EE359 Discussion Session 8
Beamforming, Diversity-multiplexing tradeoff, MIMO receiver design, Multicarrier modulation

November 29, 2017

Outline

1. MIMO concepts
   - Beamforming
   - Diversity multiplexing tradeoff for point to point MIMO
2. MIMO Decoding
   - Linear Decoders
   - Sphere Decoding
3. Multicarrier modulation

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Brief recap of the notation

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$
$$\tilde{\mathbf{y}} = \mathbf{\Sigma}\tilde{\mathbf{x}} + \tilde{\mathbf{n}}$$

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathbf{H}} = \sum_i \sigma_i \mathbf{u}_i {\mathbf{v}^{\mathbf{H}}}_i$$
$$\mathbf{x} = \mathbf{V}\tilde{\mathbf{x}} \qquad \tilde{\mathbf{y}} = \mathbf{U}^{\mathbf{H}}\mathbf{y}$$

- N_t transmit antennas and N_r receive antennas
- Decomposition into parallel channels with perfect CSIT and CSIR
- $\sigma_1 > \sigma_2 > \dots$

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Beamforming

Idea

If CSIT available, simply transmit along vector with the largest singular value, i.e. make

$$\tilde{x} \in \mathbb{C} \quad \text{scalar - one value}$$

Some points

- Equivalent scalar channel $\tilde{y} = \mathbf{u^H H v}\tilde{x} + \tilde{n}_1$
- Maximizes SNR if $\mathbf{u}$ and $\mathbf{v}$ are first singular vectors
- Optimal only if other parallel channels are "weak" (low SNR)
- Any choice of $\mathbf{u}$ and $\mathbf{v}$ other than $\mathbf{u}_1$ and $\mathbf{v}_1$ is suboptimal

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The tradeoff

Setting

CSI (channel state info) known at receiver but *is unknown* at transmitter, finite blocklengths.

Intuition

Antennas can be used for higher reliability (diversity) or rate (multiplexing)

Fineprint

- We assume i.i.d. complex normal entries for $\mathbf{H}$
- High SNR concept:
  - Multiplexing gain $r = \lim_{\mathsf{SNR}\to\infty} \frac{R(\mathsf{SNR})}{\log_2(\mathsf{SNR})}$
  - Diversity gain $d = \lim_{\mathsf{SNR}\to\infty} \frac{-\log P_e}{\log \mathsf{SNR}}$

The tradeoff

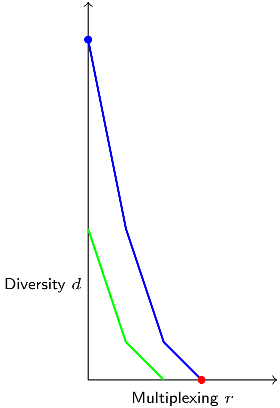


Figure: Blue curve for $N_t = 3, N_r = 3$, green for $N_t = 2, N_r = 2$

- Blue dot corresponds to low rate, high reliability transmission
- Red dot corresponds to high rate, low reliability transmission

Achievability

Any point on this tradeoff curve may be achieved in general by a suitable *space time code*

Outline

The optimal receiver

Idea

Maximum likelihood criterion: $\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathcal{X}}{\text{argmax}} \quad p(\mathbf{x}|\mathbf{H}, \mathbf{y})$

Some more details about ML decoder

- For i.i.d. Gaussian noise statistics and uniformly random MQAM signalling, $p(\mathbf{x}|\mathbf{H}, \mathbf{y}) \propto e^{-c||\mathbf{y}-\mathbf{H}\mathbf{x}||^2}$, so

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathcal{X}}{\text{argmax}} ||\mathbf{y} - \mathbf{H}\mathbf{x}||^2$$

Problem

An NP-hard combinatorial optimization problem.

Outline

Simple approximations: Zero forcing

Idea

Use matrix inversion

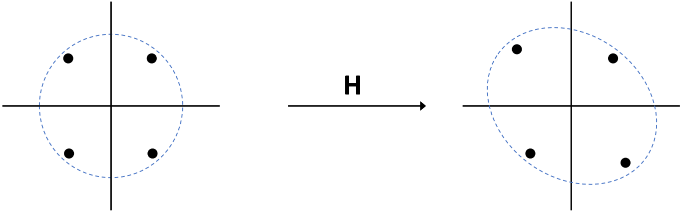
Math

$$\hat{\mathbf{x}} = \mathbf{H}^{\dagger}\mathbf{y} \text{ where } \mathbf{H}^{\dagger} = (\mathbf{H}^{\mathbf{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathbf{H}} \text{ if } \mathbf{H} \text{ is "tall"}$$

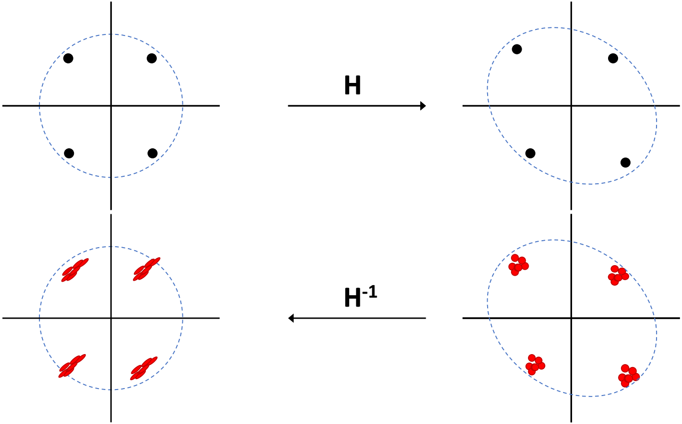
Some features

- Requires $O\left(N_t^3\right)$ operations. (Can't expect to do better than this unless H is sparse)
- Nearly optimal when condition number is close to 1. Poor performance for ill-conditioned channels.

Zero Forcing in Pictures



Zero Forcing in Pictures



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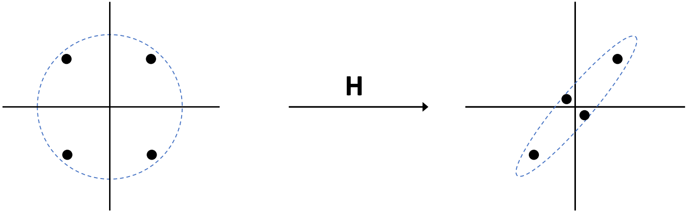

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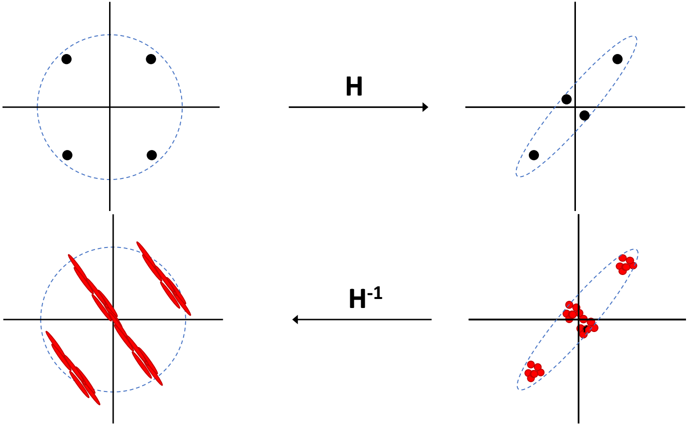


Simple approximations: Linear MMSE decoding

Idea

- Write estimate as an affine function of $\mathbf{y}$. Minimize expected squared error by choosing right affine function.
- Regular MMSE: Assume $\mathbf{x}$ to be i.i.d. multivariate Gaussian and compute optimal decoder (minimum expected mean squared error (MSE))

Math (assuming SNR $= 1/\sigma^2$)

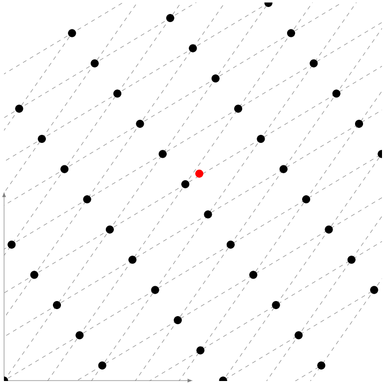
$$\hat{\mathbf{x}} = (\mathbf{H}^{\mathbf{H}}\mathbf{H} + \sigma^2\mathbf{I})^{-1}\mathbf{H}^{\mathbf{H}}\mathbf{y}$$

Some features

- Good complexity (similar to zero forcing)
- Less sensitive to ill-conditioned matrices
- In practice $\mathbf{x}$ is not Gaussian

Outline

An ML Algorithm: Enumeration



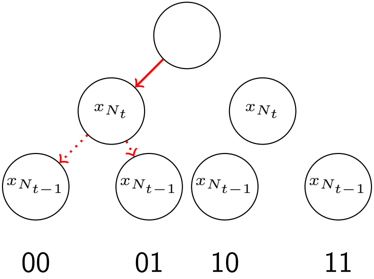
ML Decoding: Find closest point in ℓ_2 norm. How to search for close points?

Naïve approach

Check all M^n points, return closest

Enumeration

BPSK, $N_t = 2$



There is no reason two adjacent nodes are close!

Questions

- Is there are smart way to traverse this graph?
- What is our stopping criteria?
- Can we 'prune' nodes?

Better Enumeration: QR Decomposition

$$
\begin{aligned}
H &= QR \\
&= \begin{bmatrix} & & \\ & \text{Q} & \\ & & \end{bmatrix}
\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,n} \\ & \ddots & & \vdots \\ & \mathbf{0} & \ddots & \\ & & & r_{n,n} \end{bmatrix}
\end{aligned}
$$

New Basis

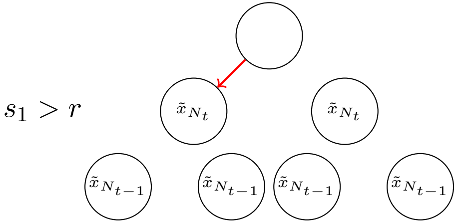
$$\tilde{x} = Rx, \quad \|y - Hx\|_2 = \|Q^H y - \tilde{x}\|_2$$

Notice: ℓ_2 norm in this basis can be considered element-wise: $x_n, \ldots, x_1$

'Partial objective': $s_m = \sum_{i=1}^{m} \left(\langle q_{n-i}, y\rangle - \tilde{x}_{n-i}\right)$

Sphere decoder

Prune branches below m if $s_m > r$



Sphere decoder

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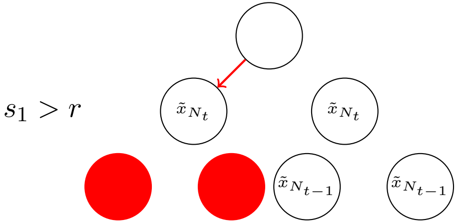
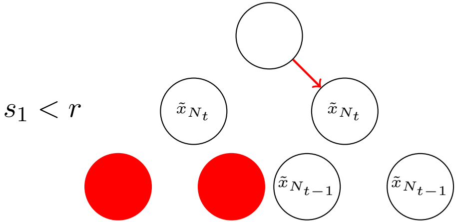


Sphere decoder

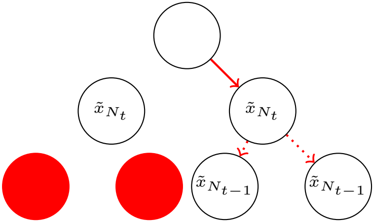
Prune branches below m if $s_m > r$



Sphere decoder

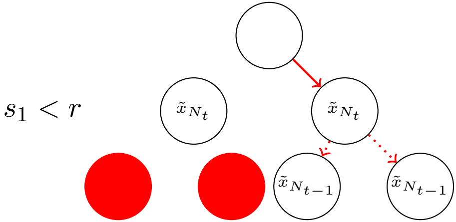
Prune branches below m if $s_m > r$

$s_1 < r$



Sphere decoder

Prune branches below m if $s_m > r$



Return minimum value of objective function at last depth.

Further notes

- If r is large enough, gives ML estimate
- If correct solution is pruned, declare error (erased symbol)
- Reducing r reduces complexity. Complexity also based on channel condition number and signal to noise ratio
- Further techniques exist improve enumeration (e.g. LLL algorithm)

Homework 7

Problem 5

Apply ML, Zero Forcing and MMSE decoder. Naïve implementation of ML is fine.

Problem 6

Simple exploration of sphere decoding. No implementation needed!

Outline

Intersymbol interference

Problem

Coherence bandwidth of channel is small, thus channel “spreads” wideband signal in time

Some common remedies

- Equalization/deconvolution/channel inversion
- Multicarrier modulation
- Spread spectrum

Multicarrier modulation

Idea

Split wideband (B) into N narrowband chunks each of bandwidth B/N, such that

$$B_n = B/N \leq B_c$$

Common approaches

- Frequency division multiplexing (FDM)
- Orthogonal FDM (OFDM)

Uses

- 4G LTE, Wifi use OFDM
- 2G standards (GSM) used FDM heavily

FDM

Idea

Pack a bunch of orthogonal basis functions in frequency domain, thereby creating *parallel channels*

Implementation issues

- Minimum carrier frequency separation with signal duration T_N is $1/T_N$
- Usually need a rolloff factor β and guard bands ϵ, thus effective occupancy $B_n = N(1 + \beta + \epsilon)/T_N$
- Need separate receiver hardware/modulation schemes at each carrier frequency

Homework 7

Problem 7

Two signals $s_i(t)$ and $s_j(t)$ over time T_N are orthogonal if $\int_{t=0}^{T_N} s_i(t)s_j(t)dt = 0$