Section 8
EE278: Introduction to Statistical Signal Processing (Fall 2020)

Monday, Nov 9, 2020 - 3 to 4 pm

1. Stationary Processes.

For each random process, sketch a few sample paths, and compute the mean function and covariance function. State whether each process is stationary or not. Each process is defined for $n = 0, 1, 2, 3, \ldots$.

1. X_n are i.i.d. $\mathcal{N}(0,1)$ random variables.
2. $X_n = A$ where $A \sim \mathcal{N}(0,1)$.
3. $X_n = nA$ where $A \sim \mathcal{N}(0,1)$.
4. $X_n = \cos(2\pi n)$.
5. $X_n = \cos(\pi n)$.
6. $X_n = \cos(\pi n + \Theta)$ where Θ is a $\mathrm{U}[0, 2\pi)$ random variable.

2. AM modulation.

Consider the discrete AM modulated random process

$$Y_n = A_n \cos(\pi n + \Theta)$$

for $n \geq 1$, where the amplitude A_n is a stationary zero-mean random process with covariance function $K_A(n_1, n_2) = e^{-|n_1 - n_2|}$, the phase Θ is a $\mathrm{U}[0, 2\pi)$ random variable, and A_n and Θ are independent.

1. Find the mean function of Y_n. Does it depend on n?
2. Find the covariance function $K_Y(n_1, n_2)$. Does it depend only on $|n_1 - n_2|$?
3. Is Y_n stationary?

The following trigonometric identities may be useful:

$$\cos(A)\cos(B) = \frac{1}{2}\left(\cos(A - B) + \cos(A + B)\right)$$
$$\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$