EE359 Discussion Session 2
Statistical Fading Models

January 22, 2020

Admin

- Please direct questions about grades to myself or Andrea and not the grader

A note on units

- dB is dimensionless
- dBm is relative to 1mW
- dBW is relative to 1 W

The time varying channel model

Basic idea

The channel is linear but may *not* be time invariant

Modelling linear time varying channel

$$r(t) = Re\left\{\left(\int_{-\infty}^{\infty} c(\tau \quad)u(t-\tau)d\tau\right)e^{j2\pi f_c t}\right\}$$

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- $c(\tau, t)$ is channel response at time t to an impulse at time $t - \tau$
- Multipath model is when

$$c(\tau, t) = \sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \delta(\tau - \tau_n(t))$$

Multipath model in more detail

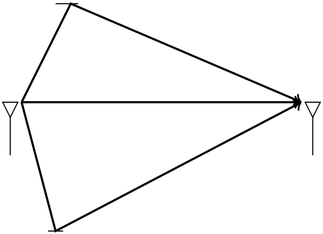

Figure: Multipath channel

Received signal

$$r(t) = Re\left\{ \left(\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} u(t - \tau_n(t)) \right) e^{j2\pi f_c t} \right\}$$

- $N(t)$ is possibly random number of multipath components

Multipath model in more detail

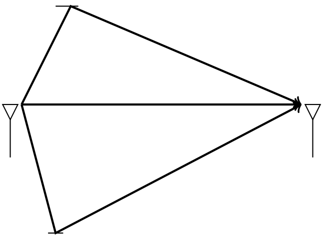

Figure: Multipath channel

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- $\tau_n(t)$ is the delay of the n^{th} multipath component

Multipath model in more detail

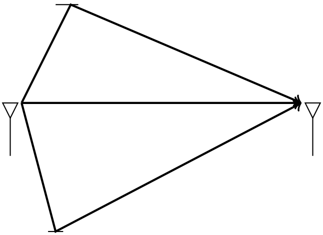

Figure: Multipath channel

Received signal

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- $\phi_n(t) = 2\pi f_c \tau_n - \int_t 2\pi f_D(\tau) d\tau - \phi_0$ is the phase due to time delay and doppler in the n^{th} multipath component

Most Modern Wireless Channels

Block Fading model $\longrightarrow$ approximate time-invariance.

$$r(t) = \text{Re}\left\{ \left(\int_{-\infty}^{\infty} c(\tau)u(t-\tau)d\tau \right) e^{j2\pi f_c t} \right\}$$

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Assumptions:

- 'Under spread': period of Doppler spread is small compaired to TX block
- Movement of scatterers, terminals small within TX block

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Implication:

- Only care about statistics block-to-block

Narrowband approximation

A measure of the "spread" of $\tau_n(t)$

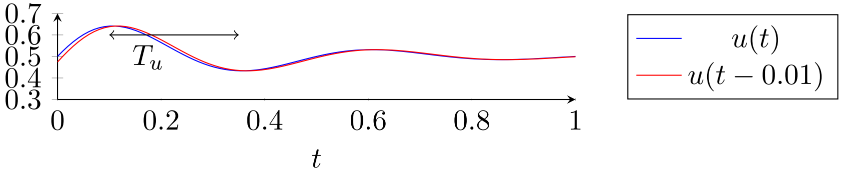
Non random: $T_m = \max_n \tau_n(t) - \min_n \tau_n(t)$
Random: $T_m = \mathrm{stddev}(\tau_1, ..., \tau_{N(t)})$

Idea: Narrowband assumption

If the "spread" T_m is such that

$$T_m \ll T_u,$$

the signals "overlap", i.e. $u(t-\tau_n) \approx u(t)$



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Question

- What happens if the above assumption is not true?
- How do you express the above in terms of the signal bandwidth (instead of T_u) ?

Some implications of narrowband assumption

- Signal suffers only scaling by a complex factor

$$Re\left\{ u(t) \left(\sum_{n=1}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \right) e^{j2\pi f_c t} \right\}$$

- If the above scaling factor is $r_I(t) + jr_Q(t)$, then the in-phase and the quadrature components are

$$r_I(t) = \sum_{n=1}^{N(t)} \alpha_n(t) \cos(\phi_n(t))$$

$$r_Q(t) = -\sum_{n=1}^{N(t)} \alpha_n(t) \sin(\phi_n(t))$$

Rayleigh fading

Observation so far

Both in-phase and quadrature components are zero mean gaussian and independent (according to model)

Some implications for $Z = r_I + jr_Q$

- The amplitude $|Z|$ is Rayleigh distributed

$$|Z| \sim \frac{2|Z|}{P_r} e^{\frac{-|Z|^2}{P_r}}$$

- The phase $\angle Z$ is uniform
- Known as *Rayleigh* fading

Ricean fading

Difference from Rayleigh

Either the in-phase or the quadrature component has non zero mean (i.e., it has a line of sight or LOS component)

Some implications

- The amplitude $|Z|$ follows a Ricean distribution

$$|Z| \sim \frac{|Z|}{\sigma^2} e^{-\frac{|Z|^2+s^2}{2\sigma^2}} I_0\left(\frac{|Z|s}{\sigma^2}\right)$$

  where $2\sigma^2$ is power in non LOS and s^2 is power in LOS
- Often specified by a K parameter where $K = \frac{s^2}{2\sigma^2}$

Nakagami fading

- Parameterized by received power P_r and m,i.e.

$$|Z| \sim \frac{2m^m|Z|^{2m-1}}{\Gamma(m)P_r^m}e^{\frac{-m|Z|^2}{P_r}} \ , m > 0.5$$

- Useful for deriving closed form BER expressions
- $m = 1$ corresponds to Rayleigh fading
- $m = \frac{(K+1)^2}{2K+1}$ is approximately Ricean