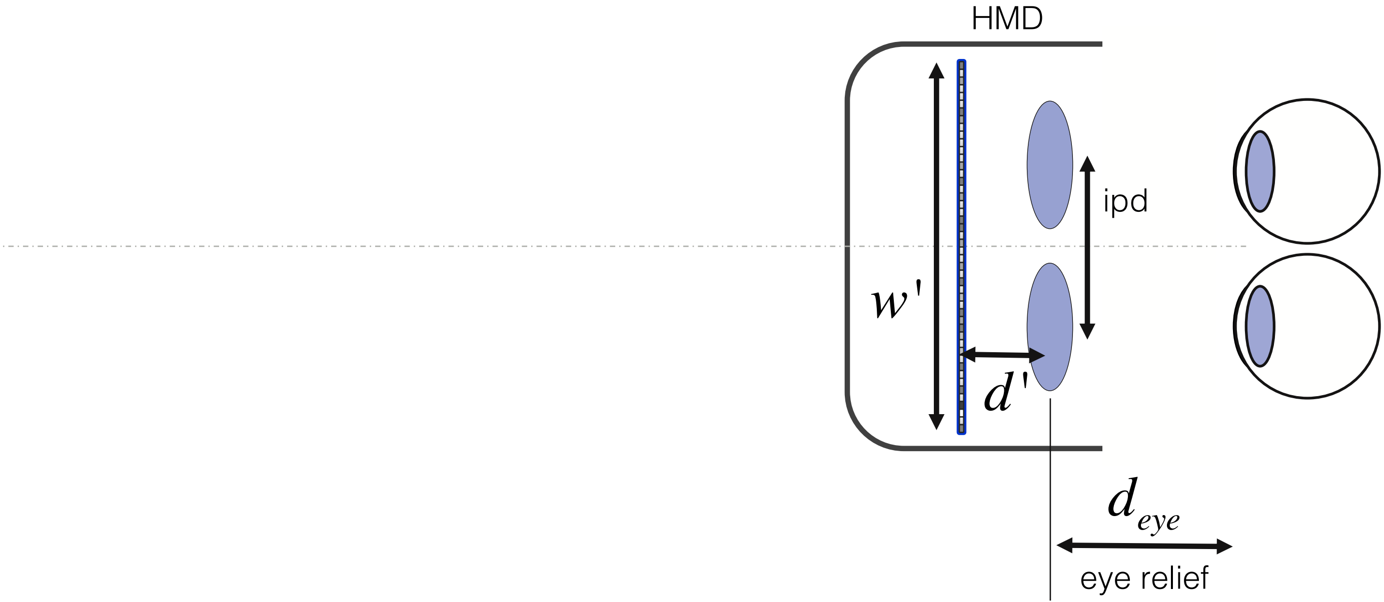



similar triangles:

$$top = z_{near} \frac{h}{2\left(d + d_{eye}\right)}$$

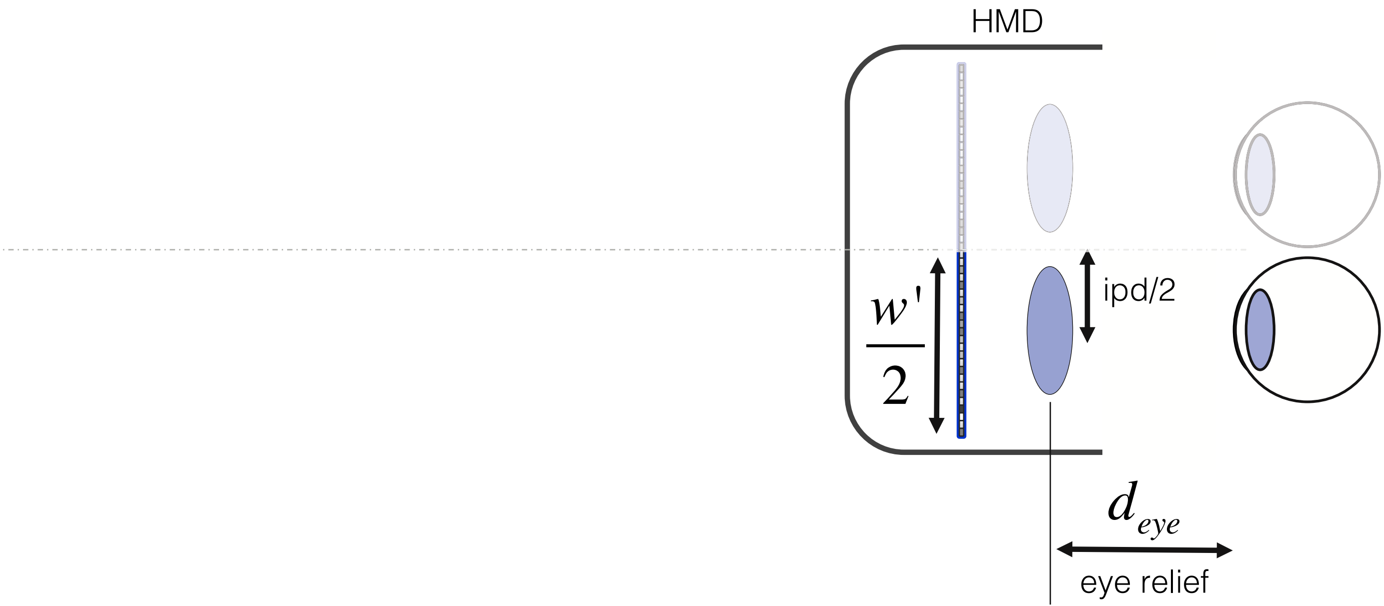
$$bottom = -z_{near} \frac{h}{2\left(d + d_{eye}\right)}$$

# Image Formation



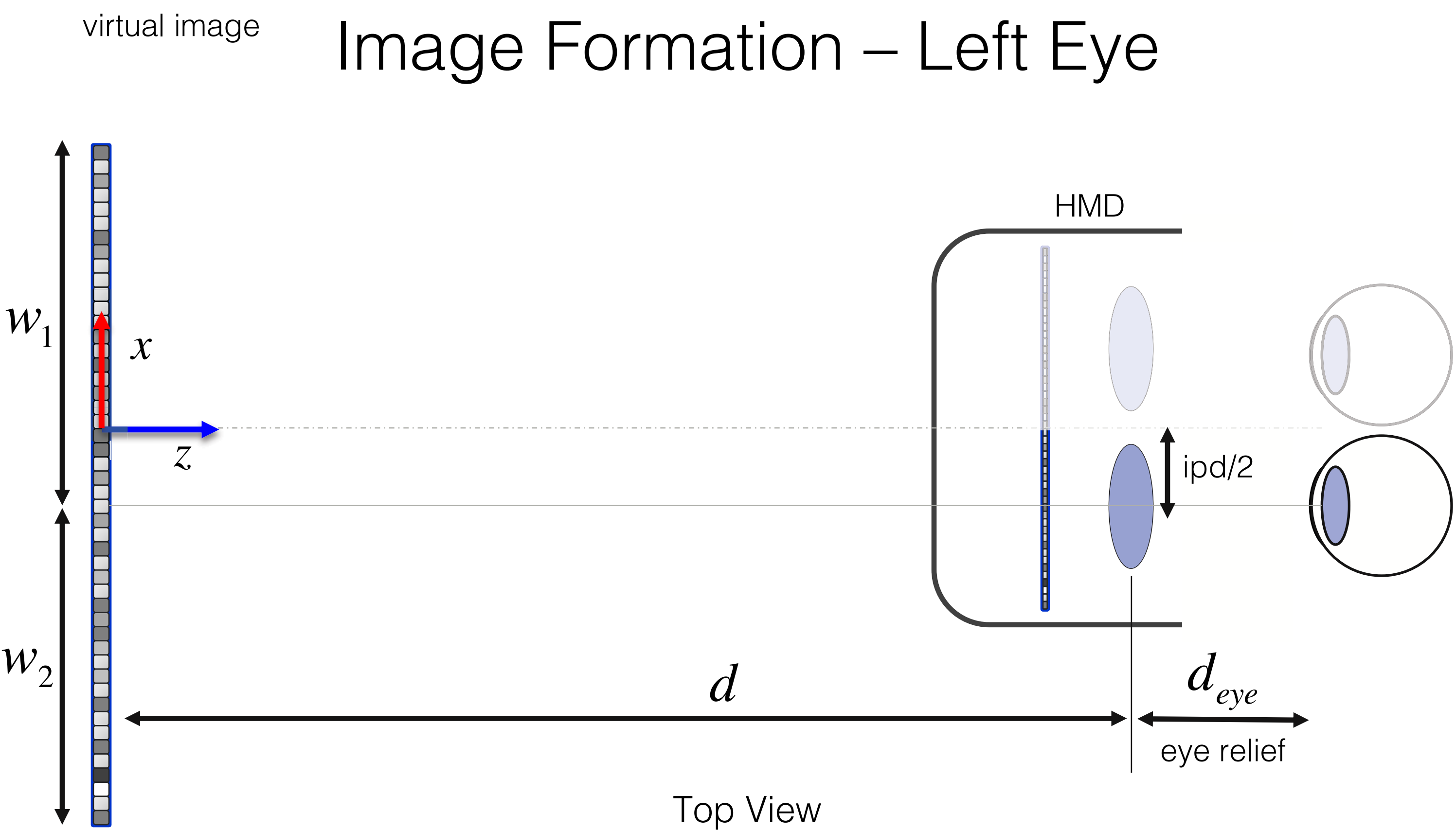

Top View

# Image Formation – Left Eye



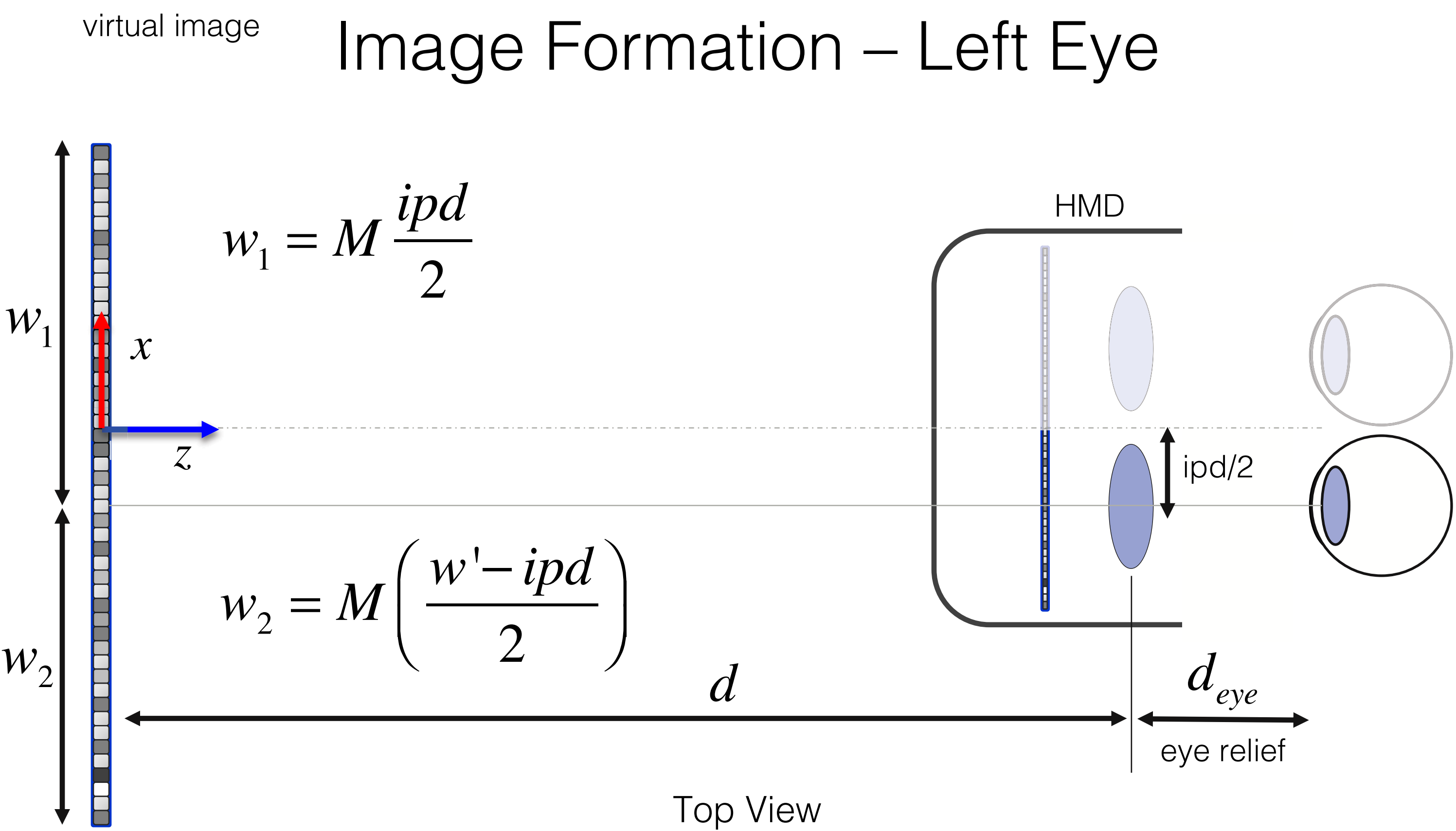

Top View

# Image Formation – Left Eye




Top View

# Image Formation – Left Eye

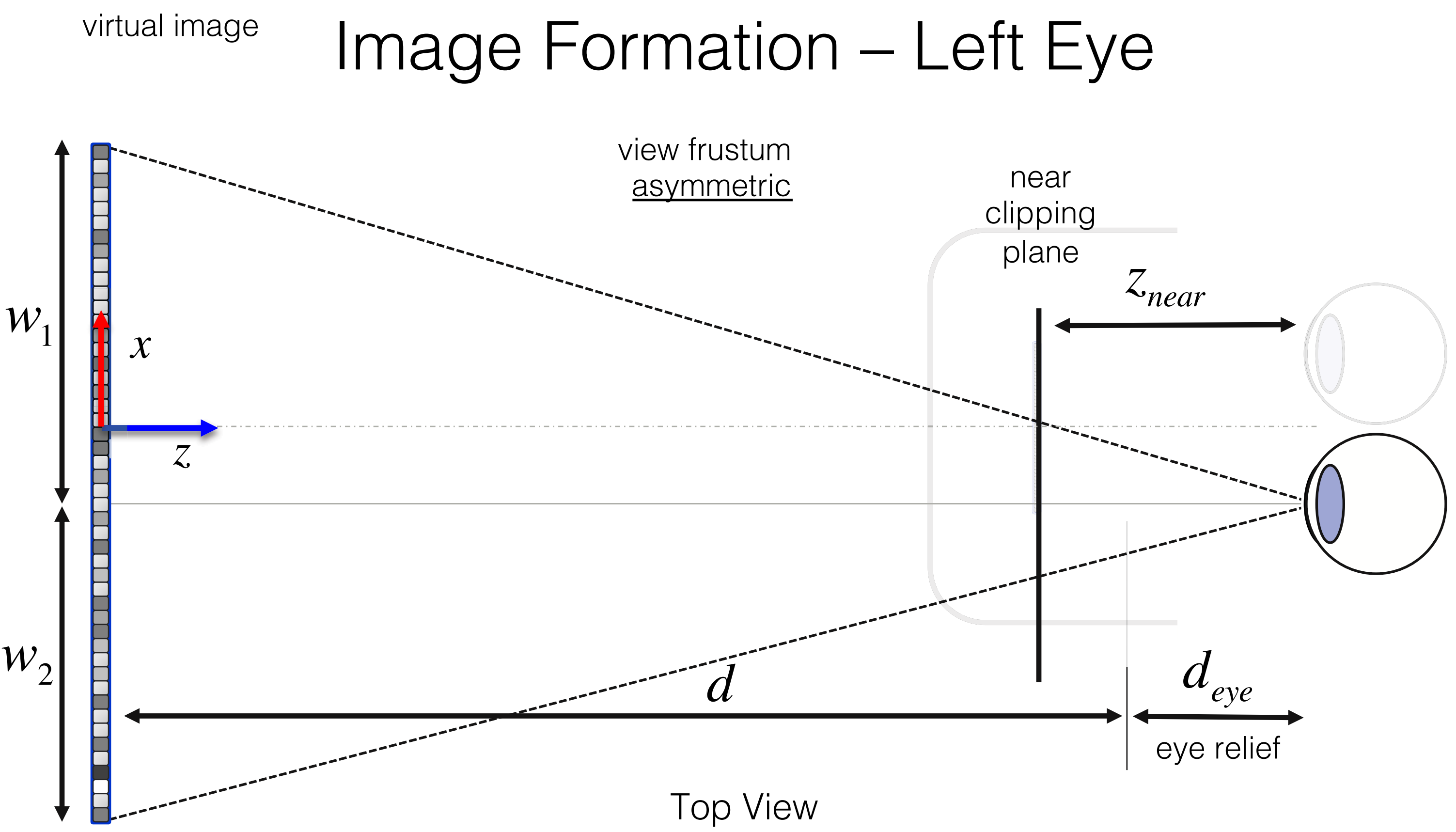



$$w_1 = M\frac{ipd}{2}$$

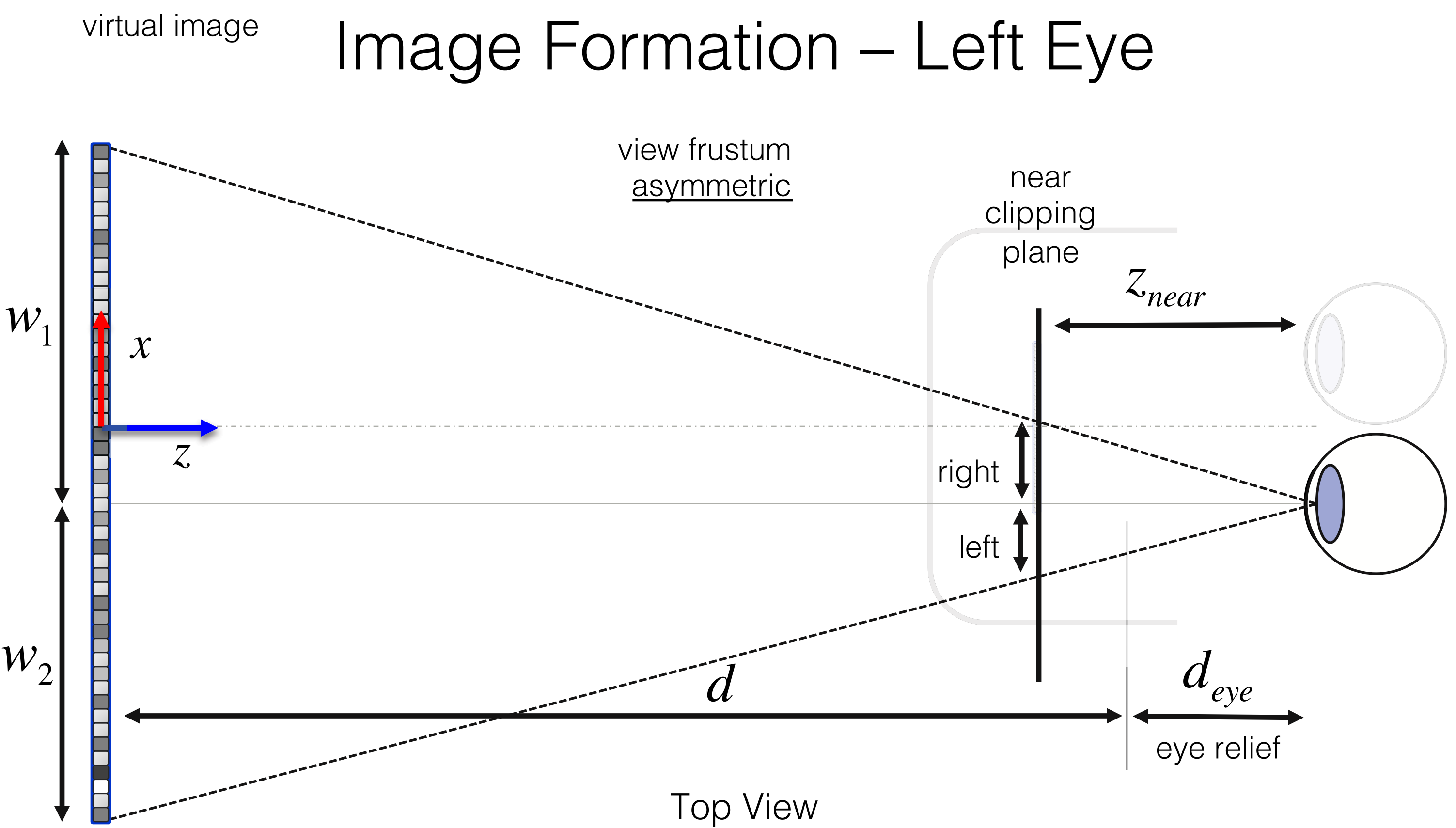
$$w_2 = M\left(\frac{w' - ipd}{2}\right)$$

Top View

# Image Formation – Left Eye

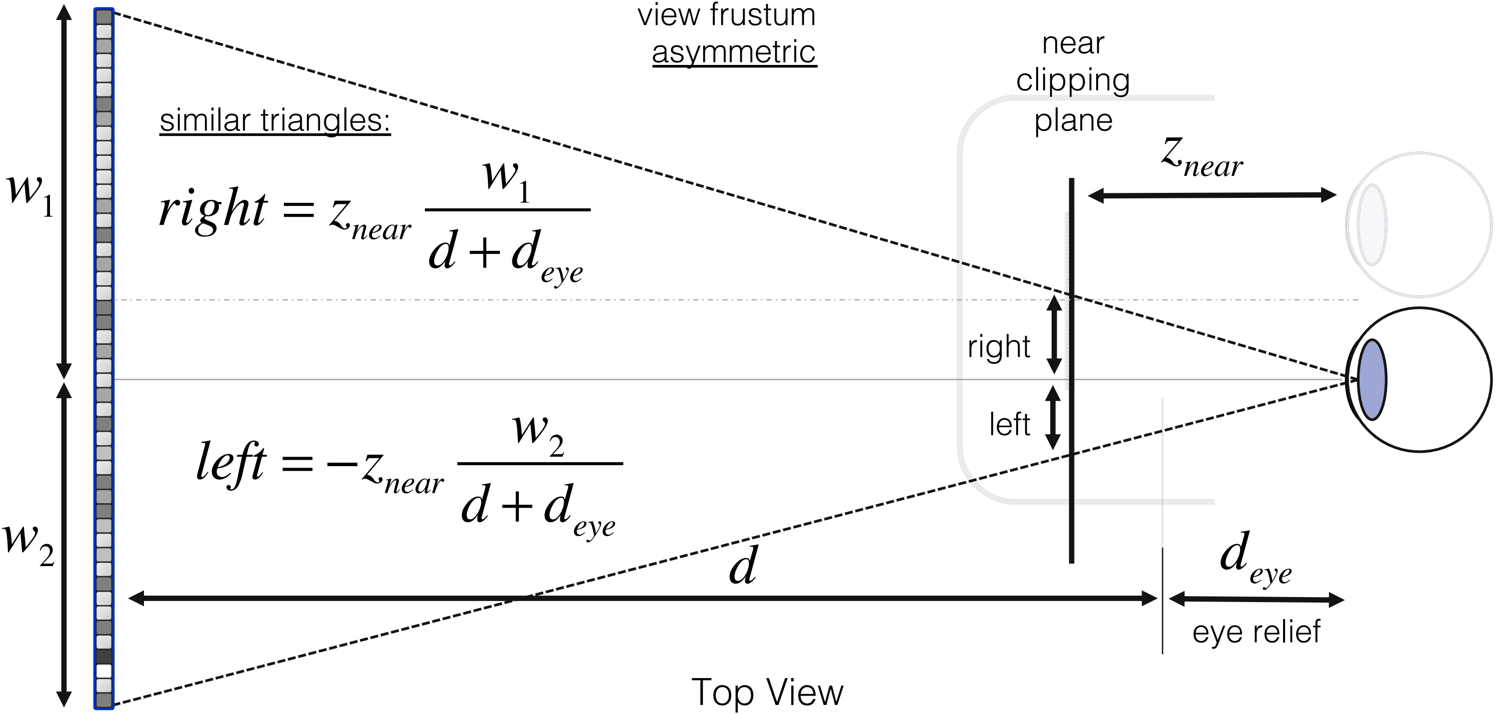


# Image Formation – Left Eye




Top View

# Image Formation – Left Eye

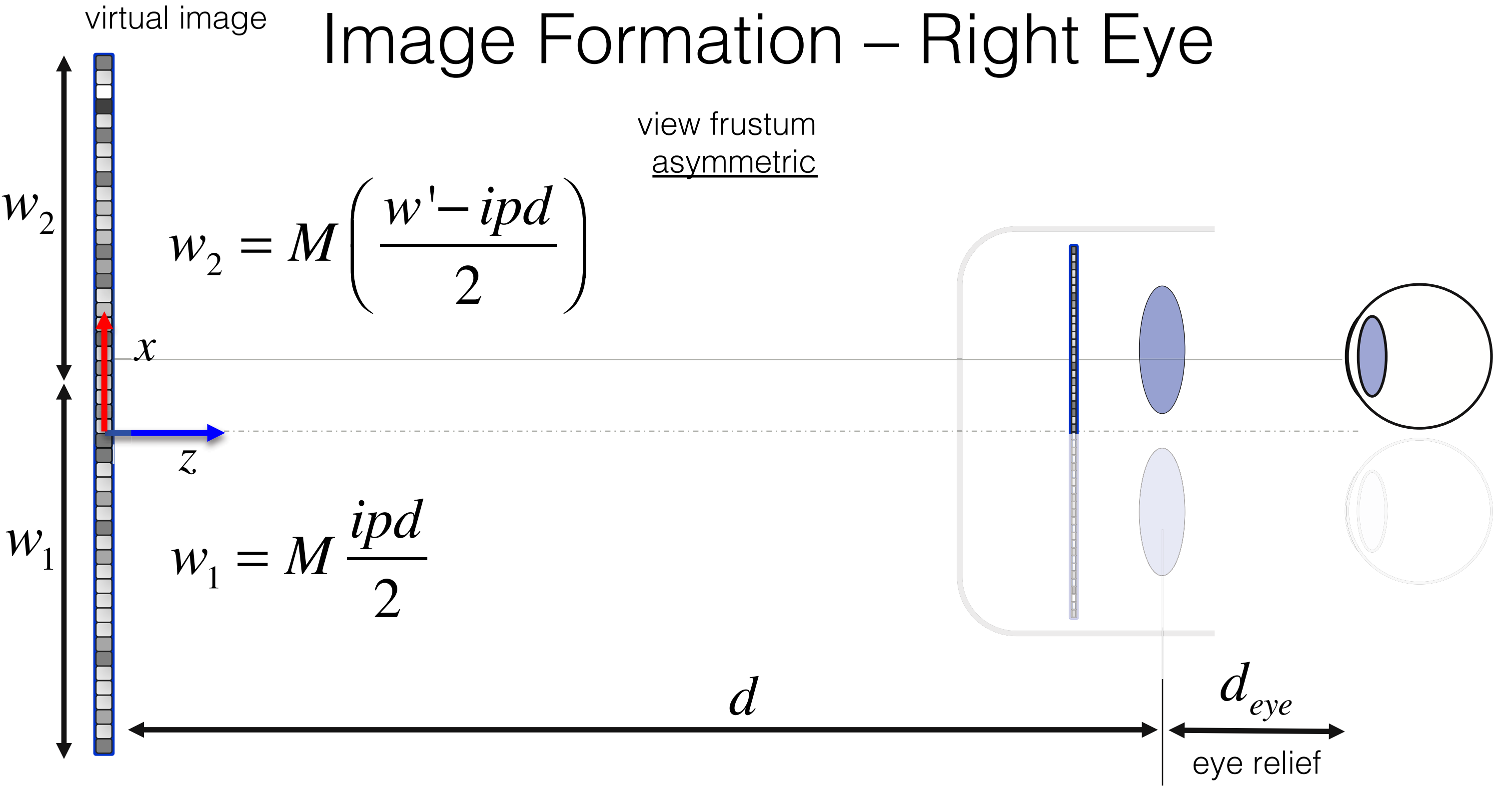



similar triangles:

$$right = z_{near} \frac{w_1}{d + d_{eye}}$$

$$left = -z_{near} \frac{w_2}{d + d_{eye}}$$

# Image Formation – Right Eye

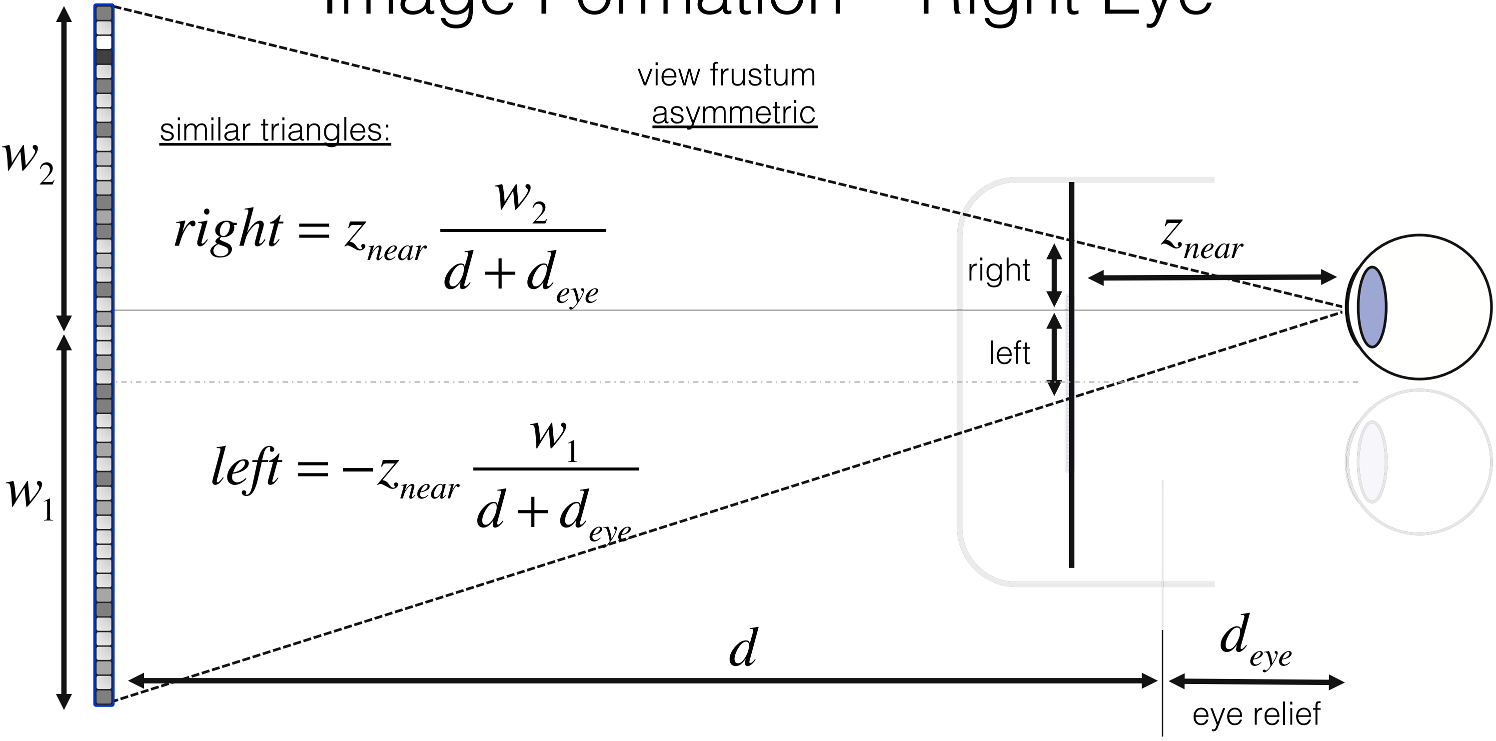



$$w_2 = M\left(\frac{w' - ipd}{2}\right)$$

$$w_1 = M\,\frac{ipd}{2}$$

Top View

# Image Formation – Right Eye

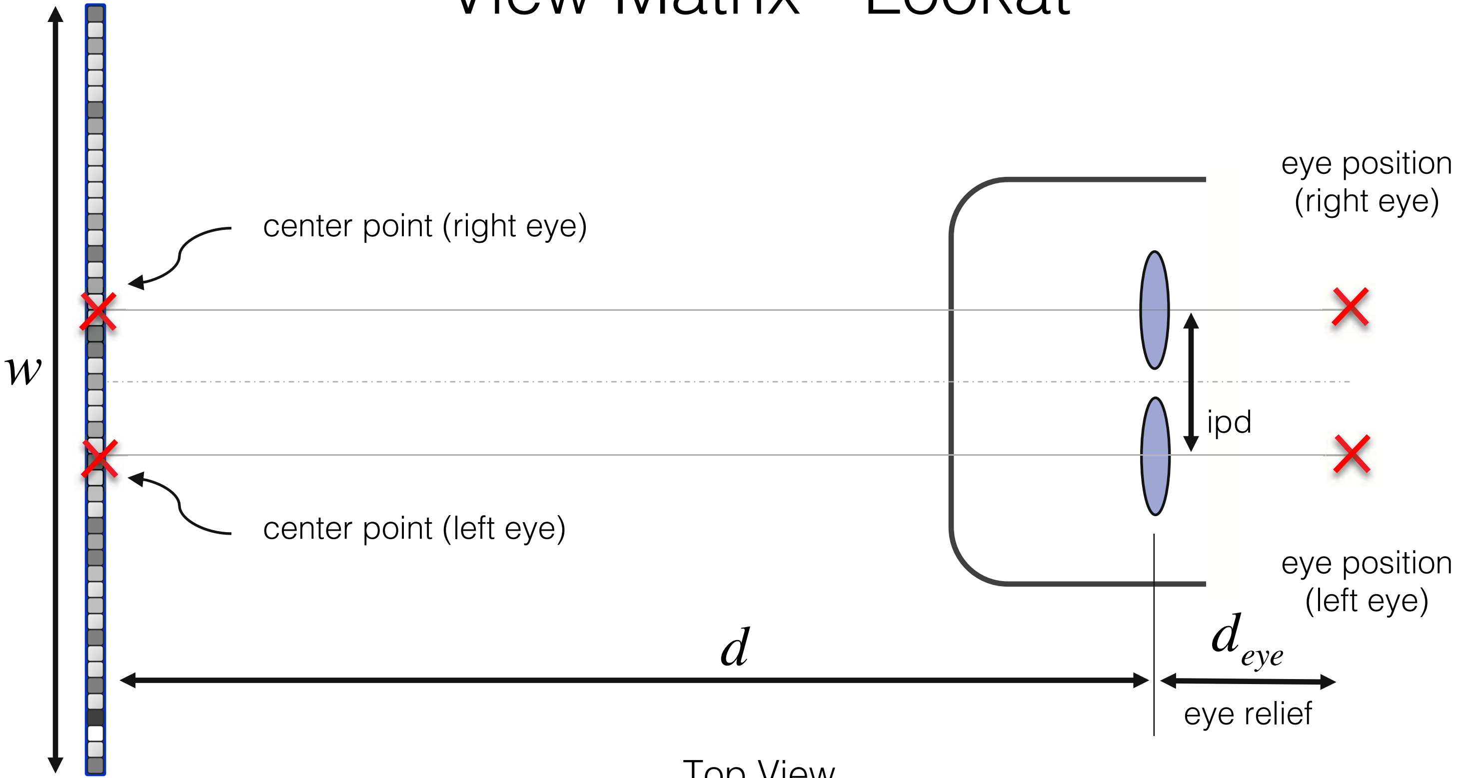



$$right = z_{near} \frac{w_2}{d + d_{eye}}$$

$$left = -z_{near} \frac{w_1}{d + d_{eye}}$$

Top View

# View Matrix - Lookat




Top View

# Prototype Specs – View-Master Deluxe VR Viewer

- roughly follows Google Cardboard 2.0:
  - lenses focal length: 40 mm
  - lenses diameter: 34 mm
  - inter-lens distance: 64 mm
  - screen to lens distance: 39 mm
  - eye relief: 18 mm
- Topfoison 6” LCD: width 132.5 mm, height 74.5 mm; 1920x1080 px OR
- Topfoison 5.5” LCD: width 120.96 mm, height 68.03 mm; 1920x1080 px

# Image Formation

- use these formulas to compute the perspective matrix in WebGL

- you can use:

```
THREE.Matrix4().makePerspective(left,right,top,bottom,near,far)

THREE.Matrix4().lookAt(eye,center,up) – attention: this only does
                                        rotation, not the translation,
                                        which is required in addition
                                        to the rotation!
```

- that's all you need to render stereo images on the HMD

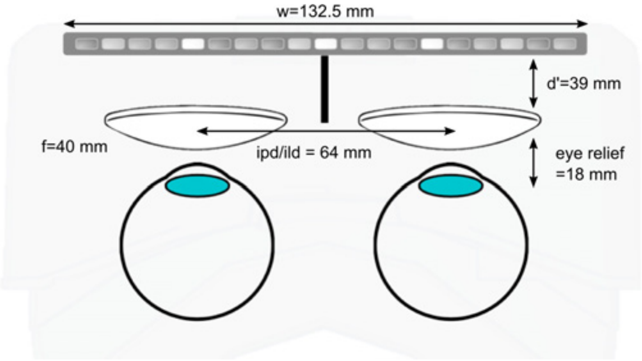
# Image Formation for More Complex Optics

- especially important in free-form optics, off-axis optical configurations & AR
- use ray tracing – some nonlinear mapping from view frustum to microdisplay pixels
- much more computationally challenging & sensitive to precise calibration; our HMD and most magnifier-based designs will work with what we discussed so far

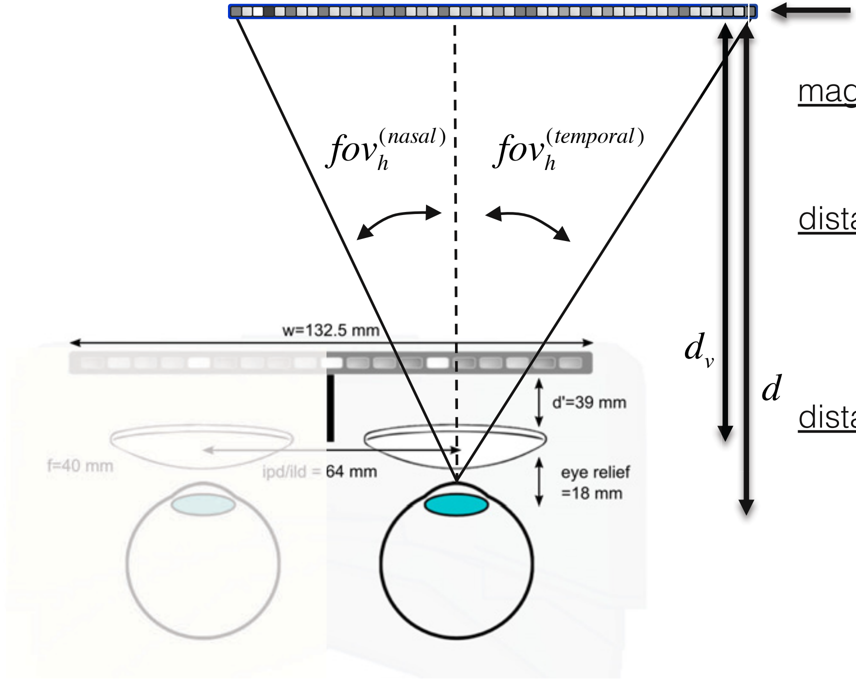
# Field of View and Visual Field

# Example Calculations for Field of View

- use Google Cardboard 2 lenses (f=40mm, d'=39mm, interpupillary/interlens distance = 64mm, eye relief = 18mm)
- Topfoison 6" LCD panel (132.5 x 74.5 mm)



# Example Calculations for Field of View




magnification: $M = \frac{f}{f - d'} = 40$

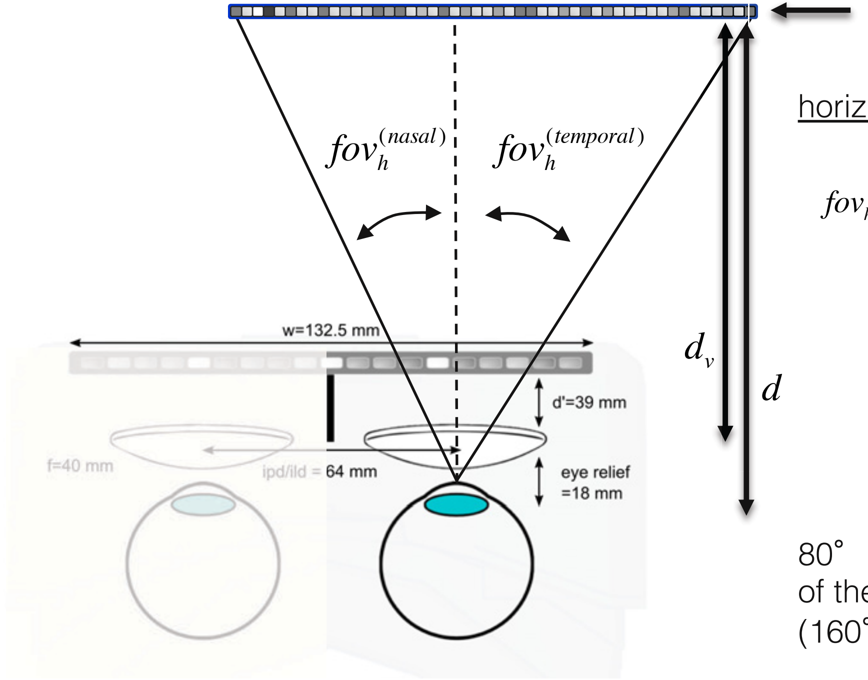
distance lens-virtual image:

$$d_v = \left| \frac{1}{\frac{1}{f} - \frac{1}{d'}} \right| = 1{,}560\,mm$$

distance eye-virtual image:

$$d = d_v + d_{eye} = 1{,}578\,mm$$

# Example Calculations for Field of View




horizontal field of view:

$$fov_h = fov_h^{(nasal)} + fov_h^{(temporal)}$$

$$= \tan^{-1}\left(\frac{M \, ipd/2}{d}\right) + \tan^{-1}\left(\frac{M\,(w - ipd)/2}{d}\right)$$

$$= 39^\circ + 41^\circ = 80^\circ$$

$80^\circ$ horizontal field of view is approx. 50% of the horizontal visual field of a single eye ($160^\circ$ total)

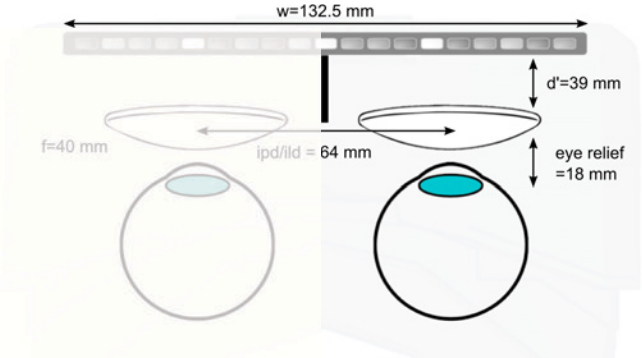
# Example Calculations for Field of View

virtual image of right display side

<u>vertical field of view</u>:

$$fov_v = fov_v^{(\text{superior})} + fov_h^{(\text{inferior})}$$

$$= 2\tan^{-1}\left(\frac{M\, h/2}{d}\right) = \boxed{87^\circ}$$



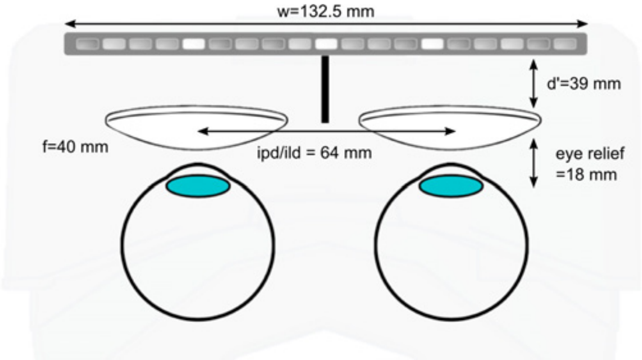

87° vertical field of view is approx. 64% of the vertical visual field of a single eye (135° total)

# Example Calculations for Field of View

total monocular field of view of both eyes:

$$fov_h^{(total)} = 2\,fov_h^{(\mathrm{temporal})} = 82^\circ$$

82° monocular field of view is approx. 41% of the full monocular visual field of both eyes (200° total)




binocular field of view of both eyes:

$$fov_h^{(total)} = 2\,fov_h^{(\mathrm{nasal})} = 78^\circ$$

78° binocular field of view is approx. 65% of the binocular visual field of both eyes (120° total)

# Lens Distortion Correction

*All lenses introduce image distortion, chromatic aberrations, and other artifacts – we need to correct for them as best as we can in software!*

# Lens Distortion

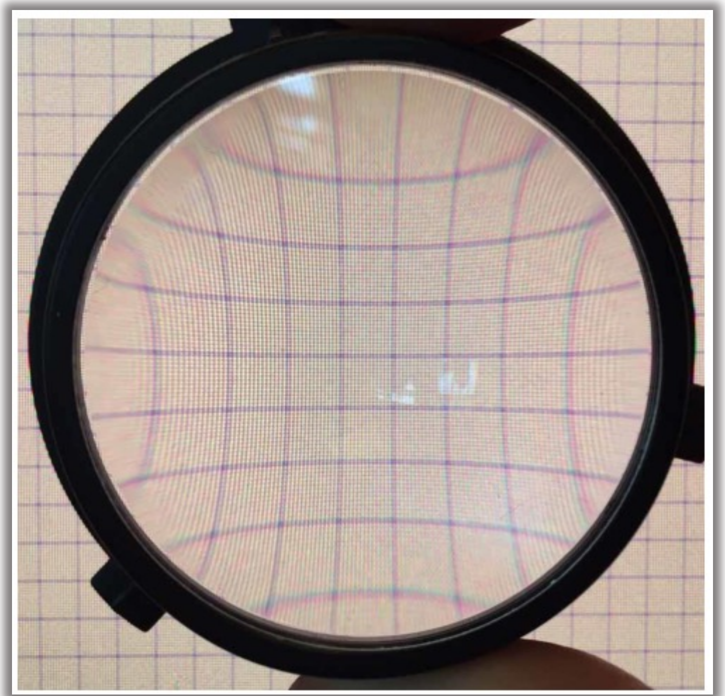
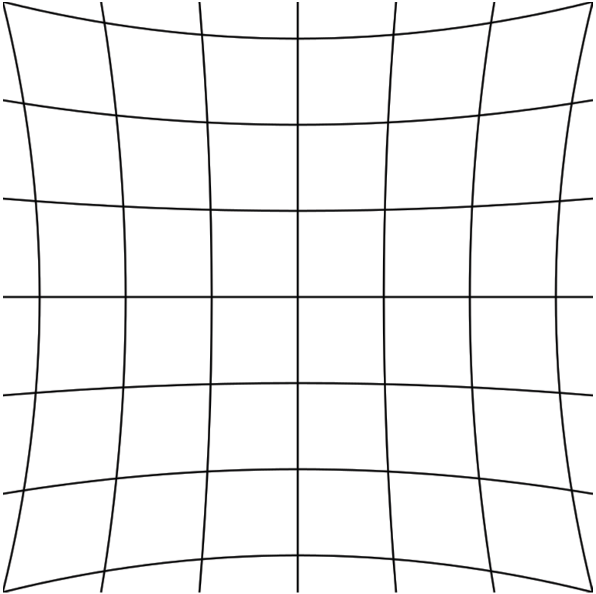


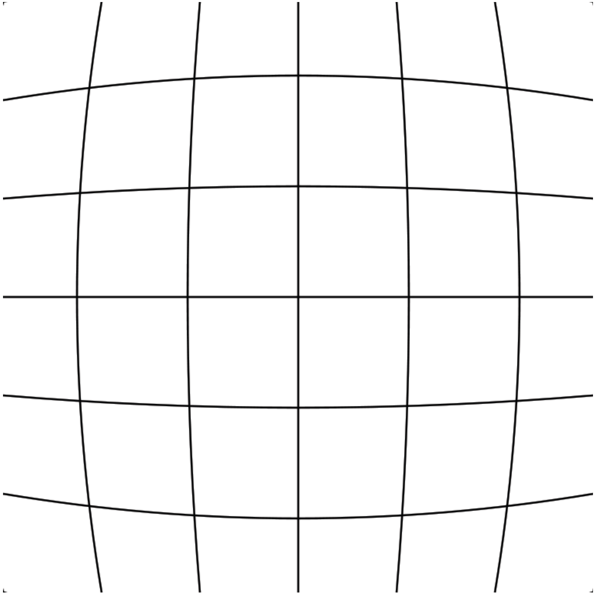
image from: https://www.slideshare.net/Mark_Kilgard/nvidia-opengl-in-2016

- grid seen through HMD lens
- lateral (xy) distortion of the image
- chromatic aberrations: distortion is wavelength dependent!

# Lens Distortion

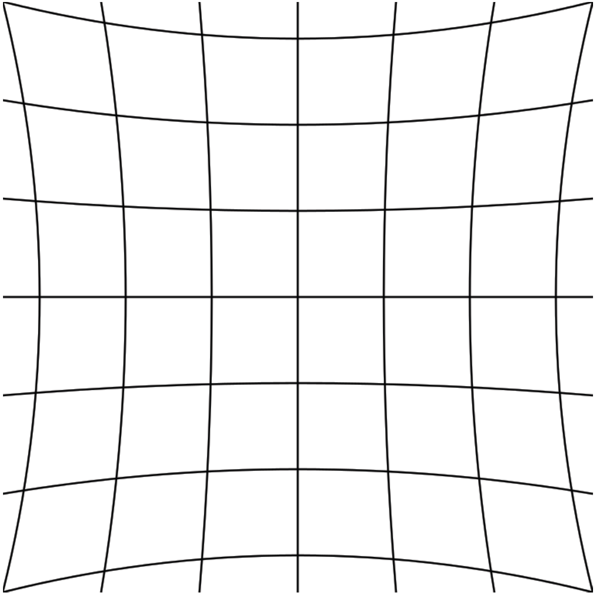


Pincussion Distortion



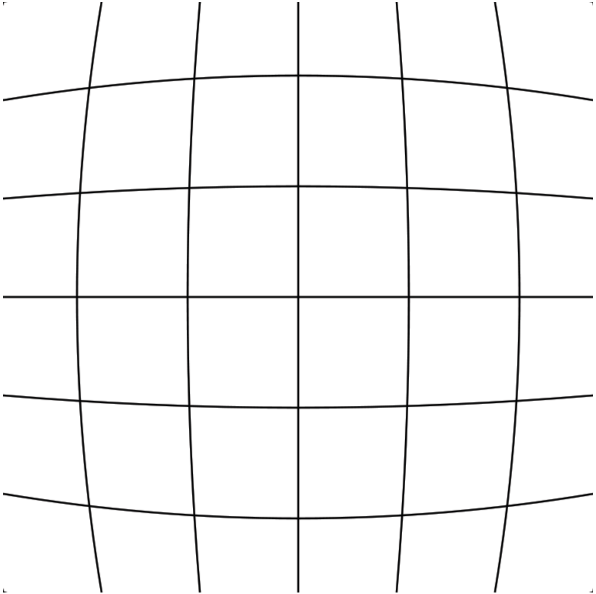
Barrel Distortion

# Lens Distortion



Pincussion Distortion

optical



Barrel Distortion

digital correction

# Lens Distortion

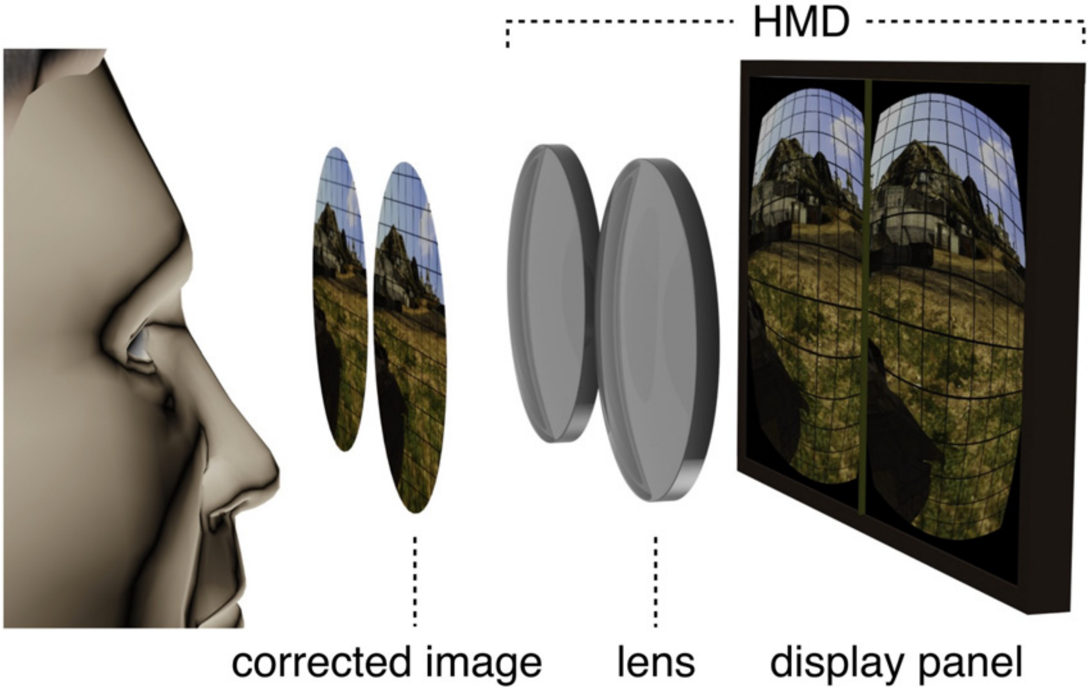
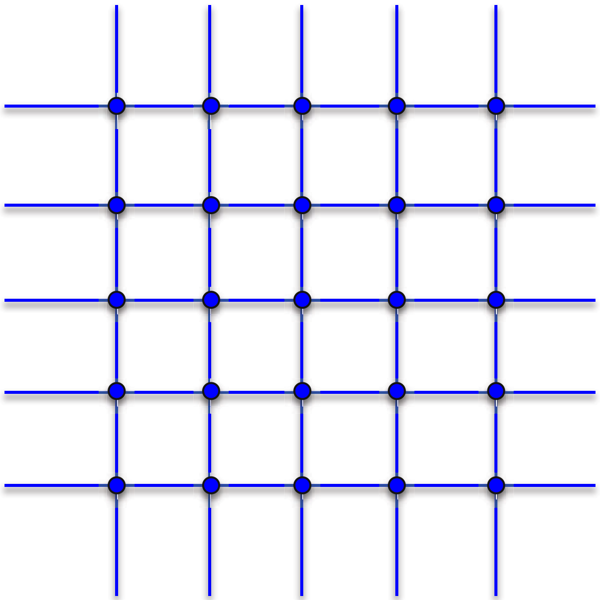



image from: https://www.slideshare.net/Mark_Kilgard/nvidia-opengl-in-2016

# Lens Distortion

- $x_u, y_u$ undistorted point



# Lens Distortion

- $x_u, y_u$ undistorted point
- $x_d \approx x_u\left(1 + K_1 r^2 + K_2 r^4\right)$

  $y_d \approx y_u\left(1 + K_1 r^2 + K_2 r^4\right)$

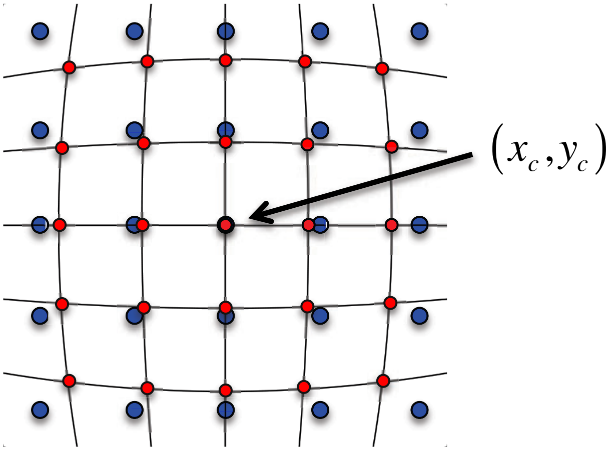
$x_d, y_d$ distorted point coordinates

$K_1, K_2$ distortion coefficients

$r$ normalized distance from center

$x_c, y_c$ center of optical axis

→ this is the origin, i.e. all other points are defined relative to this




Barrel Distortion

digital correction

# Lens Distortion

- $x_u, y_u$ undistorted point
- $x_d \approx x_u\left(1 + K_1 r^2 + K_2 r^4\right)$

  $y_d \approx y_u\left(1 + K_1 r^2 + K_2 r^4\right)$

$x_d, y_d$ distorted point coordinates

$K_1, K_2$ distortion coefficients

$r$ normalized distance from center

$x_c, y_c$ center of optical axis

→ this is the origin, i.e. all other points are defined relative to this

NOTES:

- center is assumed to be the center point (on optical axis) on screen
- distortion is radially symmetric around center point
- easy to get confused!
- can implement in fragment shader (not super efficient, but easier for us)

# Normalizing r

- $x_u, y_u$ undistorted point
- $x_d \approx x_u\left(1 + K_1 r^2 + K_2 r^4\right)$

  $y_d \approx y_u\left(1 + K_1 r^2 + K_2 r^4\right)$

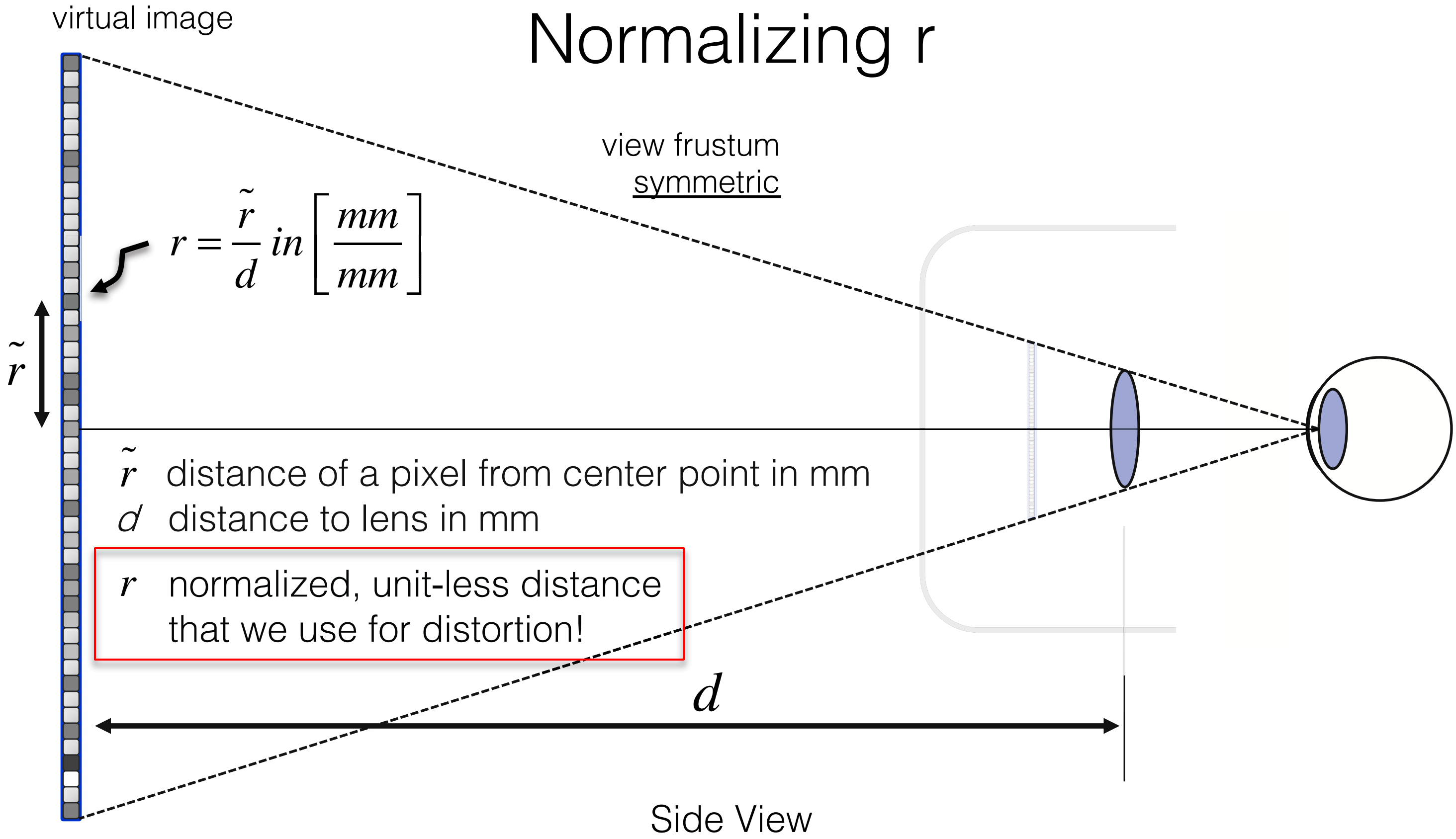
un-normalized radial distance from center:

$$\tilde{r}^2 = \left(x_u - x_c\right)^2 + \left(y_u - y_c\right)^2 \longrightarrow$$

Calculate $\tilde{r}$ in metric units, e.g. mm. Need physical size of the pixels of your screen for this!

$x_c, y_c$ center

# Normalizing r



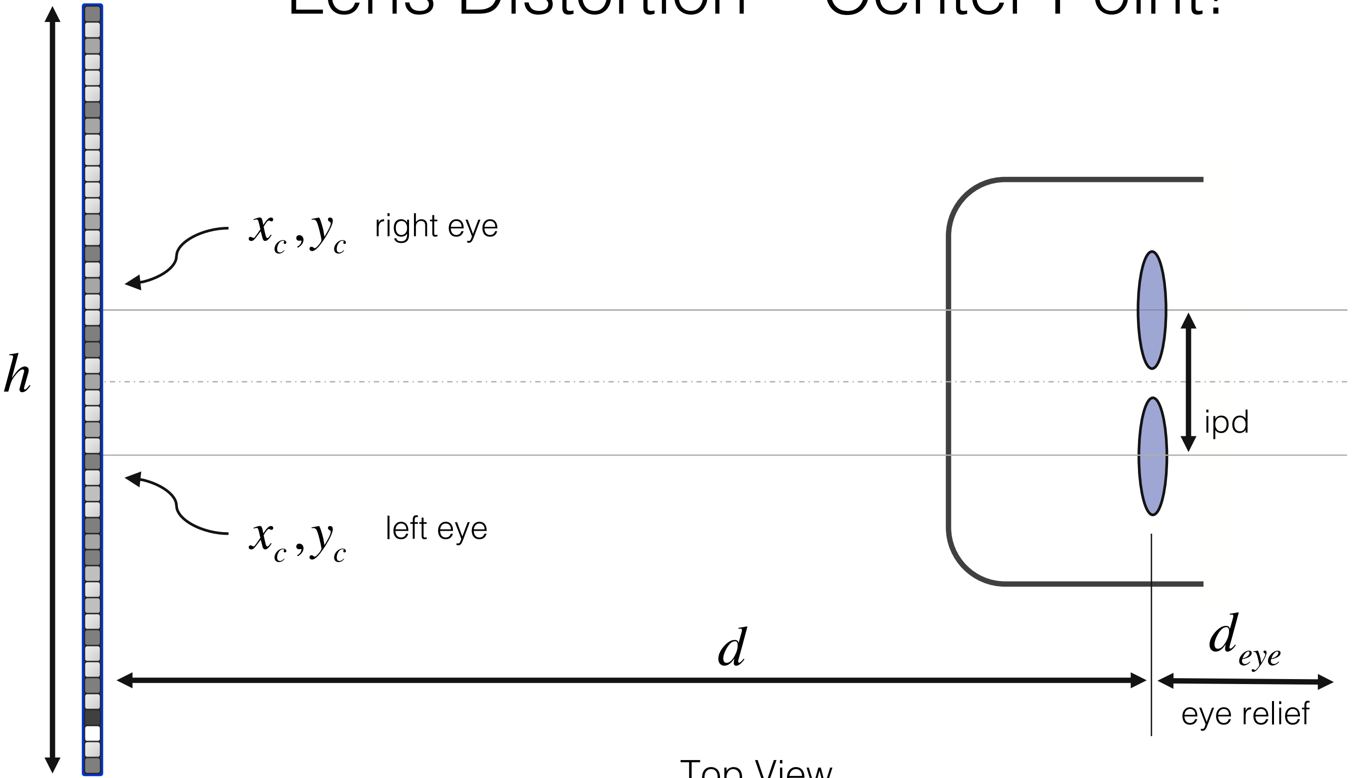

$$r = \frac{\tilde{r}}{d} \text{ in } \left[\frac{mm}{mm}\right]$$

$\tilde{r}$ distance of a pixel from center point in mm

$d$ distance to lens in mm

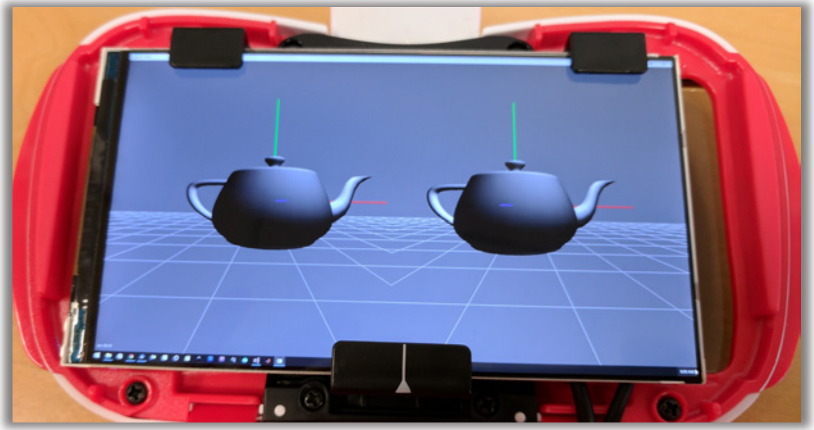
$r$ normalized, unit-less distance that we use for distortion!

# Lens Distortion – Center Point!

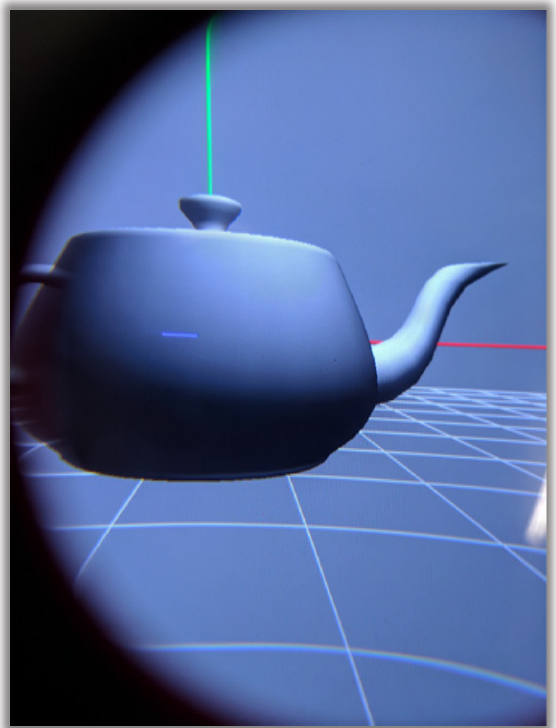


Top View

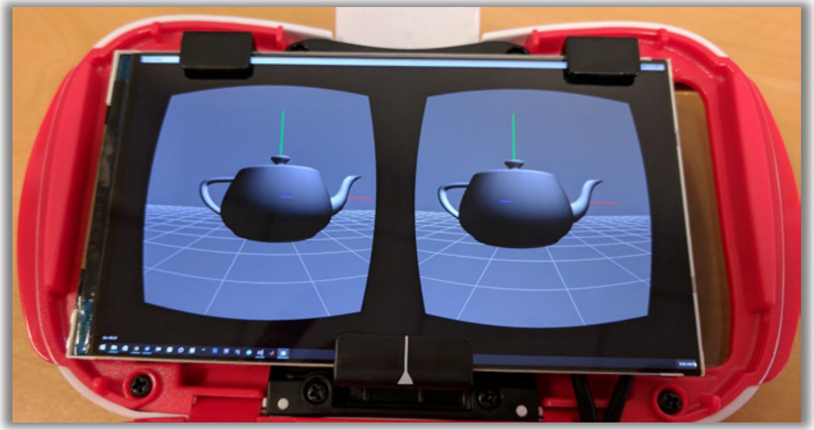
# Lens Distortion Correction Example



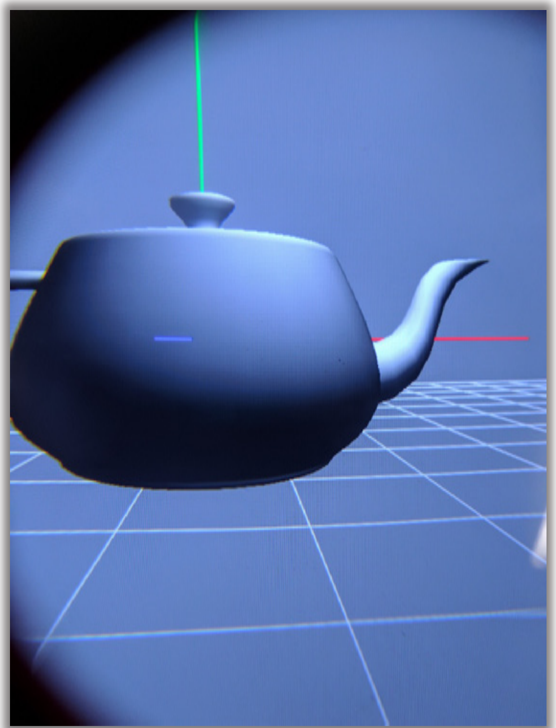
stereo rendering <u>without</u> lens distortion correction



# Lens Distortion Correction Example

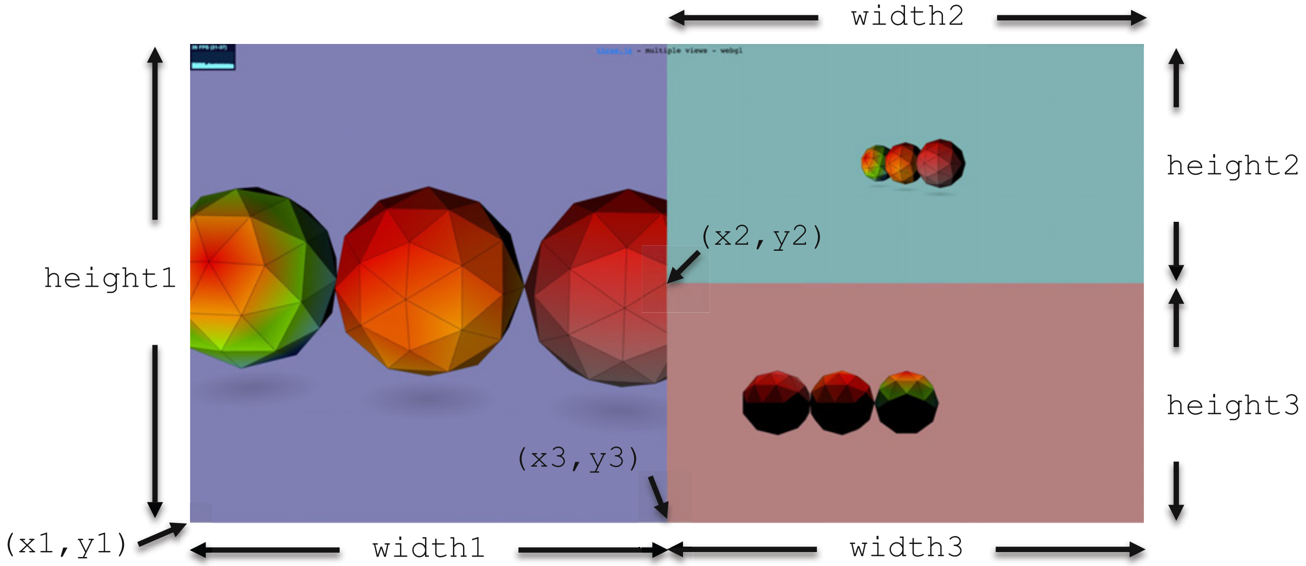


stereo rendering with lens distortion correction



# How to Render into Different Parts of the Window?

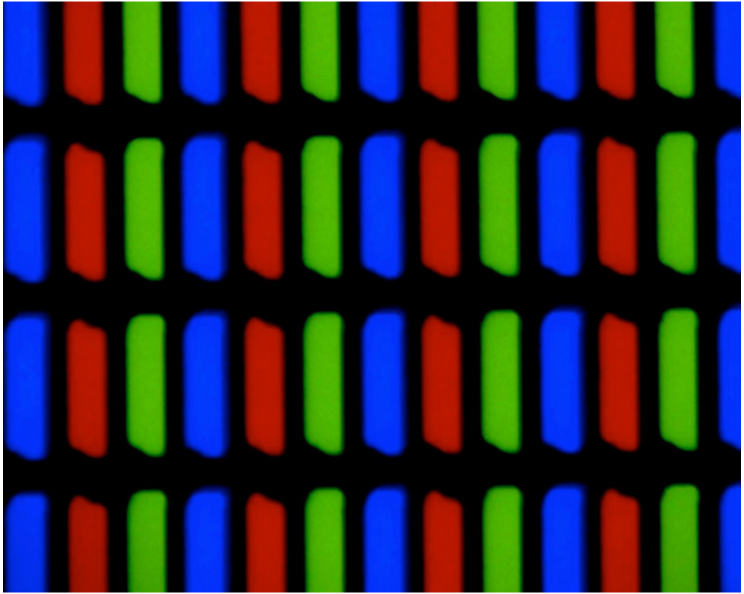
- `WebGLRenderer.setViewport(x,y,width,height)`
- `x,y` lower left corner; width, height viewport size



# Overview of Microdisplays

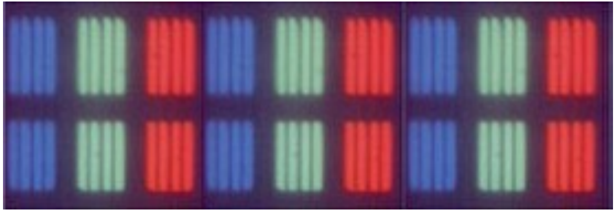
# Liquid Crystal Display (LCD) - Subpixels

wikipedia

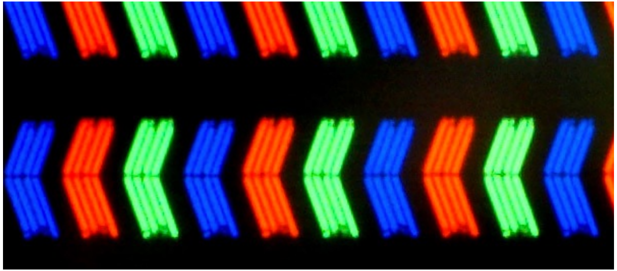


TN subpixels

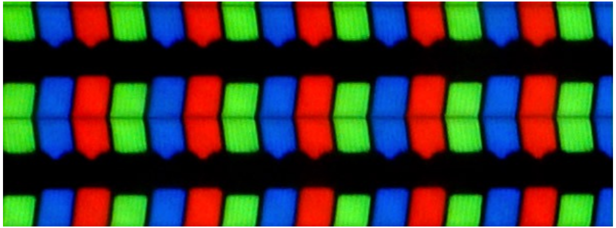
IPS



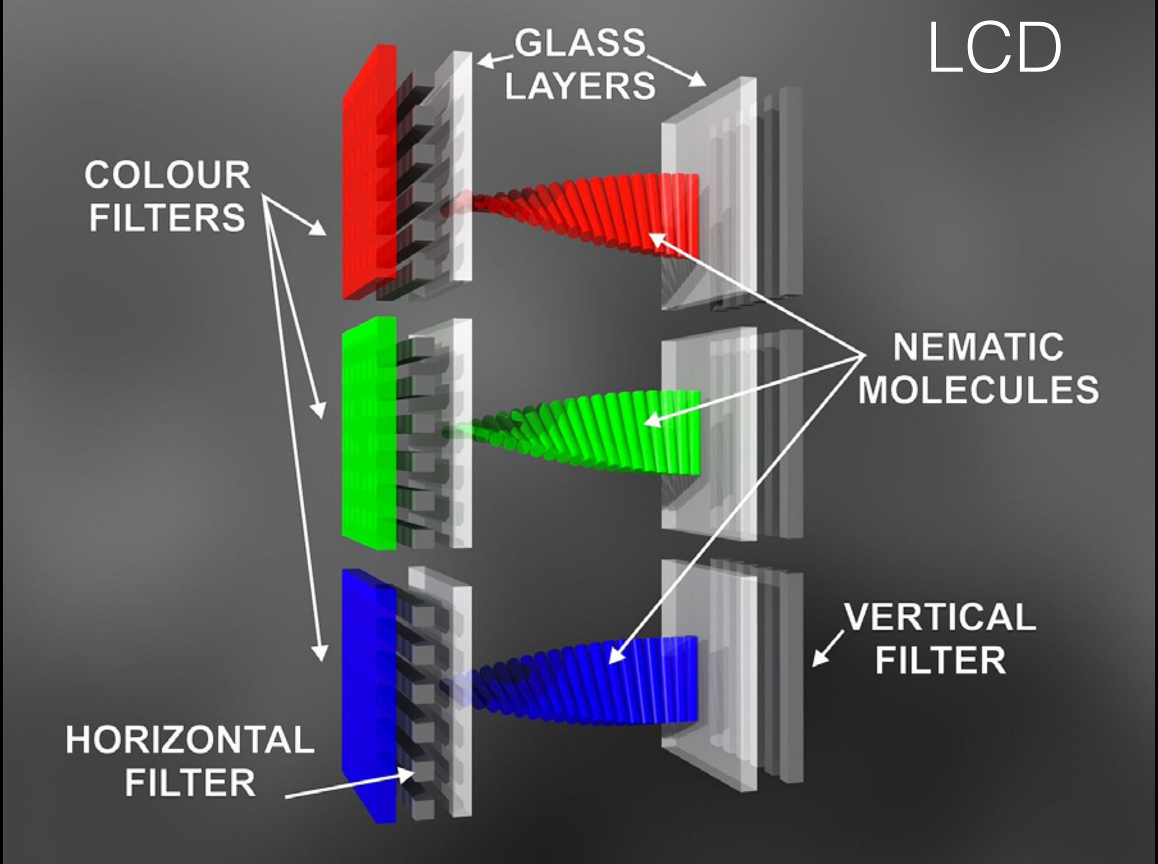
S-IPS



IPS



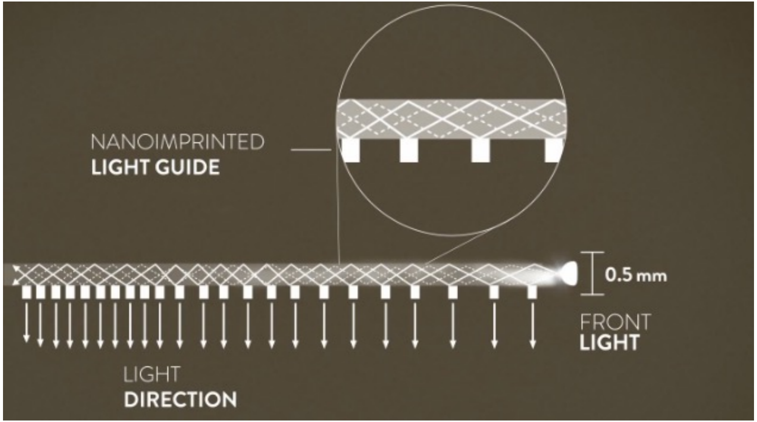
# LCD

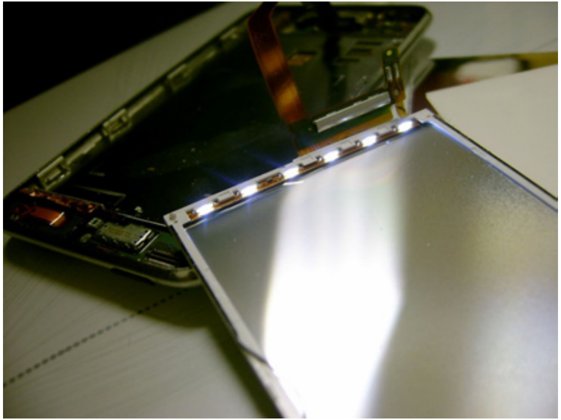



wikipedia

# LCD Backlight

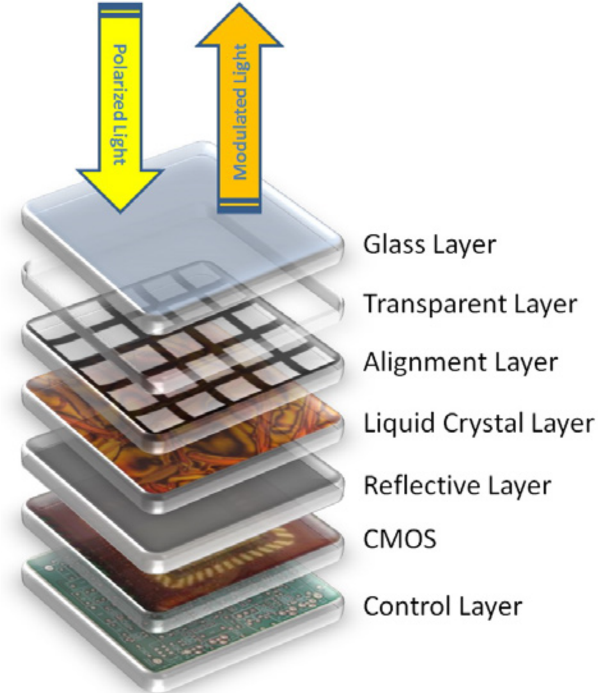

extremetech.com



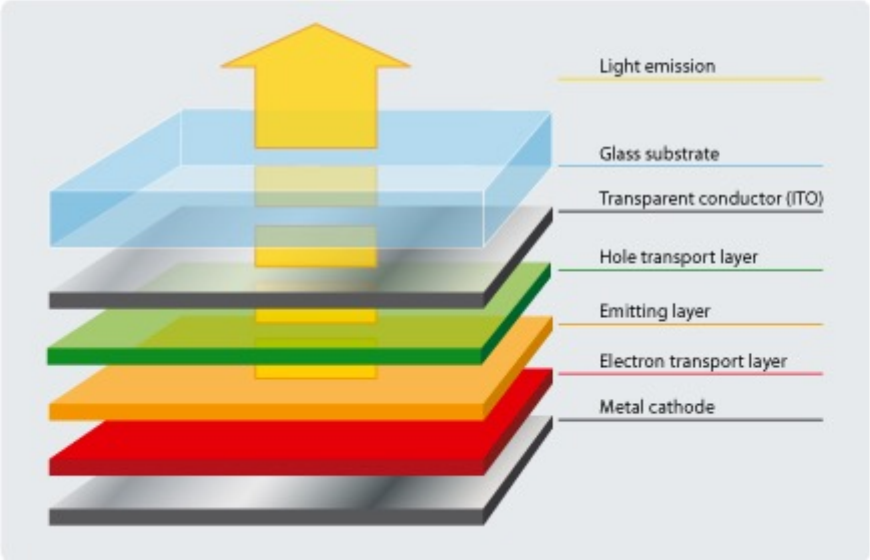
wikipedia

# Liquid Crystal on Silicon (LCoS)




- basically a reflective LCD
- standard component in projectors and head mounted displays
- used e.g. in google glass

# Organic Light Emitting Diodes (OLED)

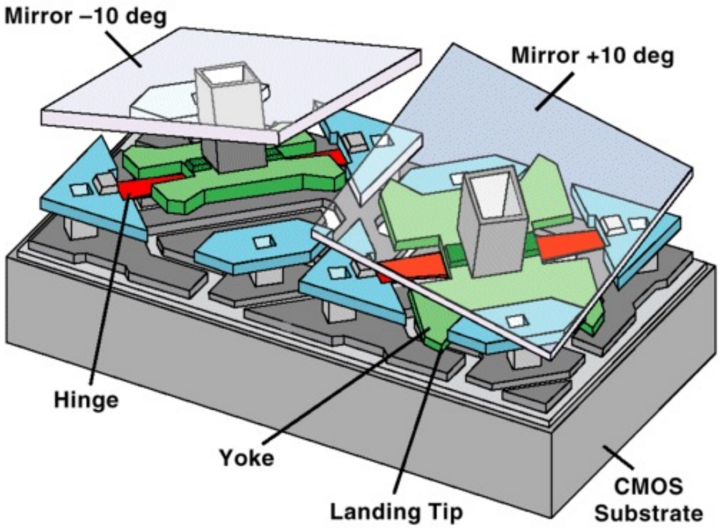



lightrabbit.co.uk

- Self emissive
- Lower persistence (can turn on and off faster than LCD/LCoS, which is great for VR)
- used e.g. VR-compatible phones, like Google's Pixel

# Digital Micromirror Device (DMD)

- developed by TI
- MEMS device
- binary states (e.g. +/- 10 degrees)
- gray-level through pulse width modulation (PWM)
- Super-fast (10-20 kHZ binary display
- More light efficient than LCD/LCoS!




Texas Instruments

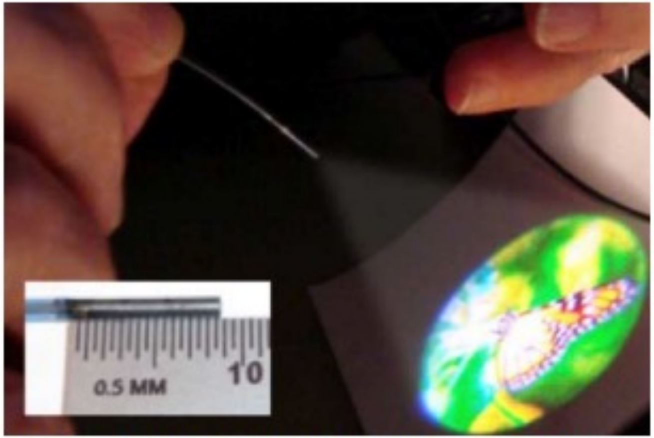
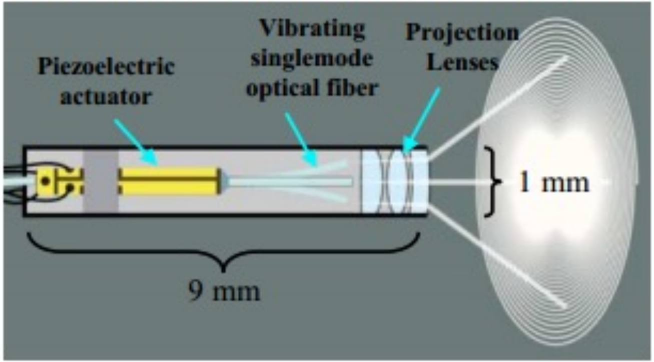


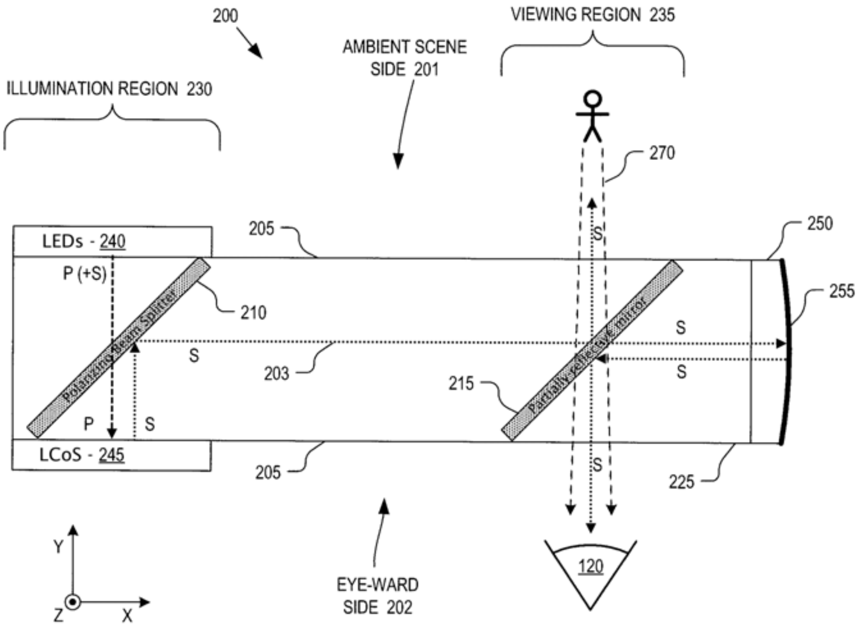

**Figure 1. 1 mm x 9 mm scanning fiber projector.**




B. T. Schowengerdt, R. Johnston, C.D. Melville, E.J. Seibel. 3D Displays Using Scanning Laser Projection. SID 2012.

# Next Lecture: HMD Displays Optics II

- advanced VR & AR optics




drawing from Google Glass patent