Inertial Measurement Units I

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Stanford University

EE 267 Virtual Reality

Lecture 9

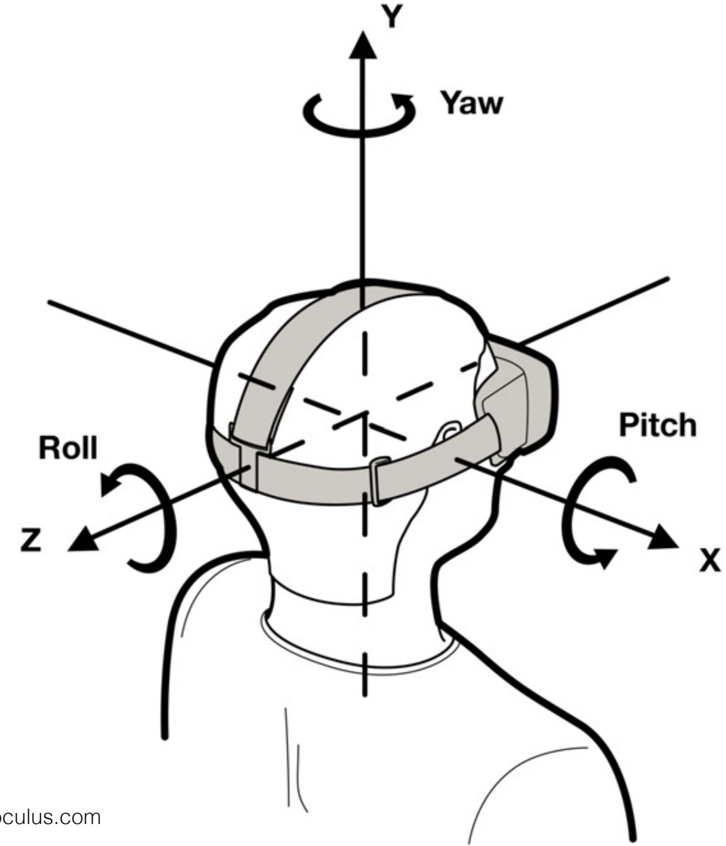
stanford.edu/class/ee267/

Logistics

- Project proposal due this Fri

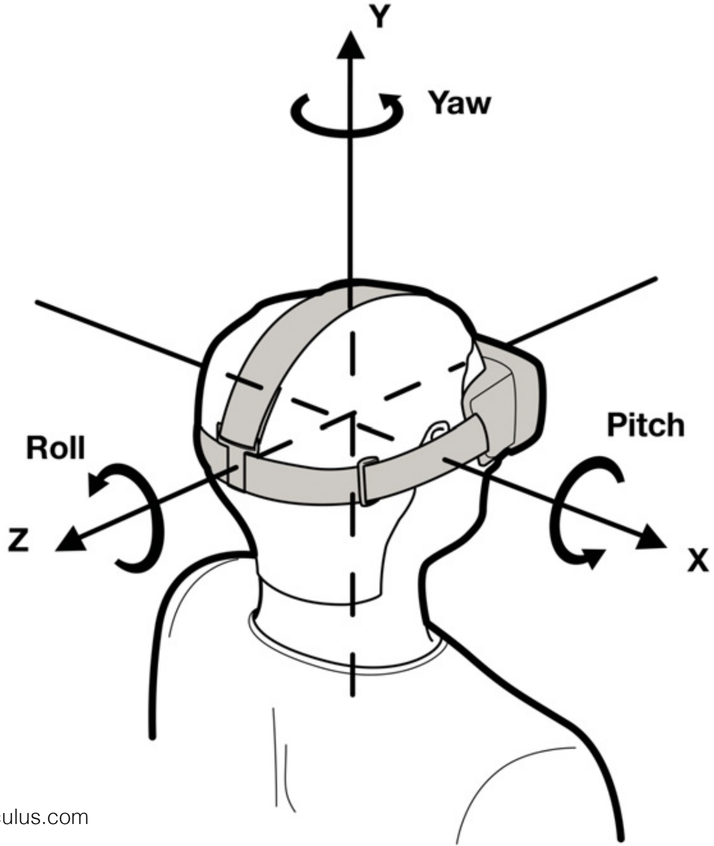
Lecture Overview

- coordinate systems (world, body/sensor, inertial, transforms)
- overview of inertial sensors: gyroscopes, accelerometers, and magnetometers
- gyro integration aka *dead reckoning*
- orientation tracking in *flatland*
- pitch & roll from accelerometer
- overview of VRduino



oculus.com

- primary goal: track orientation of head or other device
- orientation is the rotation of device w.r.t. world/earth or *inertial* frame
- rotations are represented by Euler angles (yaw, pitch, roll) or quaternions



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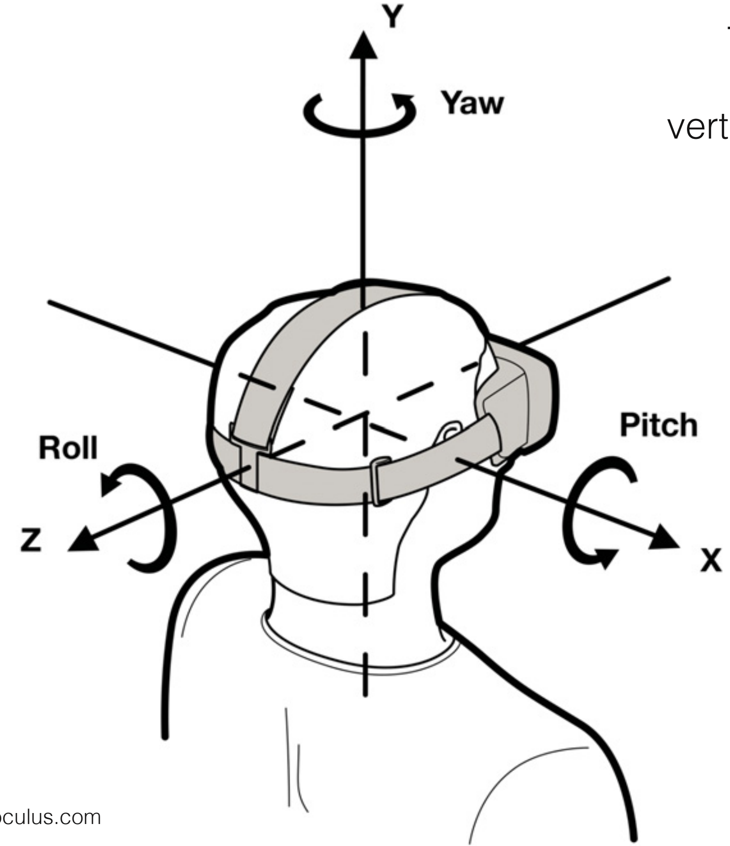
- orientation tracked with IMU models relative rotation of sensor/body frame in world/inertial coordinates
- example: person on the left looks up → pitch=90° or rotation around x-axis by 90°
- similarly, the world rotates around the sensor frame by -90° (inverse rotation)



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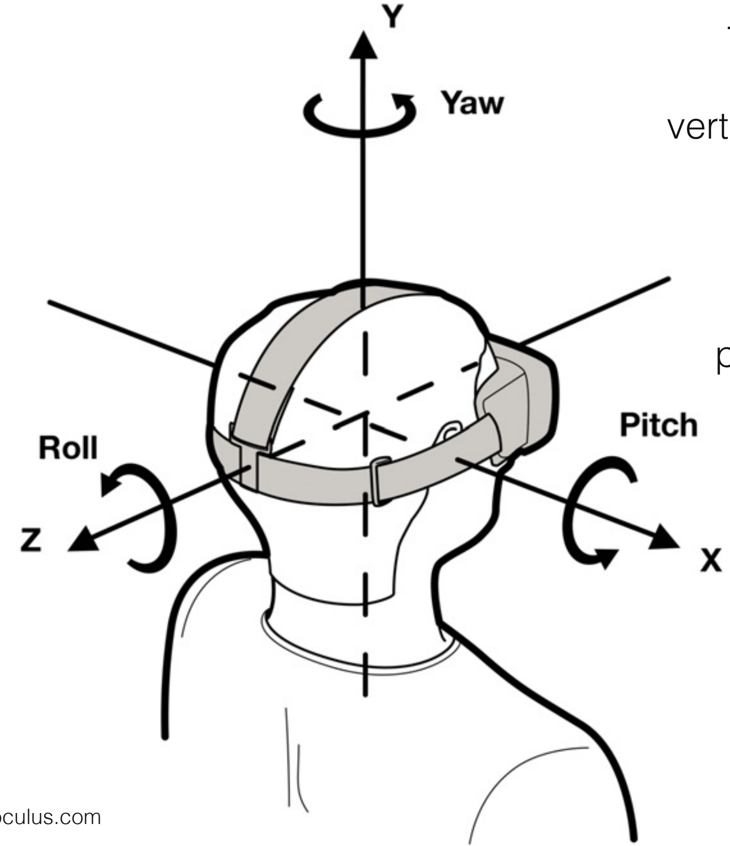
from lecture 2:

vertex in clip space

vertex

$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$



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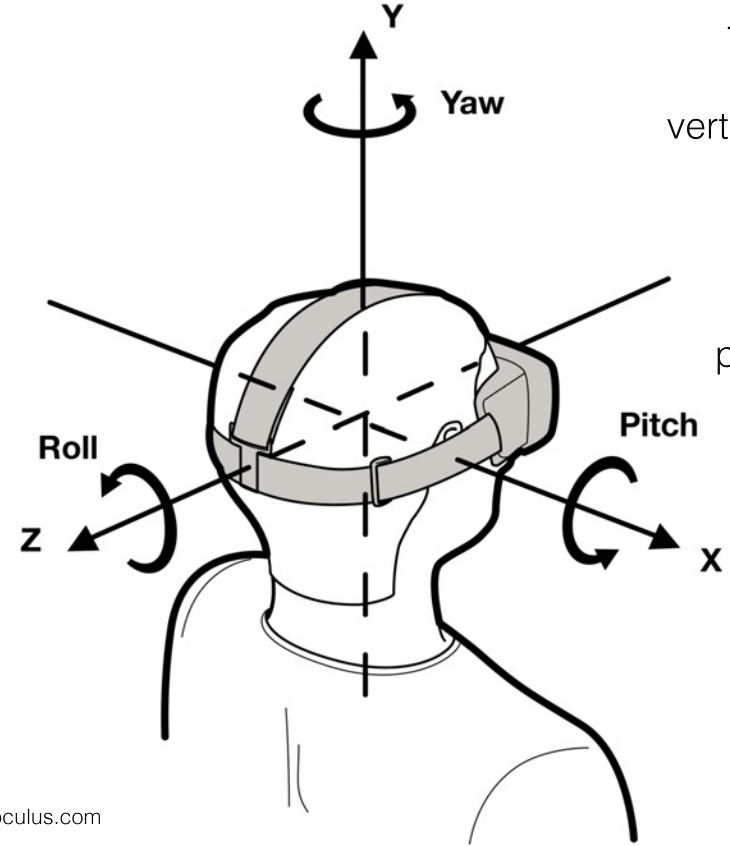
from lecture 2:

vertex in clip space ↓ ... vertex ↓

$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$

↑ projection matrix ↑ view matrix ↑ model matrix



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from lecture 2:

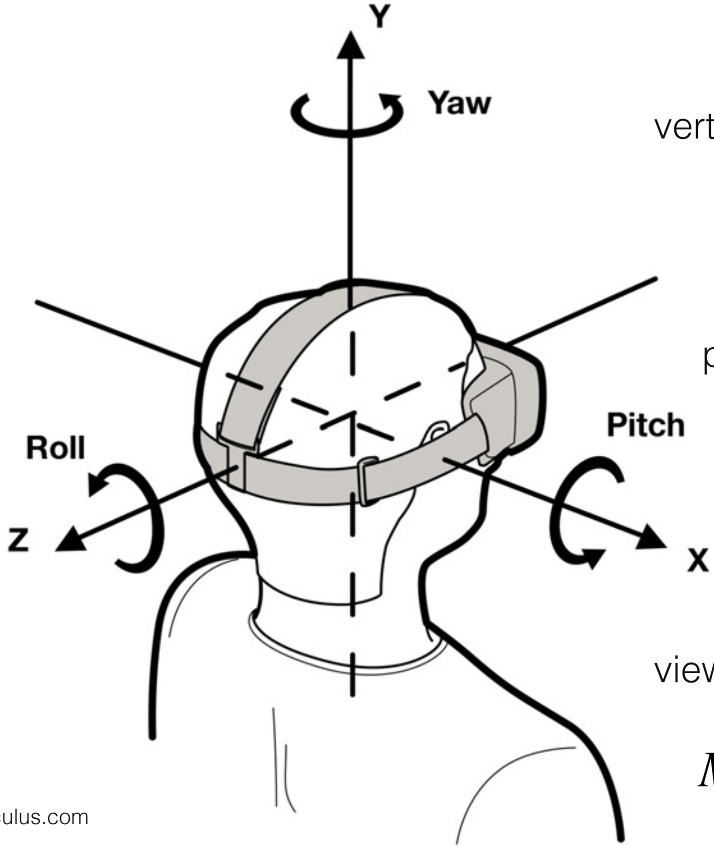
vertex in clip space → v_{clip}; vertex → v

$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$

projection matrix → M_{proj}; view matrix → M_{view}; model matrix → M_{model}

rotation → R; translation → $T(-eye)$

$$M_{view} = R \cdot T(-eye)$$



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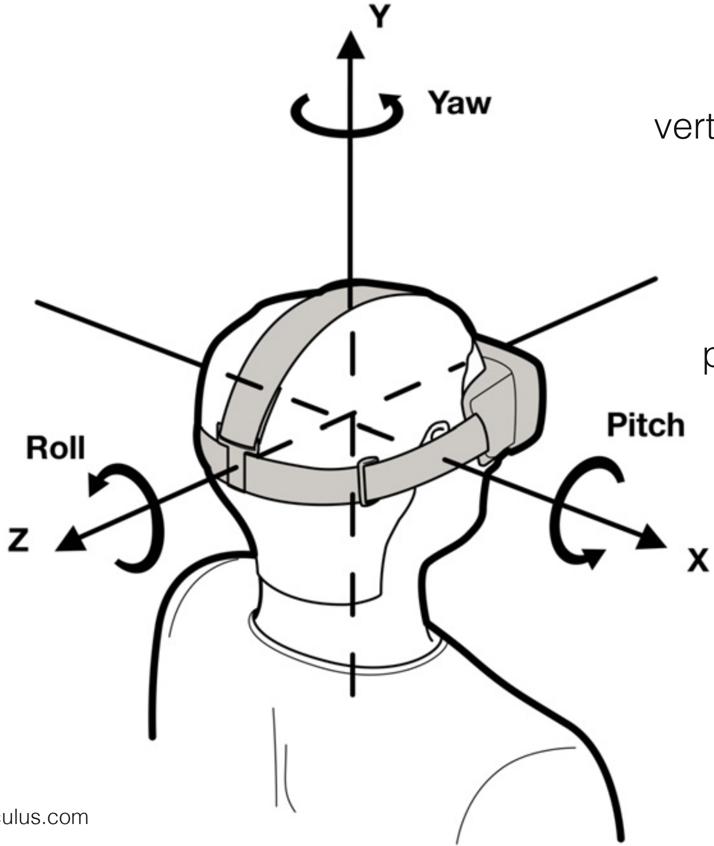
rotation → R; translation → $T(-eye)$

$$M_{view} = R \cdot T(-eye)$$

view matrix for stereo camera:

$$M_{view}^{stereo} = T\left(\pm\frac{ipd}{2}, 0, 0\right) \cdot R \cdot T(-eye)$$



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from lecture 2:

vertex in clip space ↓ … vertex ↓

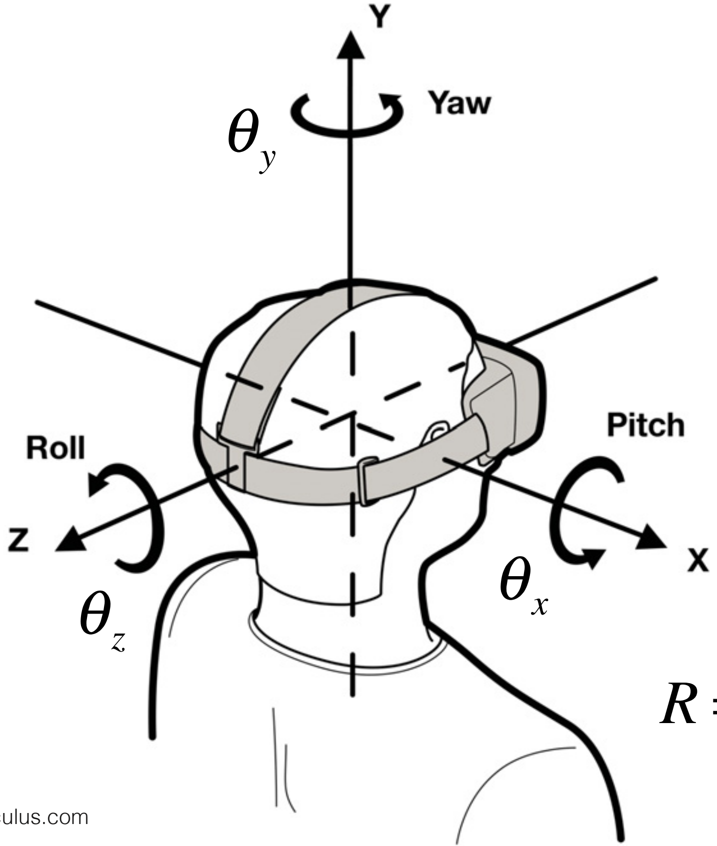
$$v_{clip} = M_{proj} \cdot M_{view} \cdot M_{model} \cdot v$$

↑ projection matrix ↑ view matrix ↑ model matrix

rotation ↓ translation ↓

$$M_{view} = R \cdot T(-eye)$$

↑ sensor/body frame ↑ world/inertial frame



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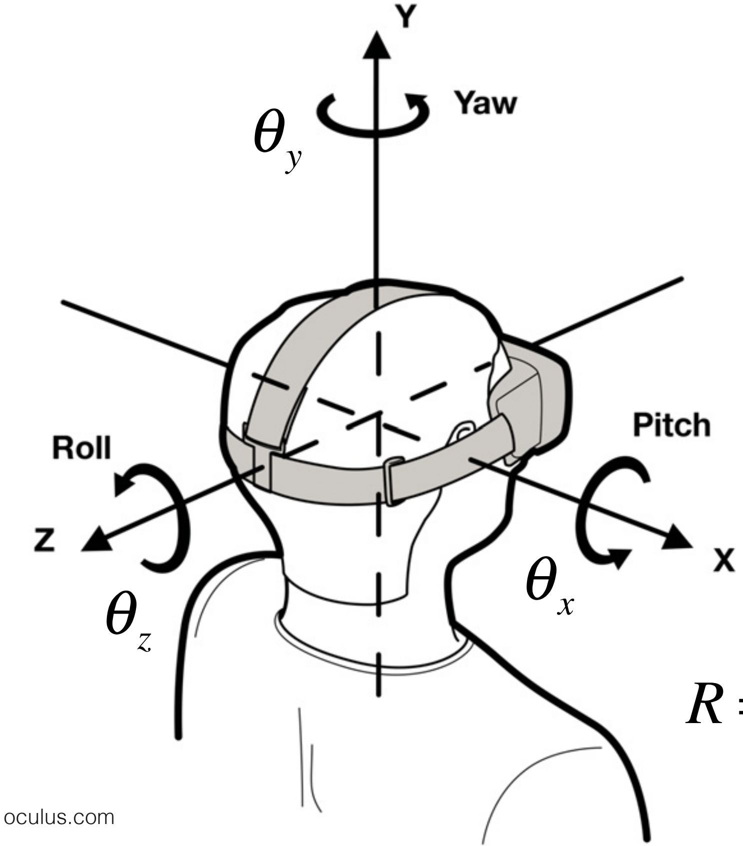
rotation ↓ translation ↓

$$M_{view} = R \cdot T(-eye)$$

roll, pitch, yaw

$$R = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y)$$

← order of rotations (world to body)

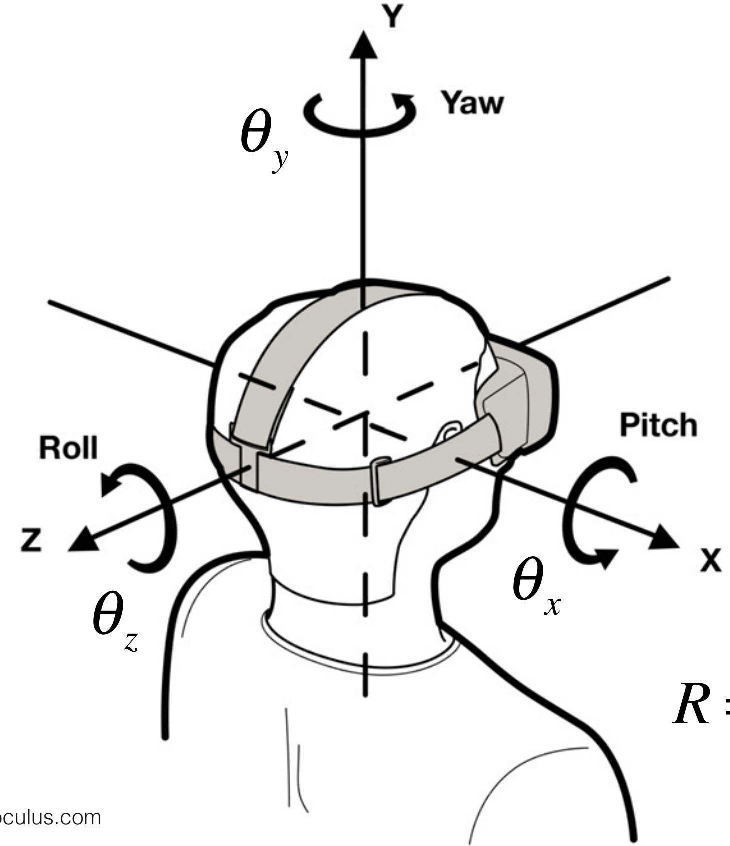


- this representation for a rotation is know as *Euler angles*

- need to specify order of rotation, e.g. yaw-pitch-roll

$$R = \underbrace{R_z(-\theta_z)}_{\text{roll}} \cdot \underbrace{R_x(-\theta_x)}_{\text{pitch}} \cdot \underbrace{R_y(-\theta_y)}_{\text{yaw}}$$

order of rotations (world to body)



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ATTENTION!

- Euler angles are usually a <u>terrible idea</u> for orientation tracking with more than 1 axis
- one of several reasons: rotations are not commutative

$$R = \underbrace{R_z(-\theta_z)}_{\text{roll}} \cdot \underbrace{R_x(-\theta_x)}_{\text{pitch}} \cdot \underbrace{R_y(-\theta_y)}_{\text{yaw}}$$

order of rotations (world to body)

What do Inertial Sensors Measure?

- gyroscope measures angular velocity $\widetilde{\omega}$ in degrees/sec
- accelerometer measures linear acceleration $\tilde{a}$ in m/s^2
- magnetometer measures magnetic field strength $\widetilde{m}$ in uT (micro Tesla) or Gauss → 1 Gauss = 100 uT

What do Inertial Sensors Measure?

- gyroscope measures angular velocity $\tilde{\omega}$ in degrees/sec
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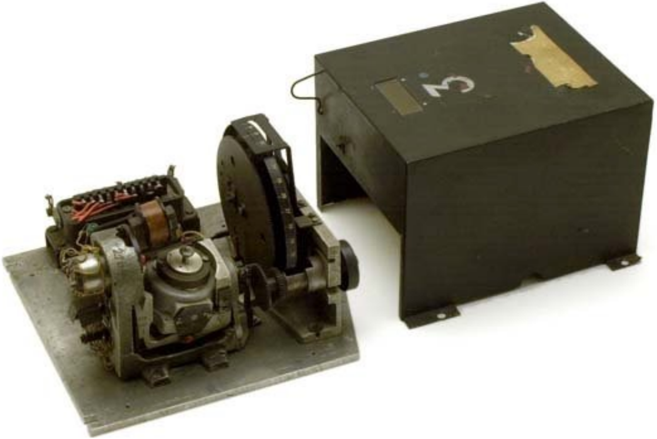
ALL MEASUREMENTS TAKEN IN SENSOR/BODY COORDINATES!

History of Gyroscopes

- critical for inertial measurements in ballistic missiles, aircrafts, drones, the mars rover, pretty much anything that moves!

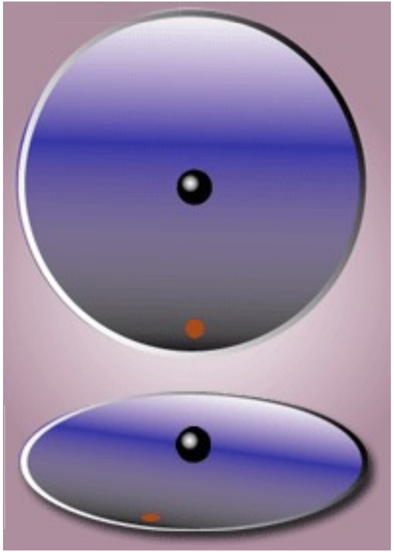


WWII era gyroscope used in the V2 rocket

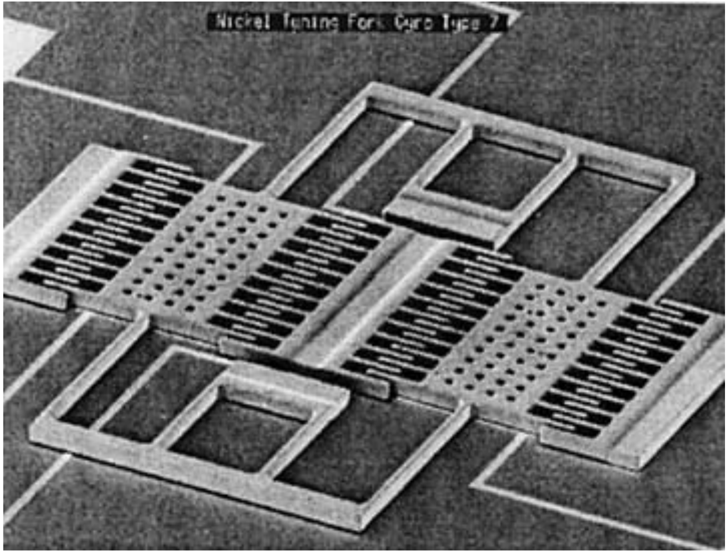
MEMS Gyroscopes

- today, we use microelectromechanical systems (MEMS)

Coriolis Force

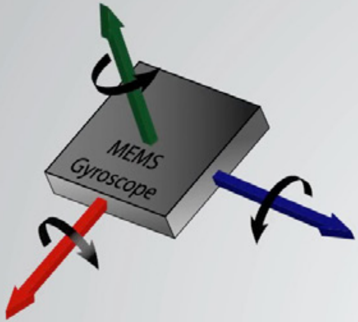


wikipedia



quora.com

MEMS Gyroscope



Gyroscopes

- gyro model: $\widetilde{\omega} = \omega + b + \eta$

Gyroscopes

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true angular velocity (↑ ω)

bias (↑ b)

additive, zero-mean Gaussian noise (↑ η)

Gyroscopes

- gyro model: $\tilde{\omega} = \omega + b + \eta$ $\eta \sim N\left(0, \sigma_{gyro}^2\right)$

  ω: true angular velocity; b: bias; η: additive, zero-mean Gaussian noise

- 3 DOF = 3-axis gyros that measures 3 orthogonal axes, assume no crosstalk
- bias is temperature-dependent and may change over time; can approximate as a constant
- additive measurement noise

Gyroscopes

- from gyro measurements to orientation – use Taylor expansion

$$\theta\left(t+\Delta t\right)\approx\theta\left(t\right)+\frac{\partial}{\partial t}\theta\left(t\right)\Delta t+\varepsilon,\quad \varepsilon\sim O\left(\Delta t^{2}\right)$$

Gyroscopes

- from gyro measurements to orientation – use Taylor expansion

$$\theta(t+\Delta t) \approx \theta(t) + \frac{\partial}{\partial t}\theta(t)\Delta t + \varepsilon, \quad \varepsilon \sim O(\Delta t^2)$$

$$\frac{\partial}{\partial t}\theta(t) = \omega$$

want: angle at current time step → $\theta(t+\Delta t)$

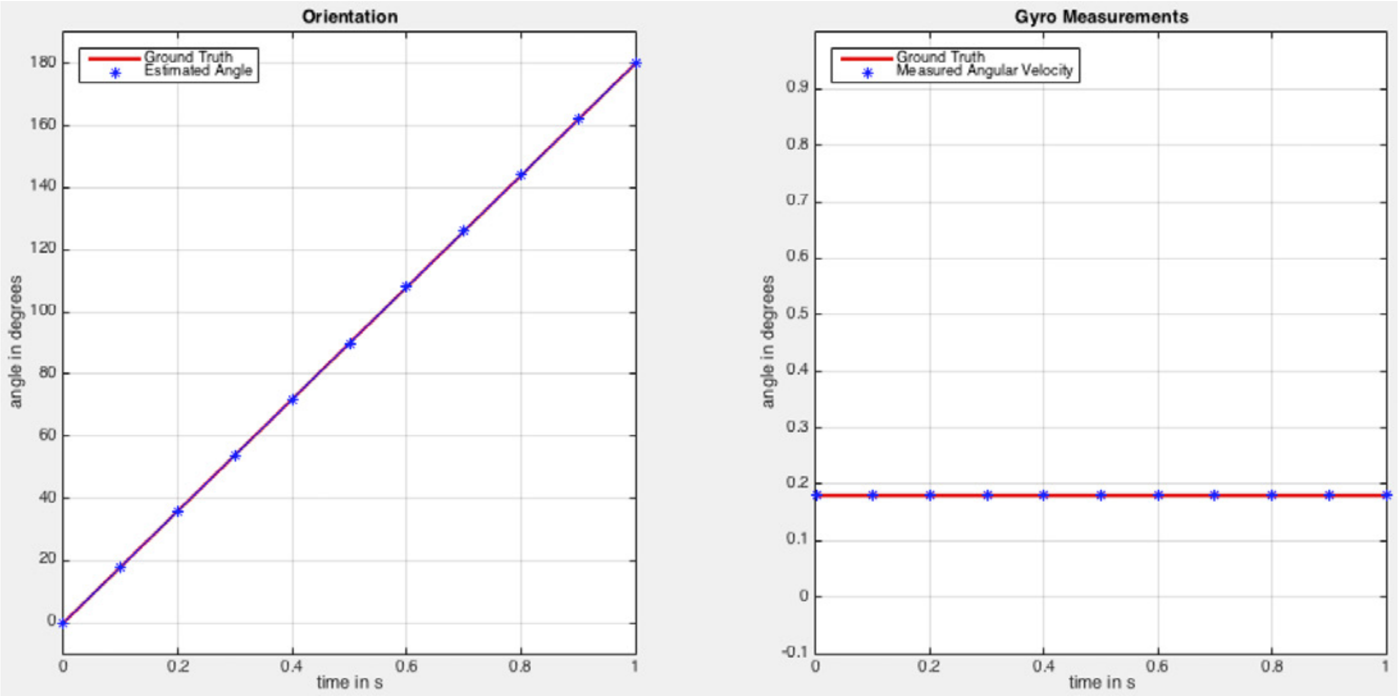
have: angle at last time step → $\theta(t)$

have: gyro measurement (angular velocity) → ω

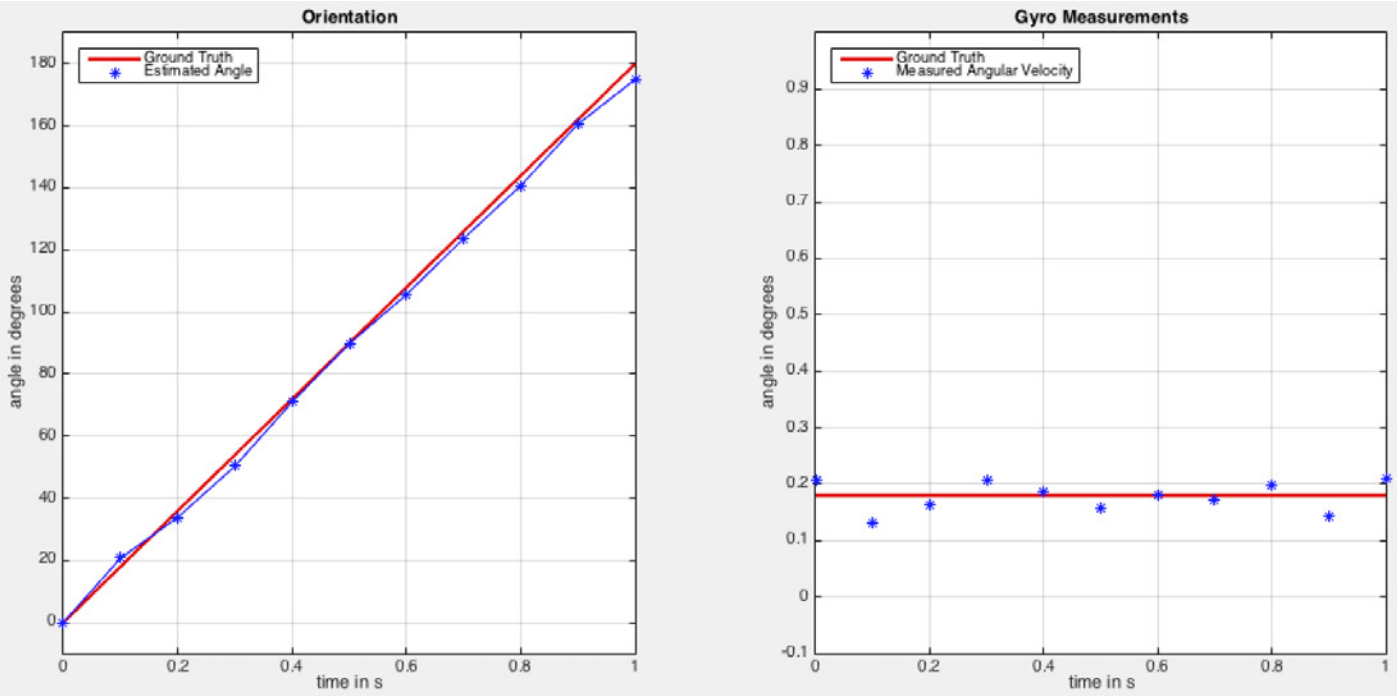
have: time step → Δt

approximation error! → ε

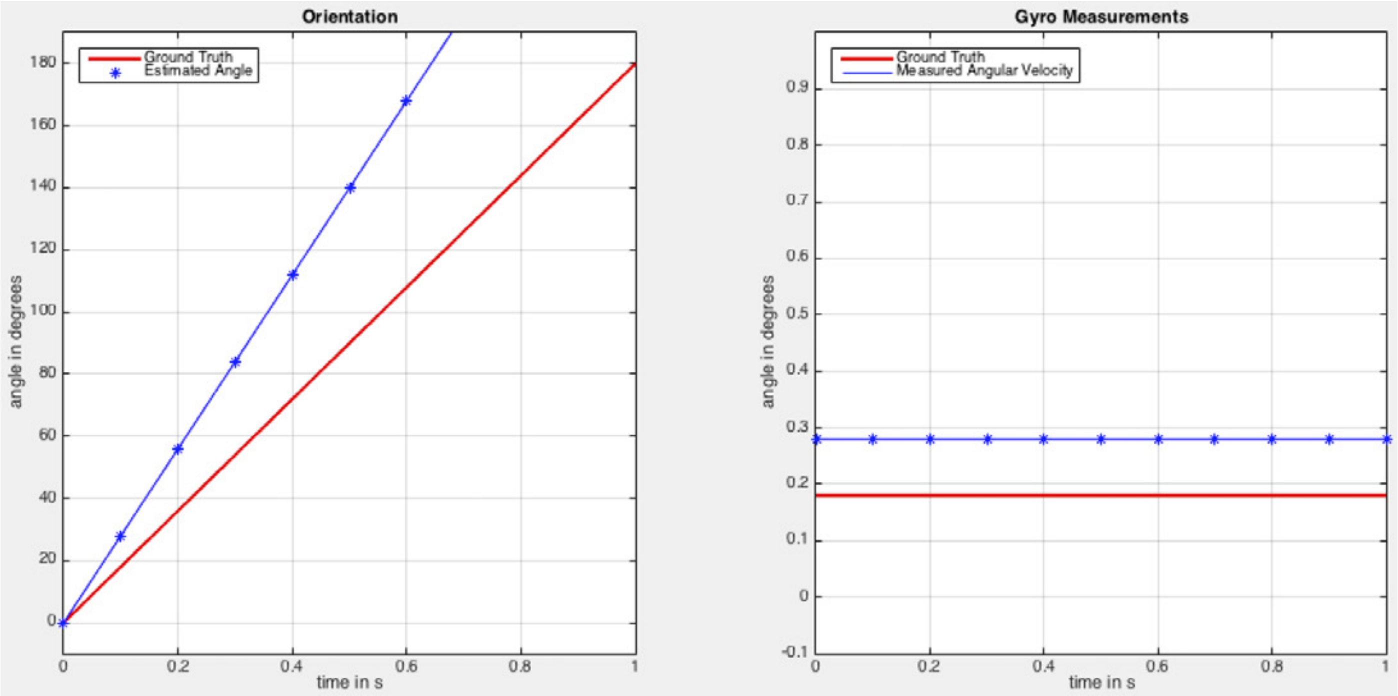
Gyro Integration: linear motion, no noise, no bias



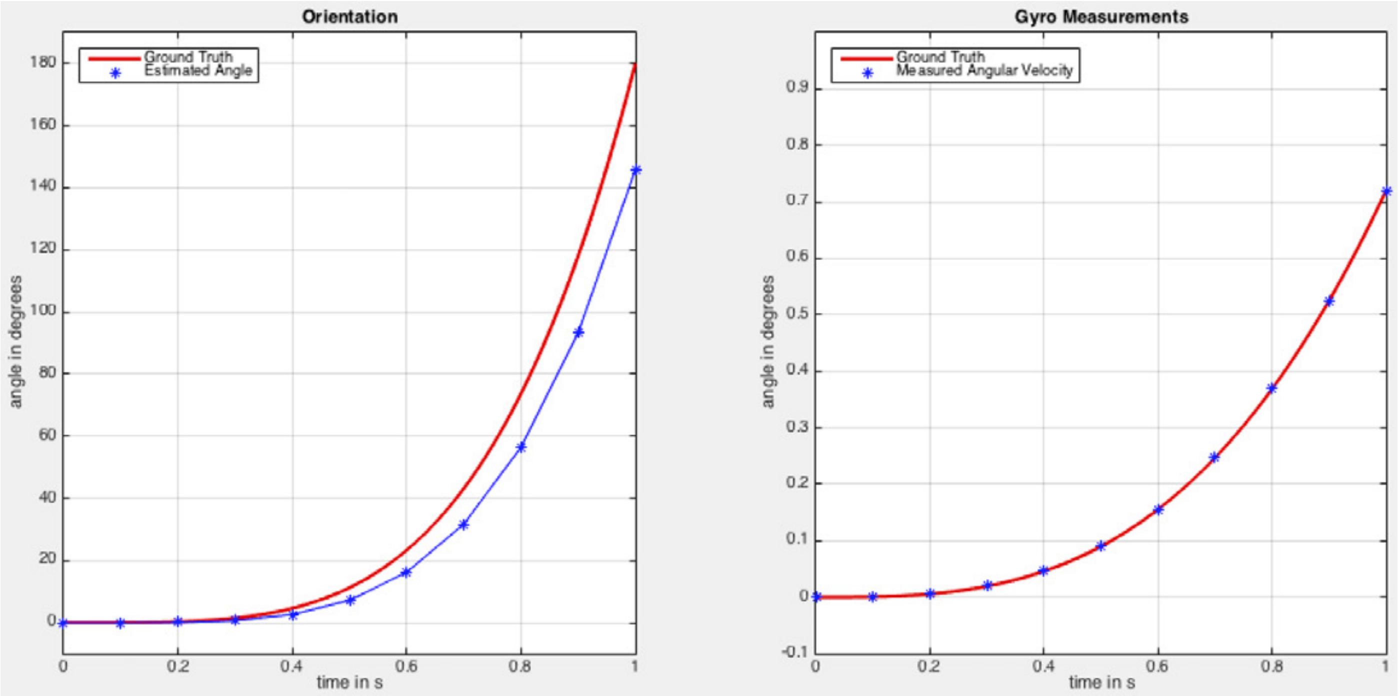
Gyro Integration: linear motion, noise, no bias



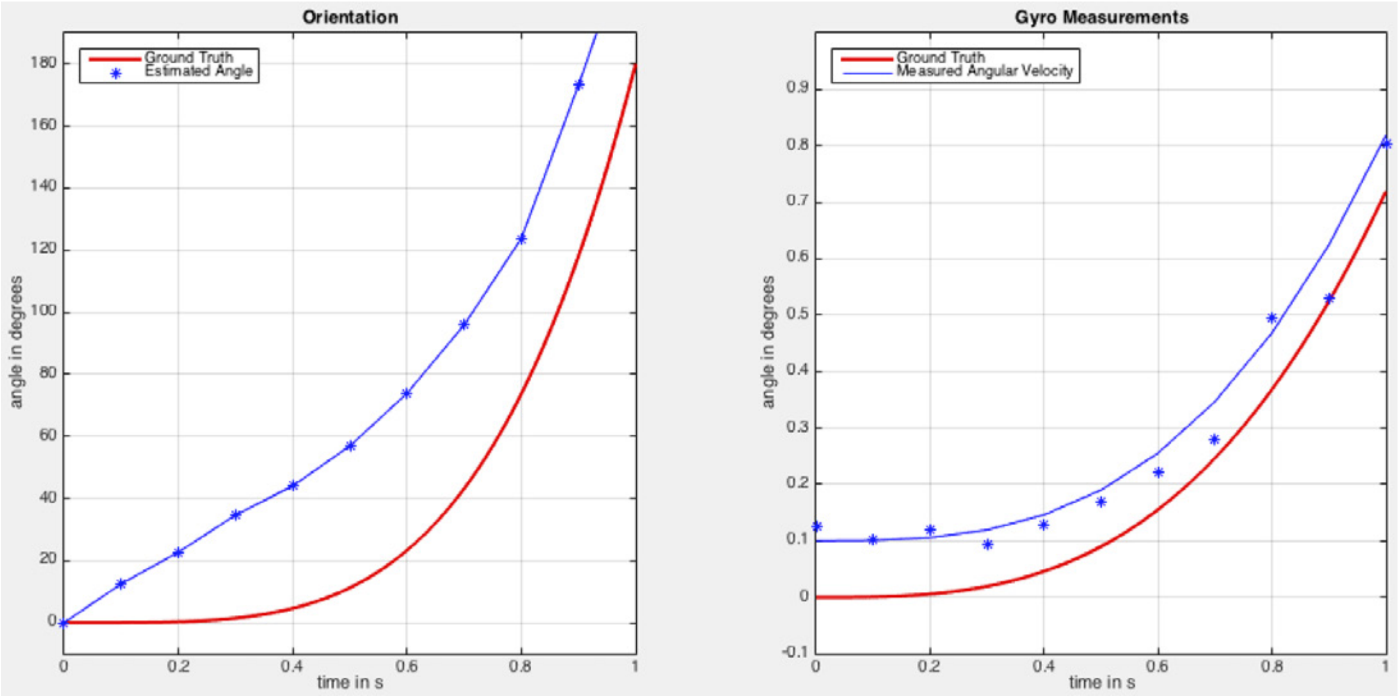
Gyro Integration: linear motion, no noise, bias



Gyro Integration: <u>nonlinear</u> motion, no noise, no bias



Gyro Integration: <u>nonlinear</u> motion, <u>noise</u>, <u>bias</u>



Gyro Integration aka *Dead Reckoning*

- works well for linear motion, no noise, no bias = unrealistic

- even if bias is know and noise is zero → <u>drift</u> (from integration)

- bias & noise variance can be estimated, other sensor measurements used to correct for drift (sensor fusion)

- <u>accurate in short term, but not reliable in long term due to drift</u>

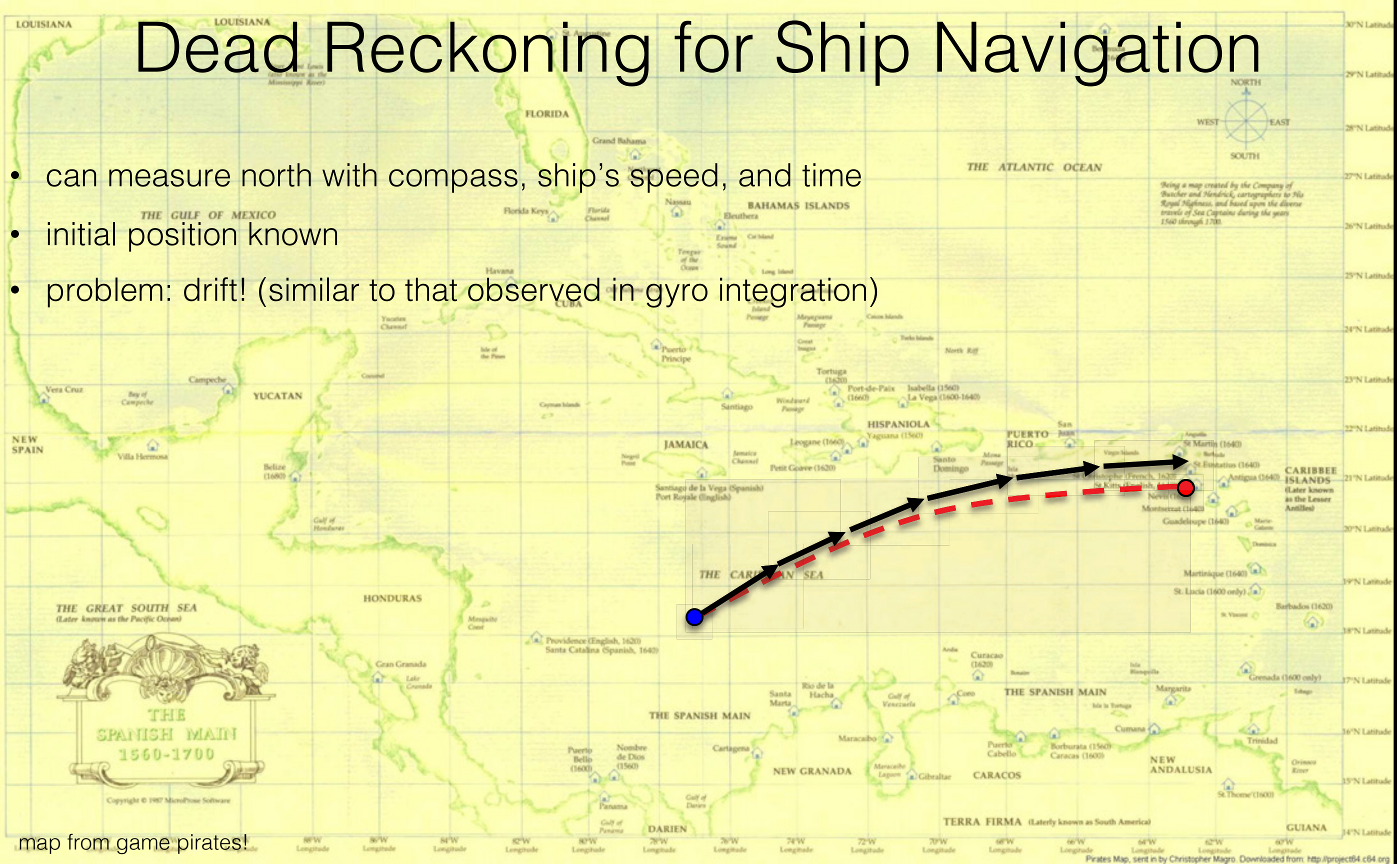
Dead Reckoning for Ship Navigation

- can measure north with compass, ship's speed, and time
- initial position known

map from game pirates!

Dead Reckoning for Ship Navigation

- can measure north with compass, ship's speed, and time
- initial position known
- problem: drift! (similar to that observed in gyro integration)



map from game pirates!

Gyro Advice

Always be aware of what units you are working with, degrees per second vs. radians per second!

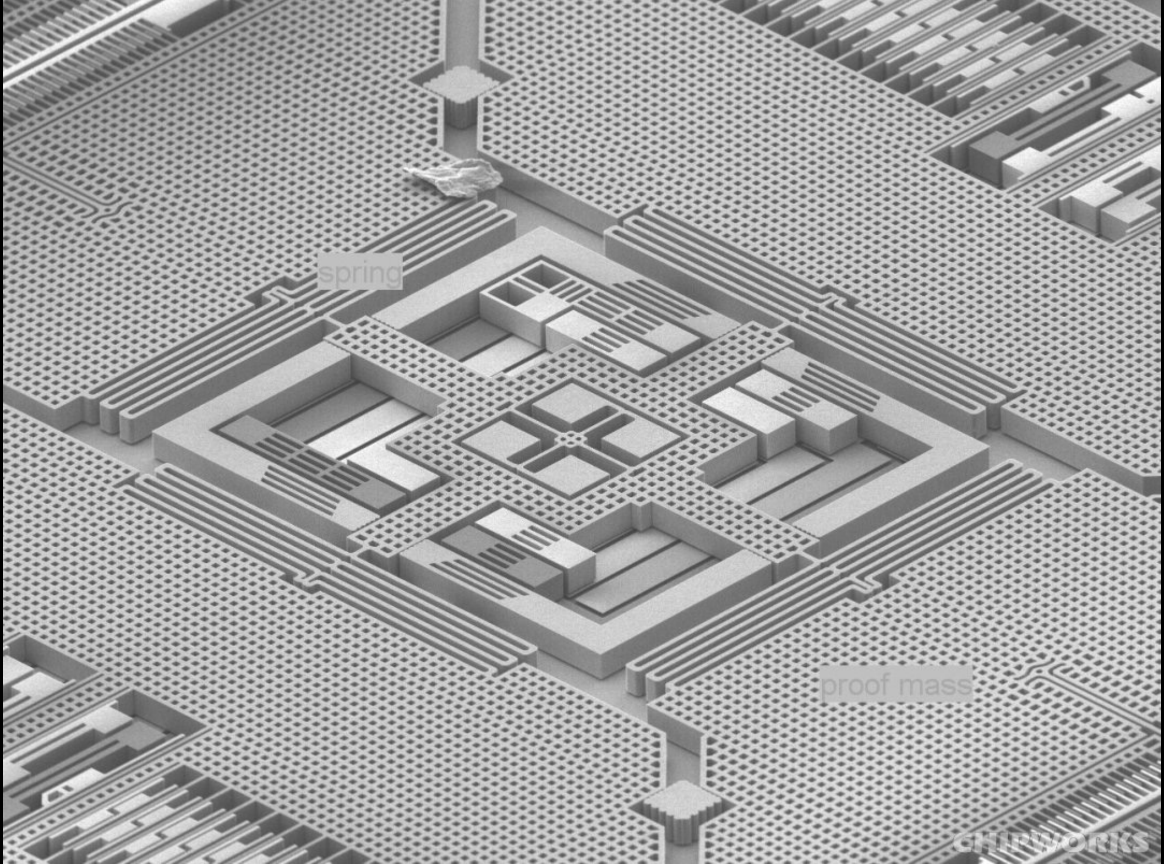
Accelerometers

- measure linear acceleration $\tilde{a} = a^{(g)} + a^{(l)} + \eta, \quad \eta \sim N\left(0, \sigma_{acc}^2\right)$

- without motion: read noisy gravity vector $a^{(g)} + \eta$ pointing UP! with magnitude 9.81 m/s^2 = 1g

- with motion: combined gravity vector and external forces $a^{(l)}$

chipworks



https://stevecurd.wordpress.com/2015/08/25/mems-and-me-how-does-my-fitbit-know-im-walking/

capacitive
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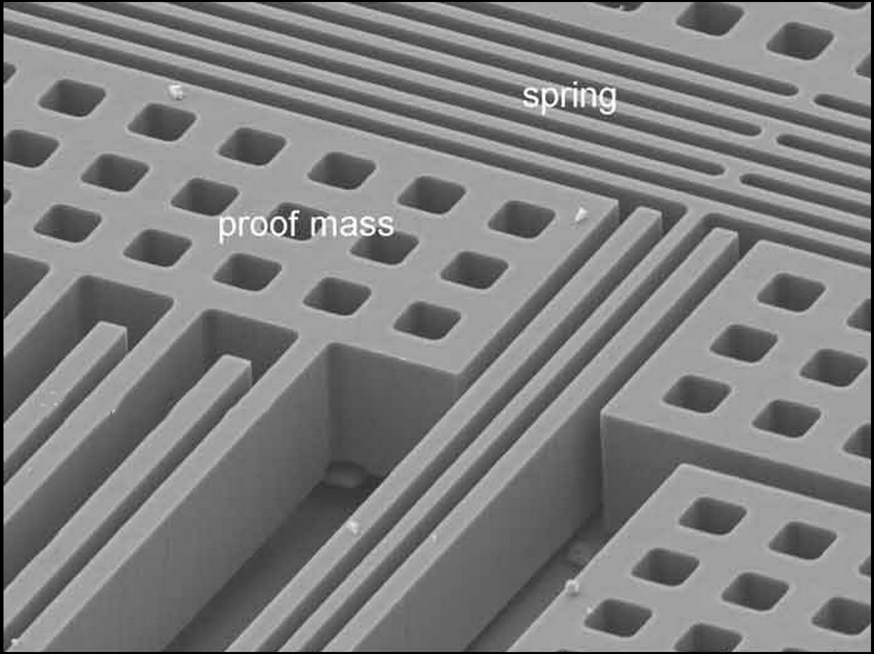
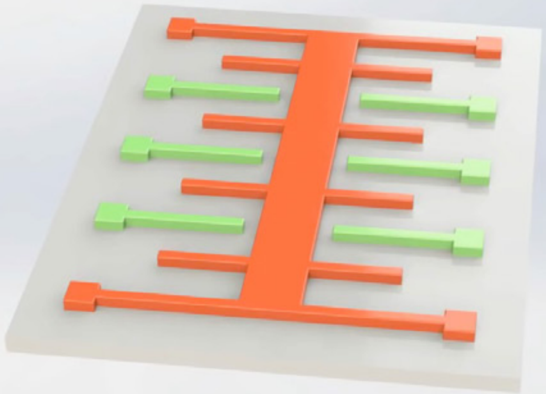


MEMS Accelerometer



Accelerometers

- advantages:
    - points up on average with magnitude of 1g
    - accurate in long term because no drift and the earth's center of gravity (usually) doesn't move
- problem:
    - noisy measurements
    - unreliable in short run due to motion (and noise)
- <u>complementary to gyro measurements!</u>

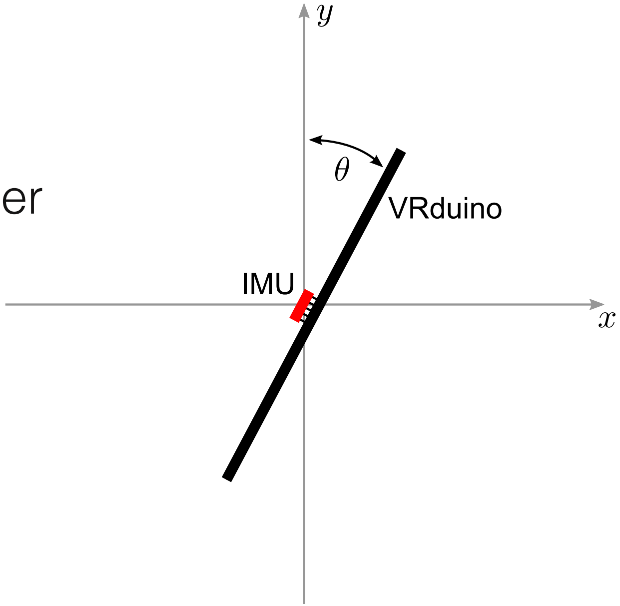
Accelerometers

- fusing gyro and accelerometer data = 6 DOF sensor fusion

- can correct tilt (i.e., pitch & roll) only – no information about yaw

Orientation Tracking in *Flatland*

- problem: track angle θ in 2D space
- sensors: 1 gyro, 2-axis accelerometer
- goal: understand sensor fusion



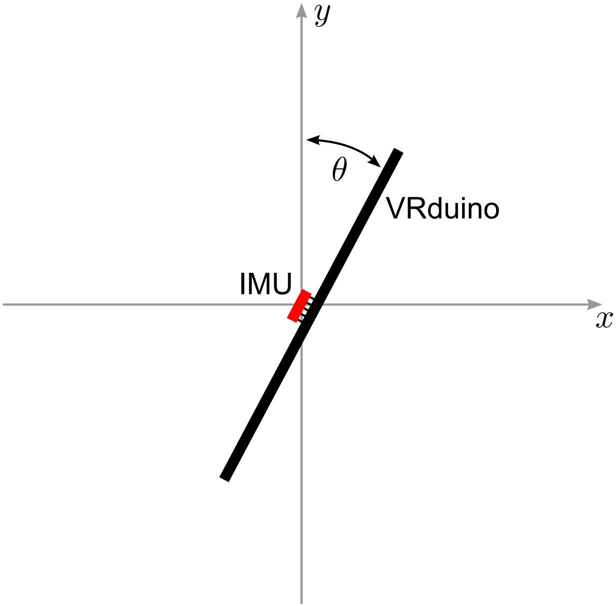
Orientation Tracking in *Flatland*

- gyro integration via Taylor series as

$$\theta_{gyro}^{(t)} = \theta_{gyro}^{(t-1)} + \tilde{\omega}\Delta t$$

- get Δt from microcontroller
- set $\theta_{gyro}^{(0)} = 0$

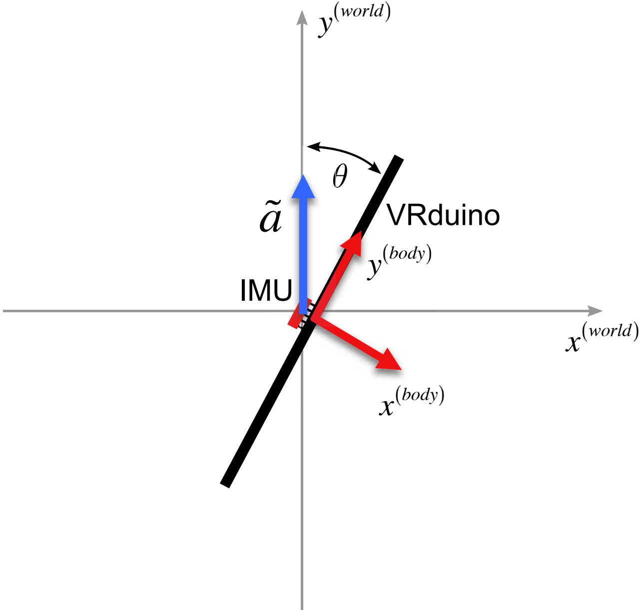
- biggest problem: drift!



Orientation Tracking in *Flatland*

- angle from accelerometer

$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right)$$

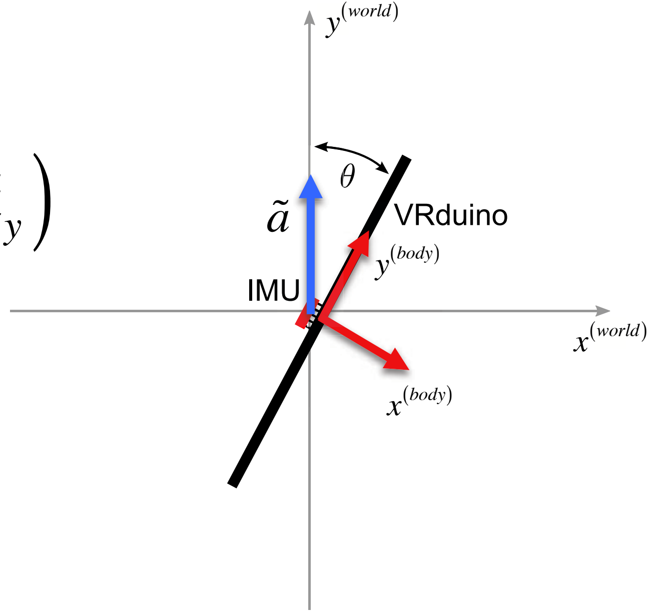


Orientation Tracking in *Flatland*

- angle from accelerometer

$$\theta_{acc} = \tan^{-1}\left(\frac{\tilde{a}_x}{\tilde{a}_y}\right) = \text{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$

handles division by 0 and proper signs, provided by most programming languages



Orientation Tracking in *Flatland*

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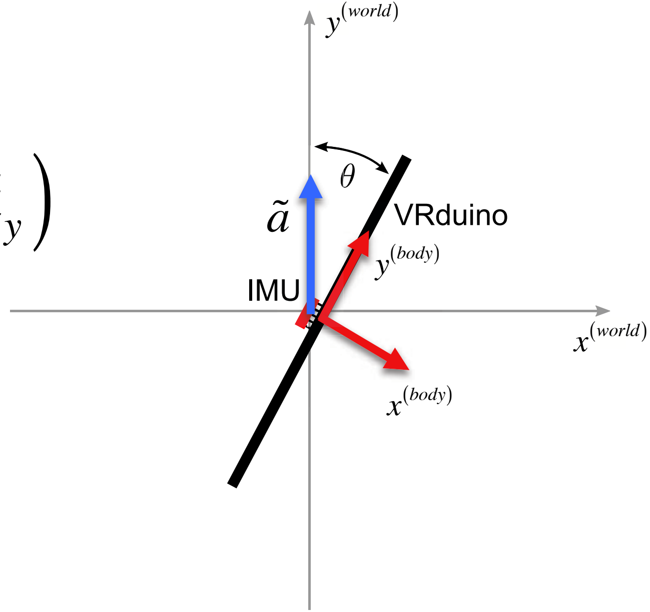
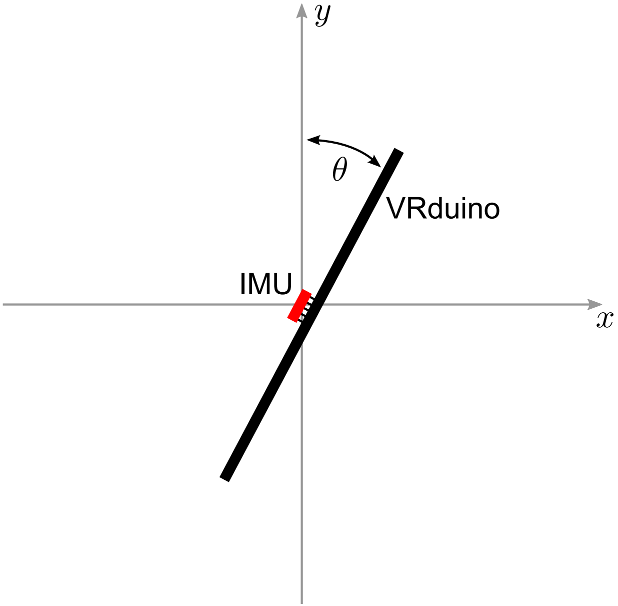


- biggest problem: noise

Orientation Tracking in *Flatland*

- sensor fusion: combine gyro and accelerometer measurements

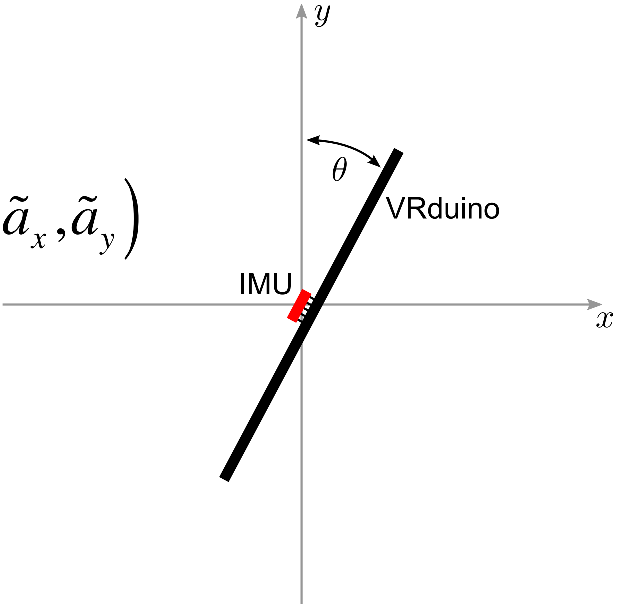
- intuition:
    - remove drift from gyro via high-pass filter
    - remove noise from accelerometer via low-pass filter



Orientation Tracking in *Flatland*

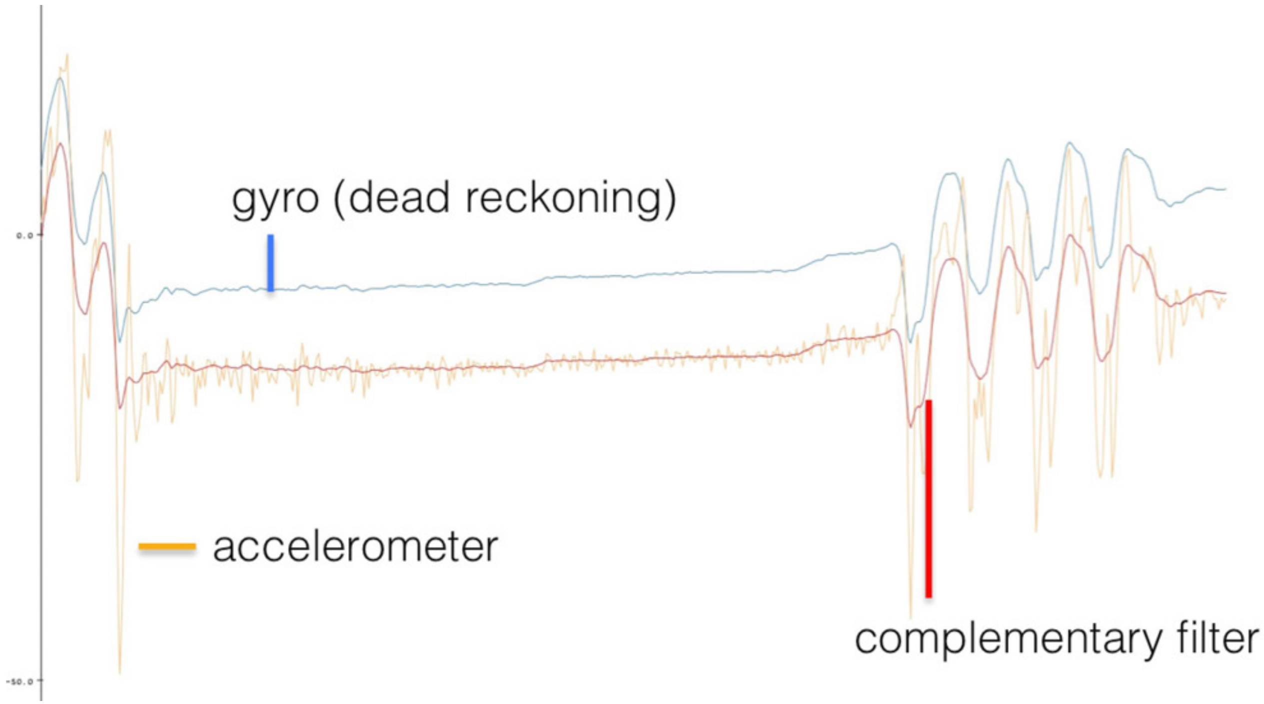
- sensor fusion with complementary filter, i.e. linear interpolation

$$\theta^{(t)} = \alpha\left(\theta^{(t-1)} + \tilde{\omega}\Delta t\right) + (1-\alpha)\,\mathrm{atan2}\left(\tilde{a}_x, \tilde{a}_y\right)$$



- no drift, no noise!

Orientation Tracking in *Flatland*



Pitch and Roll from 3-axis Accelerometer

- problem: estimate pitch and roll angles in 3D, from 3-axis accelerometer
- together, pitch & roll angles are known as *tilt*
- goal: understand tilt estimation in 3D

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

normalize gravity vector in inertial coordinates

$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = R\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = R_z(-\theta_z) \cdot R_x(-\theta_x) \cdot R_y(-\theta_y)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

normalize gravity vector rotated into sensor coordinates

Pitch and Roll from 3-axis Accelerometer

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$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = R\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = R_z(-\theta_z)\cdot R_x(-\theta_x)\cdot R_y(-\theta_y)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos(-\theta_z) & -\sin(-\theta_z) & 0 \\ \sin(-\theta_z) & \cos(-\theta_z) & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(-\theta_x) & -\sin(-\theta_x) \\ 0 & \sin(-\theta_x) & \cos(-\theta_x) \end{pmatrix}\begin{pmatrix} \cos(-\theta_y) & 0 & \sin(-\theta_y) \\ 0 & 1 & 0 \\ -\sin(-\theta_y) & 0 & \cos(-\theta_y) \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

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$$\hat{a} = \frac{\tilde{a}}{||\tilde{a}||} = \begin{pmatrix} -\cos(-\theta_x)\sin(-\theta_z) \\ \cos(-\theta_x)\cos(-\theta_z) \\ \sin(-\theta_x) \end{pmatrix}$$

Pitch and Roll from 3-axis Accelerometer

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roll

$$\frac{\hat{a}_x}{\hat{a}_y} = \frac{-\sin(-\theta_z)}{\cos(-\theta_z)} = -\tan(-\theta_z)$$

$$\theta_z = -\mathrm{atan2}(-\hat{a}_x, \hat{a}_y) \text{ in rad} \in [-\pi, \pi]$$

Pitch and Roll from 3-axis Accelerometer

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$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = \begin{pmatrix} -\cos(-\theta_x)\sin(-\theta_z) \\ \cos(-\theta_x)\cos(-\theta_z) \\ \sin(-\theta_x) \end{pmatrix}$$

pitch

$$\frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)\underbrace{\left(\sin^2(-\theta_z) + \cos^2(-\theta_z)\right)}_{=1}}}$$

$$= \frac{\sin(-\theta_x)}{\cos(-\theta_x)} = \tan(-\theta_x)$$

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
- assume no external forces (only gravity) – acc is pointing UP!

pitch

$$\hat{a} = \frac{\tilde{a}}{\|\tilde{a}\|} = \begin{pmatrix} -\cos(-\theta_x)\sin(-\theta_z) \\ \cos(-\theta_x)\cos(-\theta_z) \\ \sin(-\theta_x) \end{pmatrix}$$

$$\Rightarrow \frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)\underbrace{\left(\sin^2(-\theta_z) + \cos^2(-\theta_z)\right)}_{=1}}}$$

$$\theta_x = -\text{atan2}\left(\hat{a}_z, \sqrt{\hat{a}_x^2 + \hat{a}_y^2}\right) \text{ in rad } \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Pitch and Roll from 3-axis Accelerometer

- use only accelerometer data – can estimate pitch & roll, not yaw
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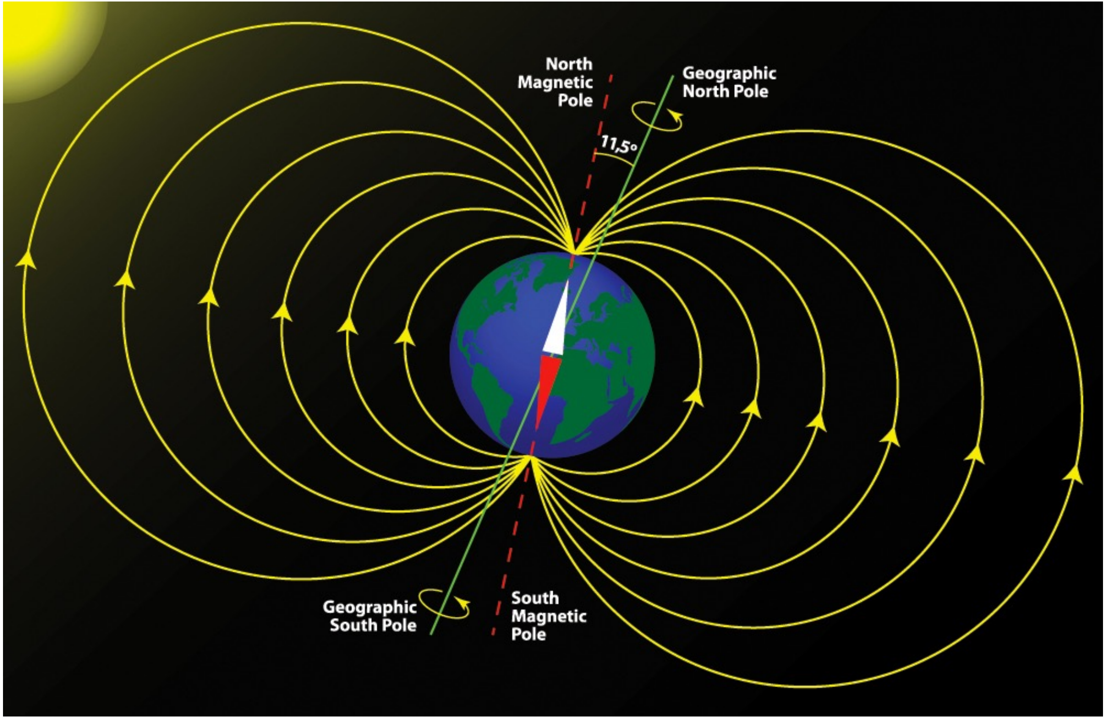
pitch

$$\frac{\hat{a}_z}{\sqrt{\hat{a}_x^2 + \hat{a}_y^2}} = \frac{\sin(-\theta_x)}{\sqrt{\cos^2(-\theta_x)\underbrace{\left(\sin^2(-\theta_z) + \cos^2(-\theta_z)\right)}_{=1}}}$$

$$\theta_x = -\mathrm{atan2}\left(\hat{a}_z, \mathrm{sign}(\hat{a}_y) \cdot \sqrt{\hat{a}_x^2 + \hat{a}_y^2}\right) \text{ in rad} \quad \in [-\pi, \pi]$$

Magnetometers

wikipedia



lifescience.com

MEMS Magnetometer

Hall Effect

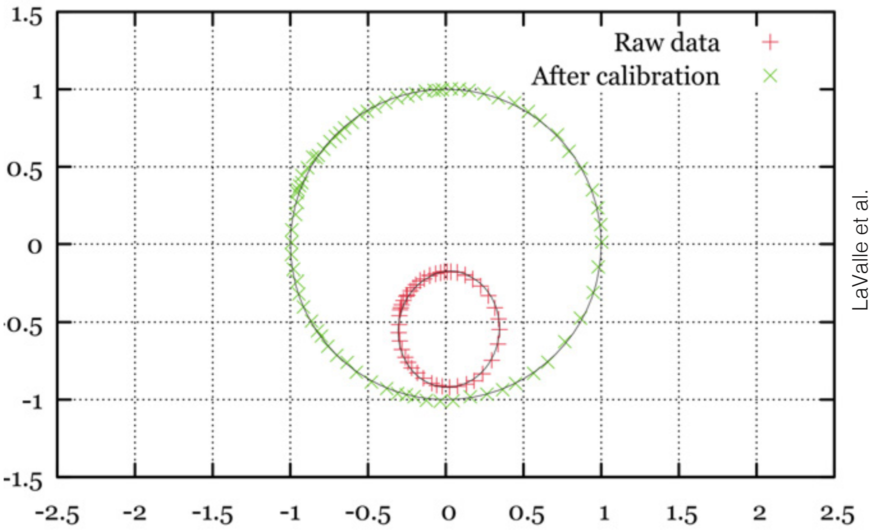
Magneto-resistive effect

Magnetometers

- measure earth's magnetic field in Gauss or uT
- 3 orthogonal axes = vector pointing along the magnetic field
- actual direction depends on latitude and longitude!
- distortions due to metal / electronics objects in the room or in HMD

Magnetometers

difficult to work with magnetometers without proper calibration → we will not use the magnetometer in the HW!



LaValle et al.

Magnetometers

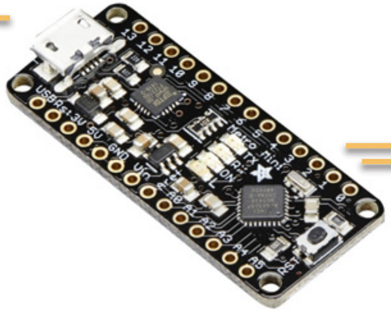
- advantages:
  - complementary to accelerometer – gives yaw (heading)
- problems:
  - affected by metal, distortions of magnetic field
  - need to know location, even when calibrated (e.g. GPS)
- together with gyro + accelerometer = 9 DOF sensor fusion

Prototype IMU

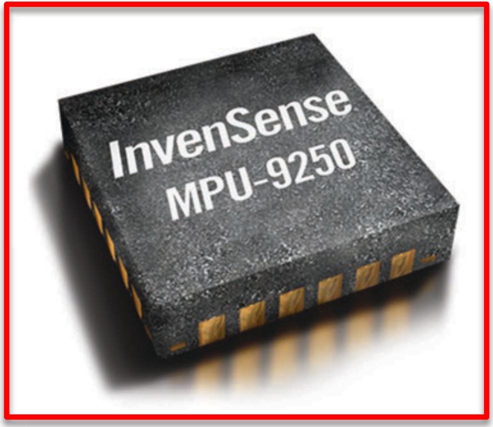
- 9 DOF IMU: InvenSense MPU-9250 = updated model of what was in the Oculus DK2
- 3-axis gyro, 3-axis accelerometer, 3-axis magnetometer all on 1 chip (we'll only use gyro and acc, but we'll give you code to read mag if you want to use it in your project)
- interface with I2C (serial bus) from Arduino

Prototype IMU

to host:
serial via USB



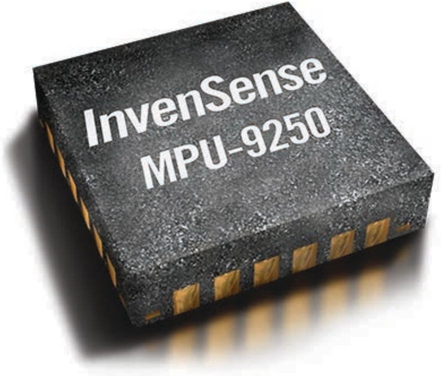
I2C



e.g. Arduino

InvenSense MPU-9250

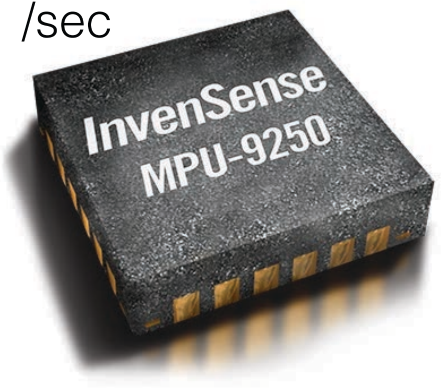
MPU-9250 Specs



- multi-chip module: 1 die houses gyro & accelerometer, the other the magnetometer
- magnetometer: Asahi Kasei Microdevices AK8963 ("3rd party device")
- 9x 16 bit ADCs for digitizing 9DOF data

MPU-9250 Specs

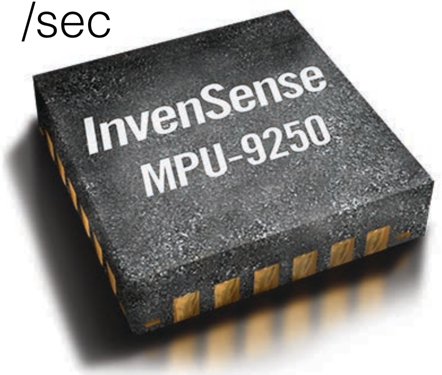
- gyro modes: ±250, ±500, ±1000, ±2000 ° /sec
- accelerometer: ±2, ±4, ±8, ±16 g
- magnetometer: ±4800 uT



- configure using registers (see specs) via I2C
- also supports on-board Digital Motion Processing™ (DMP™) sorry, we don't have access
- we'll provide starter code for Arduino in lab (easy to use for beginners, not consumer product grade!)

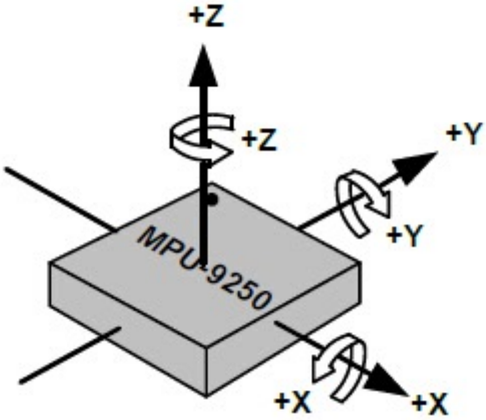
MPU-9250 Specs

- gyro modes: ±250, ±500, ±1000, ±2000 °/sec
- accelerometer: ±2, ±4, ±8, ±16 g
- magnetometer: ±4800 uT

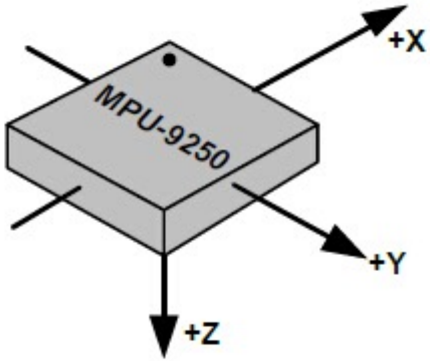


$$metric\_value = \frac{raw\_sensor\_value}{2^{15}-1} \cdot \max\_range$$

MPU-9250 Coordinate Systems



gyro & accelerometer



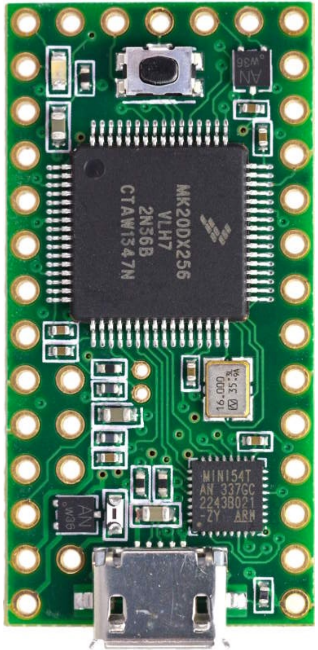
magnetometer

How to read data from IMU

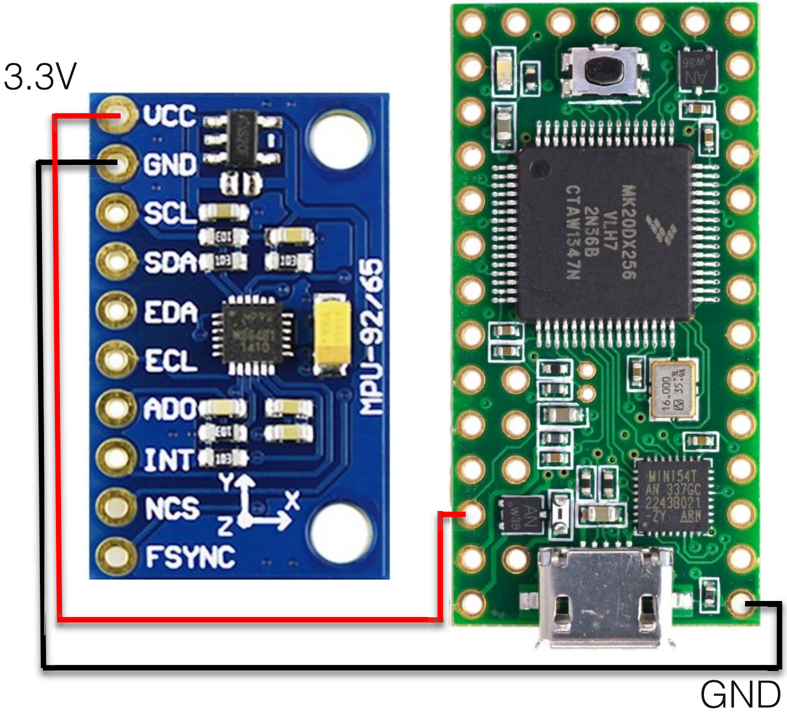
- I2C = serial interface with 2 wires (also see next lab)
- microcontroller to read, we’ll use Teensy 3.2, but any Arduino can be used, e.g. past offerings used Metro Mini

- schematics - which pins to connect where
- quick intro to Arduino
- Wire library to stream out data via serial
- serial client using node server

How to read data from IMU

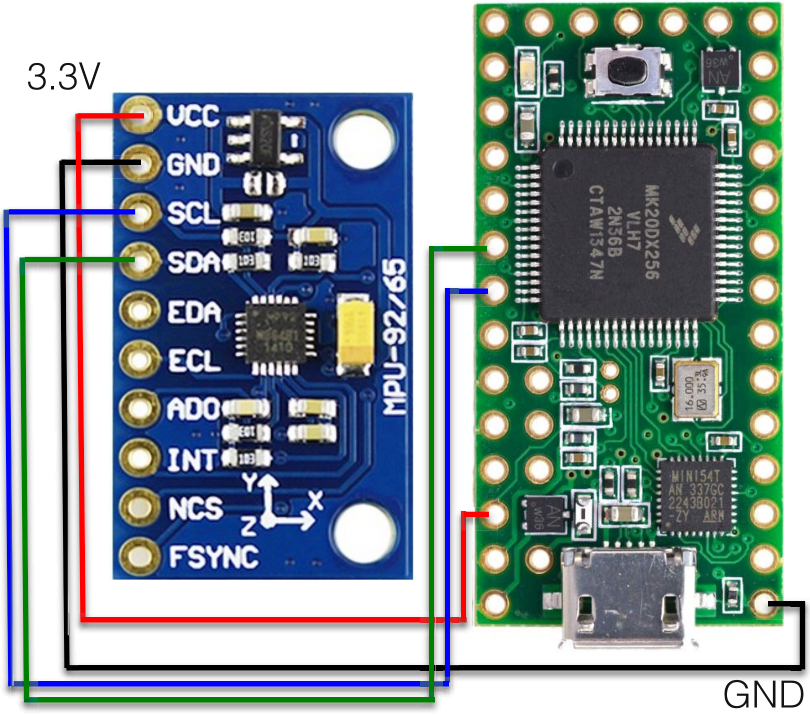


How to read data from IMU



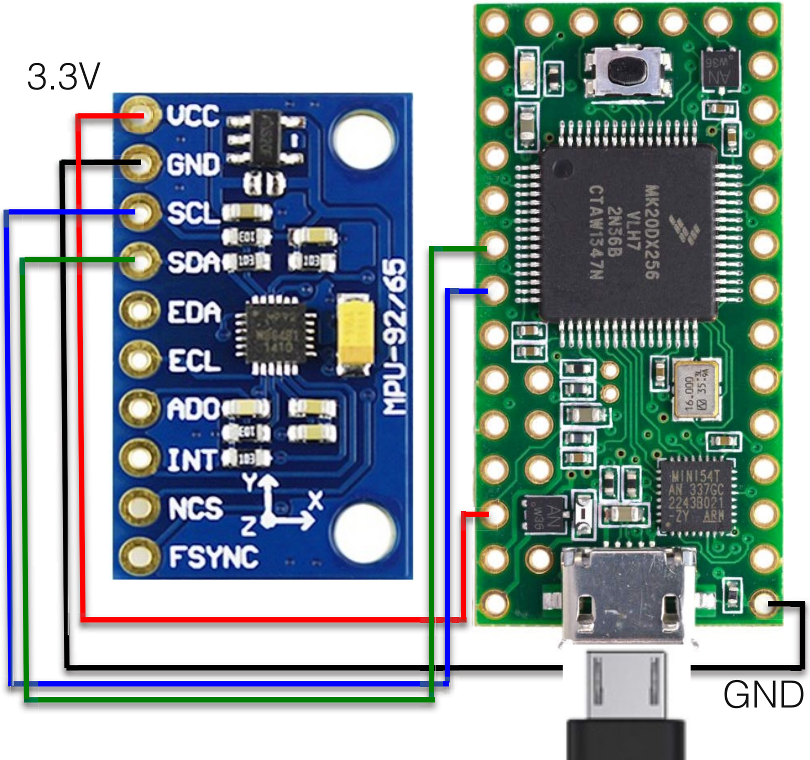
- connect power & ground

How to read data from IMU



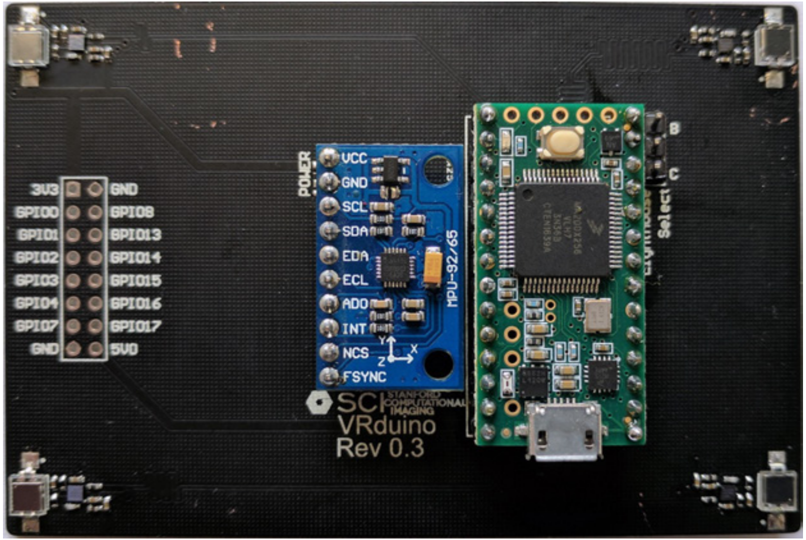
- connect power & ground
- connect I2C clock (SCL, pin19/A5) and data (SDA, pin18/A4) lines

How to read data from IMU



- connect power & ground
- connect I2C clock (SCL, pin19/A5) and data (SDA, pin18/A4) lines
- connect micro USB for power and data transfer

VRduino

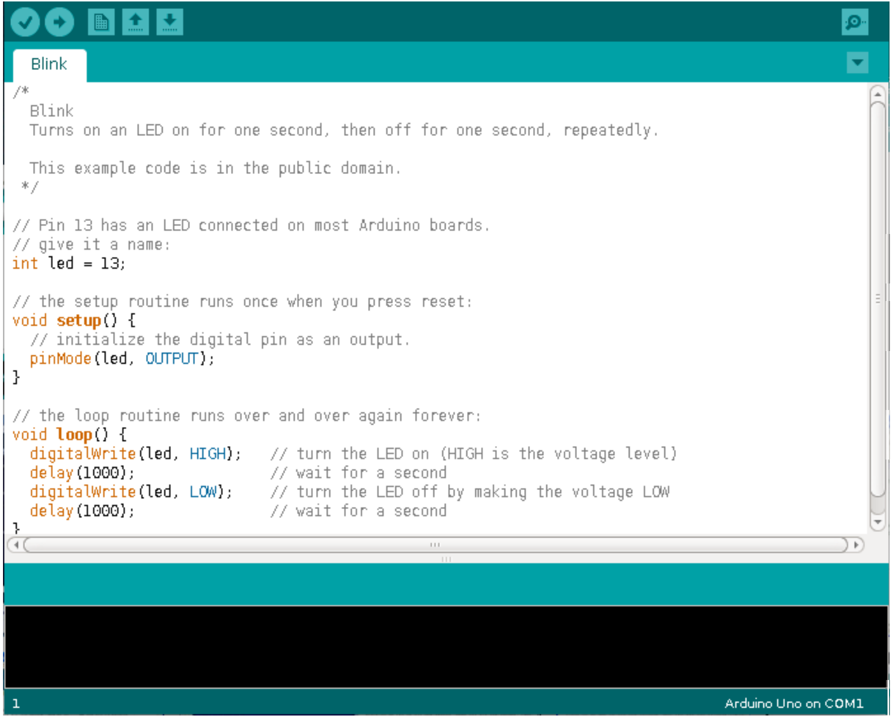


- Teensy 3.2 & IMU already connected through PCB
- also has 4 photodiodes (more details next week)
- GPIO pins for additional sensors or other add-ons

Introduction to Arduino

- open source microcontroller hardware & software
- directly interface with sensors (i.e. IMU) and process raw data
- we will be working with Teensy 3.2 (Arduino compatible)
- use Arduino IDE for all software development, installed on all lab machines
- if you want to install it on your laptop, make sure to get:
  - IDE: https://www.arduino.cc/en/Main/Software
  - Teensyduino: https://www.pjrc.com/teensy/teensyduino.html
  - Wire library (for serial & I2C): http://www.arduino.cc/en/Reference/Wire
  - FTDI drivers: http://www.ftdichip.com/Drivers/VCP.htm

Introduction to Arduino (Random Test Program)



← variable definition

← <u>setup</u> function = initialization

← <u>loop</u> function = runtime callback

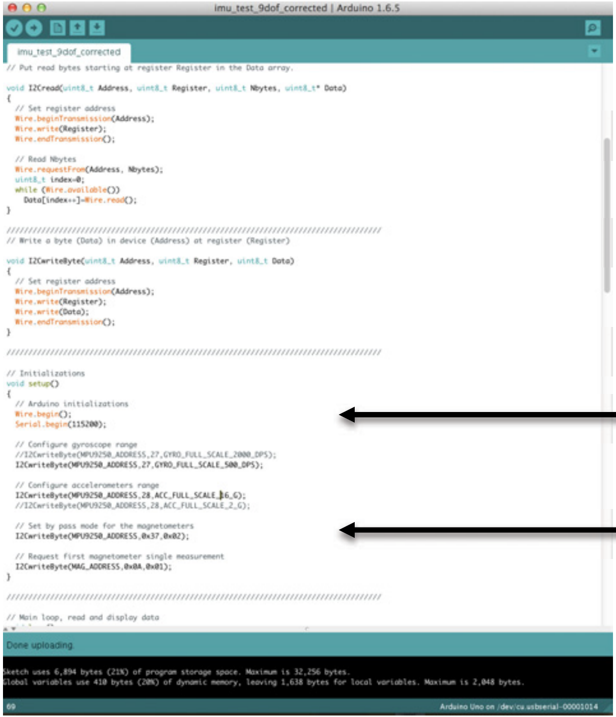
← connected to COM1 serial port

Introduction to Arduino

- need to stream data from Arduino to host PC
- use Wire library for all serial & I2C communication

- use node server to read from host PC and connect to JavaScript (see lab)

Introduction to Arduino



← read from I2C (connected to IMU)

← write to I2C (connected to IMU)

← <u>setup</u> function = one time initialization

← open serial connection to communicate with host PC

← set registers to configure IMU

Read Serial Data in Windows

- serial ports called COMx (USB serial usually COM3-COM7)
  1. establish connection to correct COM port (choose appropriate baud rate)
  2. read incoming data (in a thread)

Summary

- coordinate systems (world, body/sensor, inertial, transforms)
- overview of inertial sensors: gyroscopes, accelerometers, and magnetometers
- gyro integration aka *dead reckoning*
- orientation tracking in *flatland*
- pitch & roll from accelerometer
- overview of VRduino

Next Lecture

- quaternions and rotations with quaternions
- 6 DOF sensor fusion with quaternions & complementary filtering

Must read: course notes on IMUs!

Additional Information

- D. Sachs “Sensor Fusion on Android Devices: A Revolution in Motion Processing”, Google Tech Talks 2010, Video on youtube.com (https://www.youtube.com/watch?v=C7JQ7Rpwn2k)

- S. LaValle, A. Yershova, M. Katsev, M. Antonov “Head Tracking for the Oculus Rift”, Proc. ICRA 2014

- http://www.chrobotics.com/library