Lecture Overview

- coordinate systems (world, body/sensor, inertial, transforms)
- overview of inertial sensors: gyroscopes, accelerometers, and magnetometers
- gyro integration aka *dead reckoning*
- orientation tracking in *flatland*
- pitch & roll from accelerometer
- overview of VRduino
• primary goal: track orientation of head or other device

• orientation is the rotation of device w.r.t. world/earth or inertial frame

• rotations are represented by Euler angles (yaw, pitch, roll) or quaternions
• orientation tracked with IMU models relative rotation of sensor/body frame in world/inertial coordinates

• example: person on the left looks up → pitch=90° or rotation around x-axis by 90°

• similarly, the world rotates around the sensor frame by -90° (inverse rotation)
from lecture 2:

vertex in clip space

\[ v_{\text{clip}} = M_{\text{proj}} \cdot M_{\text{view}} \cdot M_{\text{model}} \cdot v \]
from lecture 2:

$\mathbf{v}_{\text{clip}} = M_{\text{proj}} \cdot M_{\text{view}} \cdot M_{\text{model}} \cdot \mathbf{v}$

vertex in clip space

projection matrix  view matrix  model matrix

vertex
from lecture 2:

vertex in clip space

\[ v_{\text{clip}} = M_{\text{proj}} \cdot M_{\text{view}} \cdot M_{\text{model}} \cdot v \]

projection matrix  view matrix  model matrix

rotation  translation

\[ M_{\text{view}} = R \cdot T(-\text{eye}) \]
from lecture 2:

vertex in clip space

\[ v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v \]

projection matrix | view matrix | model matrix

rotation | translation

view matrix for stereo camera:

\[ M_{view}^{stereo} = T \left( \pm \frac{ipd}{2},0,0 \right) \cdot R \cdot T\left(-\text{eye}\right) \]
from lecture 2:

vertex in clip space

\[ v_{\text{clip}} = M_{\text{proj}} \cdot M_{\text{view}} \cdot M_{\text{model}} \cdot v \]

projection matrix  
view matrix  
model matrix  

rotation  
translation  

\[ M_{\text{view}} = R \cdot T (\text{\textit{-eye}}) \]

sensor/body frame  
world/inertial frame
\[
M_{\text{view}} = R \cdot T (-\text{eye})
\]

\[
R = R_z (-\theta_z) \cdot R_x (-\theta_x) \cdot R_y (-\theta_y)
\]

order of rotations (world to body)
• this representation for a rotation is known as Euler angles

• need to specify order of rotation, e.g. yaw-pitch-roll

\[ R = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y) \]

order of rotations (world to body)
ATTENTION!

• Euler angles are usually a terrible idea for orientation tracking with more than 1 axis

• one of several reasons: rotations are not commutative

\[
R = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y)
\]

order of rotations (world to body)
What do Inertial Sensors Measure?

- gyroscope measures angular velocity $\tilde{\omega}$ in degrees/sec
- accelerometer measures linear acceleration $\tilde{a}$ in m/s$^2$
- magnetometer measures magnetic field strength $\tilde{m}$ in uT (micro Tesla) or Gauss $\rightarrow$ 1 Gauss = 100 uT
What do Inertial Sensors Measure?

- Gyroscope measures angular velocity \( \omega \) in degrees/sec

- Accelerometer measures linear acceleration \( \dot{a} \) in m/s\(^2\)

- Magnetometer measures magnetic field strength \( \tilde{m} \) in \( \mu \text{T} \) (micro Tesla) or Gauss \( \Rightarrow 1 \text{ Gauss} = 100 \mu \text{T} \)

**ALL MEASUREMENTS TAKEN IN SENSOR/ BODY COORDINATES!**
History of Gyroscopes

- critical for inertial measurements in ballistic missiles, aircrafts, drones, the mars rover, pretty much anything that moves!

WWII era gyroscope used in the V2 rocket
MEMS Gyroscopes

- today, we use microelectromechanical systems (MEMS)

Coriolis Force
MEMS Gyroscope
Gyroscopes

- gyro model: \( \tilde{\omega} = \omega + b + \eta \)
Gyroscopes

- gyro model: \( \tilde{\omega} = \omega + b + \eta \)   \( \eta \sim N\left(0, \sigma_{\text{gyro}}^2\right) \)

  true angular velocity  
  additive, zero-mean Gaussian noise  
  bias
Gyroscopes

• gyro model: \( \hat{\omega} = \omega + b + \eta \)

  \[ \eta \sim N(0, \sigma_{gyro}^2) \]

  true angular velocity \quad additive, zero-mean Gaussian noise

  bias

• 3 DOF = 3-axis gyros that measures 3 orthogonal axes, assume no crosstalk

• bias is temperature-dependent and may change over time; can approximate as a constant

• additive measurement noise
Gyroscopes

- from gyro measurements to orientation – use Taylor expansion

\[ \theta(t + \Delta t) \approx \theta(t) + \frac{\partial}{\partial t} \theta(t) \Delta t + \varepsilon, \quad \varepsilon \sim O(\Delta t^2) \]
Gyroscopes

- from gyro measurements to orientation – use Taylor expansion

\[
\theta(t + \Delta t) \approx \theta(t) + \frac{\partial}{\partial t} \theta(t) \Delta t + \varepsilon, \quad \varepsilon \sim O(\Delta t^2)
\]

want: angle at current time step

have: angle at last time step

have: gyro measurement (angular velocity)

approximation error!
Gyro Integration: linear motion, no noise, no bias
Gyro Integration: linear motion, noise, no bias
Gyro Integration: linear motion, no noise, bias
Gyro Integration: **nonlinear** motion, no noise, no bias
Gyro Integration: **nonlinear motion, noise, bias**
Gyro Integration aka *Dead Reckoning*

- works well for linear motion, no noise, no bias = unrealistic
- even if bias is know and noise is zero → *drift* (from integration)
- bias & noise variance can be estimated, other sensor measurements used to correct for drift (sensor fusion)
- accurate in short term, but not reliable in long term due to drift
Dead Reckoning for Ship Navigation

- can measure north with compass, ship’s speed, and time
- initial position known
Dead Reckoning for Ship Navigation

- can measure north with compass, ship’s speed, and time
- initial position known
- problem: drift! (similar to that observed in gyro integration)
Always be aware of what units you are working with, degrees per second v radians per second!
Accelerometers

- measure linear acceleration \( \tilde{a} = a^{(g)} + a^{(l)} + \eta, \quad \eta \sim N\left(0, \sigma_{\text{acc}}^2\right) \)

- without motion: read noisy gravity vector \( a^{(g)} + \eta \) pointing UP!
  with magnitude 9.81 m/s\(^2\) = 1g

- with motion: combined gravity vector and external forces \( a^{(l)} \)
capacitive plates

spring

proof mass
Accelerometers

- advantages:
  - points up on average with magnitude of $1g$
  - accurate in long term because no drift and the earth’s center of gravity (usually) doesn’t move

- problem:
  - noisy measurements
  - unreliable in short run due to motion (and noise)

- complementary to gyro measurements!
Accelerometers

- fusing gyro and accelerometer data = 6 DOF sensor fusion

- can correct tilt (i.e., pitch & roll) only – no information about yaw
Orientation Tracking in *Flatland*

- problem: track angle $\theta$ in 2D space
- sensors: 1 gyro, 2-axis accelerometer
- goal: understand sensor fusion
Orientation Tracking in *Flatland*

- gyro integration via Taylor series as

\[ \theta_{gyro}^{(t)} = \theta_{gyro}^{(t-1)} + \tilde{\omega} \Delta t \]

- get $\Delta t$ from microcontroller
- set $\theta_{gyro}^{(0)} = 0$

- biggest problem: drift!
Orientation Tracking in *Flatland*

- angle from accelerometer

\[
\theta_{acc} = \tan^{-1}\left( \frac{\tilde{\alpha}_x}{\tilde{\alpha}_y} \right)
\]
Orientation Tracking in *Flatland*

- angle from accelerometer

\[ \theta_{acc} = \tan^{-1} \left( \frac{\tilde{a}_x}{\tilde{a}_y} \right) = \text{atan2}(\tilde{a}_x, \tilde{a}_y) \]

handles division by 0 and proper signs, provided by most programming languages
Orientation Tracking in *Flatland*

- angle from accelerometer

\[ \theta_{acc} = \tan^{-1}\left( \frac{\tilde{a}_x}{\tilde{a}_y} \right) = \text{atan2}(\tilde{a}_x, \tilde{a}_y) \]

- biggest problem: noise
Orientation Tracking in *Flatland*

- sensor fusion: combine gyro and accelerometer measurements

- intuition:
  - remove drift from gyro via high-pass filter
  - remove noise from accelerometer via low-pass filter
Orientation Tracking in *Flatland*

- sensor fusion with complementary filter, i.e. linear interpolation

\[
\theta^{(t)} = \alpha \left( \theta^{(t-1)} + \tilde{\omega} \Delta t \right) + (1 - \alpha) \arctan(\tilde{a}_x, \tilde{a}_y)
\]

- no drift, no noise!
Orientation Tracking in *Flatland*

- gyro (dead reckoning)
- accelerometer
- complementary filter
Pitch and Roll from 3-axis Accelerometer

- problem: estimate pitch and roll angles in 3D, from 3-axis accelerometer

- together, pitch & roll angles are known as \textit{tilt}

- goal: understand tilt estimation in 3D
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

normalize gravity vector in inertial coordinates

\[
\hat{a} = \frac{\tilde{a}}{||\tilde{a}||} = R \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}
\]

normalize gravity vector rotated into sensor coordinates
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{|\tilde{a}|} = R \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y) \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}
\]

\[
= \begin{pmatrix} \cos(-\theta_z) & -\sin(-\theta_z) & 0 \\ \sin(-\theta_z) & \cos(-\theta_z) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta_x) & -\sin(-\theta_x) \\ 0 & \sin(-\theta_x) & \cos(-\theta_x) \end{pmatrix} \begin{pmatrix} \cos(-\theta_y) & 0 & \sin(-\theta_y) \\ 0 & 1 & 0 \\ -\sin(-\theta_y) & 0 & \cos(-\theta_y) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{|\tilde{a}|} = \begin{pmatrix}
-\cos(-\theta_x)\sin(-\theta_z) \\
\cos(-\theta_x)\cos(-\theta_z) \\
\sin(-\theta_x)
\end{pmatrix}
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{|\tilde{a}|} = \begin{pmatrix}
-\cos(-\theta_x)\sin(-\theta_z) \\
\cos(-\theta_x)\cos(-\theta_z) \\
\sin(-\theta_x)
\end{pmatrix}
\]

\[
\hat{a}_x = \frac{-\sin(-\theta_z)}{\cos(-\theta_z)} = -\tan(-\theta_z)
\]

\[
\theta_z = -\text{atan2}(-\hat{a}_x, \hat{a}_y) \text{ in rad } \in [-\pi, \pi]
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{||\tilde{a}||} = \begin{pmatrix}
-\cos(-\theta_x)\sin(-\theta_z) \\
\cos(-\theta_x)\cos(-\theta_z) \\
\sin(-\theta_x)
\end{pmatrix}
\]

\[
\text{pitch} = \frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)(\sin^2(-\theta_z) + \cos^2(-\theta_z))}} = 1
\]

\[
= \frac{\sin(-\theta_x)}{\cos(-\theta_x)} = \tan(-\theta_x)
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\hat{a} = \frac{\tilde{a}}{|\tilde{a}|} = \begin{pmatrix}
-\cos(-\theta_x)\sin(-\theta_z) \\
\cos(-\theta_x)\cos(-\theta_z) \\
\sin(-\theta_x)
\end{pmatrix} = \begin{pmatrix}
-\sin(\theta_x) \\
\cos(\theta_x)\cos(\theta_z) \\
\sin(\theta_x)
\end{pmatrix}
\]

\[
pitch \quad \hat{a}_z = \frac{\sin(-\theta_x)}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)(\sin^2(-\theta_z) + \cos^2(-\theta_z))}} = 1
\]

\[
\theta_x = -\text{atan2}\left(\hat{a}_z, \sqrt{\hat{a}_x^2 + \hat{a}_y^2}\right) \text{ in rad} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]
\]
Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

\[
\begin{align*}
pitch = & \frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)(\sin^2(-\theta_z) + \cos^2(-\theta_z))}} \\
= & 1
\end{align*}
\]

\[
\theta_x = -\text{atan2}\left(\hat{a}_z, \text{sign}(\hat{a}_y) \cdot \sqrt{\hat{a}_x^2 + \hat{a}_y^2}\right) \text{ in rad } \in [-\pi, \pi]
\]
Magnetometers
MEMS Magnetometer

Hall Effect

Magneto-resistive effect
Magnetometers

- measure earth’s magnetic field in Gauss or uT

- 3 orthogonal axes = vector pointing along the magnetic field

- actual direction depends on latitude and longitude!

- distortions due to metal / electronics objects in the room or in HMD
Magnetometers

difficult to work with magnetometers without proper calibration → we will not use the magnetometer in the HW!
Magnetometers

• advantages:
  • complementary to accelerometer – gives yaw (heading)

• problems:
  • affected by metal, distortions of magnetic field
  • need to know location, even when calibrated (e.g. GPS)

• together with gyro + accelerometer = 9 DOF sensor fusion
Prototype IMU

- 9 DOF IMU: InvenSense MPU-9250 = updated model of what was in the Oculus DK2

- 3-axis gyro, 3-axis accelerometer, 3-axis magnetometer all on 1 chip (we’ll only use gyro and acc, but we’ll give you code to read mag if you want to use it in your project)

- interface with I2C (serial bus) from Arduino
Prototype IMU

I2C

e.g. Arduino

InvenSense MPU-9250

to host:
serial via USB
MPU-9250 Specs

- multi-chip module: 1 die houses gyro & accelerometer, the other the magnetometer

- magnetometer: Asahi Kasei Microdevices AK8963 ("3\textsuperscript{rd} party device")

- 9x 16 bit ADCs for digitizing 9DOF data
MPU-9250 Specs

- gyro modes: ±250, ±500, ±1000, ±2000 °/sec
- accelerometer: ±2, ±4, ±8, ±16 g
- magnetometer: ±4800 uT

- configure using registers (see specs) via I2C
- also supports on-board Digital Motion Processing™ (DMP™)
- sorry, we don’t have access
- we’ll provide starter code for Arduino in lab (easy to use for beginners, not consumer product grade!)
MPU-9250 Specs

- gyro modes: ±250, ±500, ±1000, ±2000 °/sec
- accelerometer: ±2, ±4, ±8, ±16 g
- magnetometer: ±4800 uT

\[
\text{metric\_value} = \frac{\text{raw\_sensor\_value}}{2^{15} - 1} \cdot \text{max\_range}
\]
MPU-9250 Coordinate Systems

gyro & accelerometer

magnetometer
How to read data from IMU

• I2C = serial interface with 2 wires (also see next lab)
• microcontroller to read, we’ll use Teensy 3.2, but any Arduino can be used, last year: Metro Mini

• schematics - which pins to connect where
• quick intro to Arduino
• Wire library to stream out data via serial
• serial client using node server
How to read data from IMU
How to read data from IMU

• connect power & ground

3.3V
How to read data from IMU

- connect power & ground
- connect I2C clock (SCL, D19) and data (SDA, D18) lines
How to read data from IMU

- connect power & ground
- connect I2C clock (SCL,A5) and data (SDA,A4) lines
- connect micro USB for power and data transfer
VRduino

- Teensy 3.2 & IMU already connected through PCB
- also has 4 photodiodes (more details next week)
- GPIO pins for additional sensors or other add-ons
Introduction to Arduino

• open source microcontroller hardware & software
• directly interface with sensors (i.e. IMU) and process raw data
• we will be working with Teensy 3.2 (Arduino compatible)
• use Arduino IDE for all software development, installed on all lab machines
• if you want to install it on your laptop, make sure to get:
  • IDE: https://www.arduino.cc/en/Main/Software
  • Teensyduino: https://www.pjrc.com/teensy/teensyduino.html
  • FTDI drivers: http://www.ftdichip.com/Drivers/VCP.htm
Introduction to Arduino (Random Test Program)

```c
/* Blink
  * Turns on an LED on for one second, then off for one second, repeatedly.
  *
  * This example code is in the public domain.
  */

// Pin 13 has an LED connected on most Arduino boards.
// Give it a name:
int led = 13;

// the setup routine runs once when you press reset:
void setup() {
  // initialize the digital pin as an output.
  pinMode(led, OUTPUT);
}

// the loop routine runs over and over again forever:
void loop() {
  digitalWrite(led, HIGH); // turn the LED on (HIGH is the voltage level)
  delay(1000); // wait for a second
  digitalWrite(led, LOW); // turn the LED off by making the voltage LOW
  delay(1000); // wait for a second
}
```

- **variable definition**
- **setup function** = initialization
- **loop function** = runtime callback
- connected to COM1 serial port
Introduction to Arduino

- need to stream data from Arduino to host PC
- use Wire library for all serial & I2C communication
- use node server to read from host PC and connect to JavaScript (see lab)
Introduction to Arduino

- **setup function = one time initialization**
- **read from I2C (connected to IMU)**
- **write to I2C (connected to IMU)**
- open serial connection to communicate with host PC
- set registers to configure IMU
Read Serial Data in Windows

- serial ports called COMx (USB serial usually COM3-COM7)
  1. establish connection to correct COM port (choose appropriate baud rate)
  2. read incoming data (in a thread)
Summary

• coordinate systems (world, body/sensor, inertial, transforms)
• overview of inertial sensors: gyroscopes, accelerometers, and magnetometers
• gyro integration aka *dead reckoning*
• orientation tracking in *flatland*
• pitch & roll from accelerometer
• overview of VRduino
Next Lecture

- quaternions and rotations with quaternions
- 6 DOF sensor fusion with quaternions & complementary filtering
Must read: course notes on IMUs!
Additional Information


- S. LaValle, A. Yershova, M. Katsev, M. Antonov “Head Tracking for the Oculus Rift”, Proc. ICRA 2014

- http://www.chrobotics.com/library
Positional Tracking with Accelerometers

• goal: track 3D position from accelerometer measurements

• question: is that even possible?
Positional Tracking with Accelerometers

- why not double-integrate acceleration for positional tracking?
- assume you can remove gravity vector (which is not trivial)
- two ODEs to estimate linear velocity and position:

\[
\frac{\partial}{\partial t} x(t + 1) \approx \frac{\partial}{\partial t} x(t) + \frac{\partial^2}{\partial t^2} x(t) \Delta t
\]

\[
x(t + \Delta t) \approx x(t) + \frac{\partial}{\partial t} x(t) \Delta t
\]

\(a^{(l)}\) from accelerometer
Positional Tracking with Accelerometers

- error grows quadratically!
Positional Tracking with Accelerometers

- question: is that even possible?
- answer: usually not

- two key challenges:
  1. error in Taylor series for double integration grows quadratically
  2. very difficult to remove gravity from accelerometer measurements