Section 7
EE278: Introduction to Statistical Signal Processing (Fall 2020)

Monday, Nov 2, 2020 - 3 to 4 pm

1. Kalman filter for location tracking

Consider a truck on frictionless, straight rails [1]. Initially, the truck is stationary at location $L_0 = 0$ and moving with velocity $V_0 = 1$, but it is buffeted by random uncontrolled forces. We measure the position of the truck every $\Delta t = 1$ seconds, but these measurements are imprecise; we want to maintain a model of the truck's location L_t and its velocity V_t.

Specifically, assuming at time $t = 0$, the initial state of the truck is $L_0 = 0$, $V_0 = 1$. Between $t-1$ and t, the velocity is subject to a constant acceleration $A_{t-1} \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma_a^2)$. Also at time t, we take a noisy observation $Y_t = L_t + Z_t$, where $Z_t \stackrel{i.i.d.}{\sim} \mathcal{N}(0, \sigma_z^2)$.

Formulate this problem as a vector Kalman filter estimation problem, with $\mathbf{X}_t = [L_t, V_t]^T$.

2. Kalman filter with fading coefficient

Consider the scalar Kalman filter specified by $X_1 \sim \mathcal{N}(\bar{X}, \sigma_{X_1}^2)$, and

$$X_n = \alpha X_{n-1} + W_n \text{ for } n = 2, 3, ...$$
$$Y_n = h X_n + Z_n \text{ for } n = 1, 2, 3, ...$$

where $W_n \sim \mathcal{N}(0, \sigma_w^2)$ and $Z_n \sim \mathcal{N}(0, \sigma_z^2)$ are IID and independent each other and X_1. How do the following things change from the ones derived in class, in the presence of h?

(a) The estimate $\widehat{X}_1(y_1)$ and its mean squared error, i.e. v_1^2

(b) The estimate $\widehat{X}_2(y_1, y_2)$ and its mean squared error, i.e. v_2^2

[1] Example taken from `https://en.wikipedia.org/wiki/Kalman_filter#Example_application`