EE356 Elementary Plasma Physics
Inan
Spring 2002

HOMEWORK ASSIGNMENT #2
(due Friday, April 19th)

1. Short Review Questions:

a. Mobility and conductivity. An electric field $E$ causes electrons to drift at a velocity $v = \mu E$, while the protons remain stationary. Determine the mobility, current density, and conductivity of the electrons in terms of $N_e$ and $q_e$, i.e., the electron number density and charge respectively.


c. Constant velocity distribution. A plasma with density $N_0$ consists only of particles all of which drift in the $\hat{x}$ direction with a constant velocity $v_x$. Write down the distribution function $f(r, v, t)$ describing this plasma.

d. Magnetic heating of plasma. A 1-keV proton with $v_\parallel = 0$ in a uniform magnetic field $B = 0.1$ T is accelerated as $B$ is slowly increased to 1 T. The proton subsequently makes an elastic collision with a heavy particle and changes direction so that now $v_\perp = v_\parallel$. The $B$ field is then slowly decreased back to 0.1 T. What is the final energy of the proton?

2. Plasma distribution function. Measurements made on a plasma indicate that the electron velocities in the $\hat{x}$ direction are distributed in a Gaussian manner. Further measurements reveal the same trend for the velocities in the $\hat{y}$ and $\hat{z}$ directions, so, armed with EE356 street-smarts, you write down the velocity space distribution function as

$$f(v_x, v_y, v_z) = A e^{-\frac{mv_x^2}{2k_b T}} e^{-\frac{mv_y^2}{2k_b T}} e^{-\frac{mv_z^2}{2k_b T}}$$

(a) Noting that the plasma density is $N_0$, find $A$. (b) By suitably weighting your distribution function with particle energy (in a manner similar to Equation [1.1] of Lecture 1) find the average energy of the plasma.

3. Polarization drift. An electron gyrates in a constant and uniform $B$-field with $B = 0.1 \hat{z}$ T and $v_\parallel = 0$. A uniform electric field is slowly introduced into the system, with $E_1 = 0.1 \ t \ \hat{y}$ V/m. At $t = 0$, the guiding center of the electron is located at the origin. (a) Calculate the displacement of the guiding center of the electron due only to polarization drift at $t = 10$ seconds. (b) What is the displacement of the guiding center at $t = 10$ seconds due to $E \times B$ drift? (c) At $t = 20$ seconds, the electric field switches to $E_2 = E_1(10) - 0.1 \ t \ \hat{y}$ V/m. Repeat part (a) at $t = 20$ seconds and at $t = 30$ seconds. (d) Repeat part (b) at
At $t = 20$ seconds and at $t = 30$ seconds. (e) At $t = 30$ seconds, the electric field switches to $E_3 = E_2(30) + 0.1 \, t \, \hat{y} \, \text{V/m}$. Repeat part (a) and (b) at $t = 40$ seconds. (j) At $t = 10$ seconds, what was the work done by the field on each particle? (i.e., $\int q \cdot E \cdot dl$) and what is the kinetic energy of the $E \times B$ drift?

HINT: This exercise is meant to demonstrate that the slowly increasing electric field is the agent which imparts energy to the particle, which is then stored as kinetic energy in the $E \times B$ drift. In fact, it can be shown that the energy picked up due to the polarization drift defined in equation [5.6] of Lecture #5 Notes just suffices to cover the difference in zero-order electric drift energy, due to the change in the electric field.

4. **Conductivity tensor** Apply equation [4.17] from the Lecture #4 Notes to an electron under the influence of a time-harmonic electric field of $E = E_x \hat{x} + j E_y \hat{y}$ where $E_x, E_y > 0$. Find expressions for the particle velocity and sketch its trajectory for (a) $\omega_c \gg \omega$. Does the expression look familiar? (b) for $\omega_c \ll \omega$, and (c) for $\omega_c$ slightly larger than $\omega$. How would one analytically determine the velocity of the electron and its trajectory for a general time-varying electric field $E = E_x(t) \hat{x} + E_y(t) \hat{y}$?

5. **Magnetic pumping.** A plasma confined by an axial magnetic field $B_z$ is heated by magnetic pumping using collisional distribution of energy. For this purpose, $B_z$ is increased from $B_1$ to $B_2$ in a time $\Delta t_1$, maintained at $B_2$ for a time $\Delta t_2$, decreased from $B_2$ to $B_1$ in a time $\Delta t_1$ and maintained at $B_1$ for a time $\Delta t_2$. This process is repeated periodically. The time interval $\Delta t_1$ is large compared to the gyroperiod but short compared to the time required for the plasma to attain thermal equilibrium. The time duration $\Delta t_2$ is large compared to that required to establish thermal equilibrium. Show that for each cycle of variation of the magnetic field as described, the plasma temperature increases by the factor

$$\frac{[2 + 5B_2/B_1 + 2(B_2/B_1)^2]}{9B_2/B_1}$$