PERFORMANCE ANALYSIS OF ATM SWITCHING ARCHITECTURES

The goal of the ATM switch design is to minimize the packet loss and the packet delay under a variety of input traffic patterns using minimal hardware. We will analyze the probability of packet loss for certain switch architectures under a set of assumptions.

The probability of packet loss is defined as the ratio of the average number of packets lost to the average number of packets arriving at the switch. A related parameter of interest is the normalized throughput which is the ratio of average number of packets accepted by the switch to the average number of packets arriving at the switch. These two parameters are related as follows:

\[
\text{Normalized throughput} = 1 - \Pr\{\text{packet loss}\}
\]

For a given switch architecture, the packet loss probability is strongly dependent on the input traffic pattern. A simple traffic pattern is the uniform independent traffic pattern which is characterized by the following two assumptions:

1. The packet arrival process at each input is a Bernoulli process, with parameter \( p \), independent of all other inputs.
2. The destination request of each packet is uniformly chosen among all outputs, independently for all arriving packets.

Under these assumptions, the input load (which is defined as the average fraction of the inputs with packet arrivals in a given packet slot) is also equal to \( p \).

**Crossbar and Batcher-banyan networks**

In these networks, packet loss is due to output conflicts only.

Let \( \alpha = \Pr\{\text{a particular source \textquoteleft}s\textquoteright requests a particular destination \textquoteleft d\textquoteright}\} = p/N \)

where \( p = \text{input rate} \) and \( N = \text{number of inputs/outputs} \).

Also let \( \beta(k) = \Pr\{\text{exactly \(k\) sources request destination \textquoteleft d\textquoteright}\} \)

\[
\beta(k) = \binom{N}{k} \alpha^k (1-\alpha)^{N-k}
\]

The output rate is:

\[
\gamma = \Pr\{\text{destination \textquoteleft d\textquoteright is connected}\} = 1 - \Pr\{\text{no sources request \textquoteleft d\textquoteright}\}
\]

\[
= 1 - \beta(0) = 1 - (1-\alpha)^N = 1 - (1-p/N)^N
\]

Normalized throughput = \[
\frac{\text{Average \# of packets accepted}}{\text{Average \# of packets arriving}}
\]

\[
= \frac{N\gamma}{Np} = \frac{\text{Output rate}}{\text{Input rate}}
\]

\[
= \frac{1}{p} \left\{ 1 - \left( 1 - \frac{p}{N} \right)^N \right\}
\]

If \( p = 1 \)

\[
\lim_{N \to \infty} \frac{1}{p} \left\{ 1 - \left( 1 - \frac{p}{N} \right)^N \right\} = 1 - e^{-1} = 0.632
\]

From this, the packet loss probability may be obtained as:

\[
\Pr\{\text{packet loss}\} = 1 - \text{Normalized throughput}
\]
"Banyan" networks

Consider an arbitrary 2x2 switching element in the first stage of the network

\[
\begin{array}{c}
X^i \\
X^{i+1}
\end{array}
\begin{array}{c}
2x2
\end{array}
\begin{array}{c}
Y^i \\
Y^{i+1}
\end{array}
\]

Let \(X^i, X^{i+1}, Y^i, Y^{i+1}\) be the indicator functions; i.e. \(\Pr\{X^i = 1\} = \Pr\{X^{i+1} = 1\} = p\), the input rate. We want to evaluate the output rates, \(\Pr\{Y^i = 1\}\) and \(\Pr\{Y^{i+1} = 1\}\). Now,

\[\Pr\{Y^i = 1, Y^{i+1} = 1\} = \Pr\{\text{both inputs are active}\} \text{ and } \Pr\{\text{msb’s of destination tags differ}\}
= (p^2)(1/2) = p^2/2\]

\[\Pr\{Y^i = 0, Y^{i+1} = 0\} = \Pr\{\text{both inputs are inactive}\} = (1-p)^2\]

By symmetry,

\[\Pr\{Y^i = 0, Y^{i+1} = 1\} = \Pr\{Y^i = 1, Y^{i+1} = 0\} = \frac{1}{2}\left(1 - \left[\frac{p^2}{2} + (1-p)^2\right]\right) = p - \frac{3p^2}{4}\]

Hence,

\[\Pr\{Y^i = 1\} = \Pr\{Y^{i+1} = 1\} = \frac{p^2}{2} + \left[p - \frac{3p^2}{4}\right] = p - \frac{p^2}{4}\]

Note that \(Y^i\) and \(Y^{i+1}\) are dependent, but independent of all other such pairs defined by the outputs of other switches in the first stage. The two inputs to a typical second stage switch arrive from different first stage switches and are, therefore, independent.

The structure of the banyan (or omega etc.) network is such that this independence assumption holds good for the inputs at any 2x2 switching element. Furthermore, since the destinations are uniformly distributed, so are the destination bits. Thus the requests at any 2x2 switching element are independent and uniformly distributed over the destinations.

Let

\[p_k = \Pr\{\text{active packet at an input link of the } k^{\text{th}} \text{ stage}\}, \ k = 0, 1, \ldots, n-1 \text{ with } N = 2^n\]

then,

\[p_{k+1} = p_k - \frac{p_k^2}{4}\]

with,

\[p_0 = p = \text{input rate, and}\]

\[p_n \text{ yields the output rate.}\]

Hence,

\[\text{Normalized throughput} = \frac{\text{output rate}}{\text{input rate}} = \frac{p_n}{p}\]

and

\[\Pr\{\text{packet loss}\} = 1 - \left(\frac{p_n}{p}\right)\]