The exam is open notes, open book, with no communication devices allowed (computers, cell phones, etc.).

1. Wireless Channels (30 Points)
   You are the world-famous secret agent 007.32. You have bugged the cell phone of the spy you are tracking to capture the channel characteristics between the phone and spy headquarters. In particular, you have found that the autocorrelation function of this channel at 900 MHz is
   \[ A_c(\tau, \Delta t) = \text{sinc}(W\Delta t) \frac{1}{T} e^{-\frac{\tau}{T}}, \quad \tau \geq 0 \]
   where \( W = 100 \) Hz and \( T = 10 \) microseconds.
   (a) Is the spy more likely to be indoors in his hotel room or outdoors scoping out his victim? Why?
   (b) What is the maximum speed at which the spy moves around?
   (c) Is the fading distribution of the spy’s channel closer to Rayleigh or to Rician (justify your answer)?
   (d) The spy is communicating with spy headquarters using BPSK modulation at a data rate of 1 Mbps sent over the channel. What is the appropriate performance metric for this system and why?
   (e) Over approximately what range of DPSK data rates will the spy maintain an error probability below \( 10^{-3} \)?
   (f) Over approximately what range of BPSK data rates will the spy maintain an error probability below \( 10^{-3} \) if his phone doesn’t compensate for ISI?
   (g) The spy is moving quickly between buildings in downtown Washington D.C. tracking Obama so his channel has shadowing plus Rayleigh fading. The distribution of the shadowing at the receiver is uniformly distributed between 0 and \( 2\bar{\gamma} \) (in linear units). What should \( \bar{\gamma} \) be such that the outage probability for DPSK modulation with an average error probability of \( 10^{-5} \) is five percent.

2. Kim and Kanye at Full Capacity (35 points)
   Kim Kardashian is planning the wedding of the century. She is going between the florist, caterer, and TV producers. Hence her cell phone experiences time-varying fading and AWGN with three channel states. Assuming a fixed transmit power \( P \) and a channel bandwidth of 20 MHz, Kim’s received SNR associated with each channel state is \( \gamma_1 = 0 \) dB, \( \gamma_2 = 3 \) dB, and \( \gamma_3 = 10 \) dB, respectively. The probabilities associated with each state (i.e. each location for Kim’s wedding planning) are \( p(\gamma_1) = .4, \ p(\gamma_2) = .3, \ p(\gamma_3) = .3 \). Kim’s cell phone was a high-tech present from Kanye, so both its transmitter and receiver have perfect instantaneous estimates of the channel.
   a) Find the optimal adaptive transmission strategy and associated Shannon capacity (in bps) of Kim’s channel assuming perfect instantaneous transmitter and receiver channel knowledge.
   b) Consider now a truncated channel inversion policy. Find the truncated channel inversion adaptive power policy that maximizes Kim’s outage capacity and find this maximum rate (in bps) and the associated outage probability.
   c) Kim is tired of her cell phone dying on her calls to her mom about how many cakes to order, so she switches the phone to minimum outage mode. Find the truncated channel inversion adaptive power policy that minimizes outage probability and find the associated outage capacity (in bps) and outage.
   d) Kanye buys Kim more spectrum as an engagement present. So the channel bandwidth is increased to 40 MHz but the transmit power must remain fixed at the constant value \( P \) due to wideband power amplifier limitations. Find the Shannon capacity in this case. Is it better for capacity to adapt power or use double the bandwidth (the answer is why the spectrum cost Kanye billions)?
3. You-Tube on the Cheap via Diversity (35 points)

You are designing a cell phone for poor graduate students and you want to make it both cheap (so they can still afford to eat) and high performance (so they can eat while watching high-quality You-Tube videos). You have a **brilliant idea**: why not combine the low-complexity (i.e. cheapness) of selection combining (SC) with the performance benefits of maximal-ratio combining (MRC). The idea is to design a diversity combiner that has \( N \) receiver branches with independent fading on each one but only \( K \) receivers, whereby the combiner chooses the \( K \) strongest signals from these \( N \) branches and then combines them using MRC. We denote the SNR on the \( i \)th branch as \( \gamma_i \) and the SNR for the combiner output as \( \gamma_\Sigma \). The outage probability is defined as usual: \( P_{out} = p(\gamma_\Sigma < \gamma_0) \) for some threshold \( \gamma_0 \). For this problem we will assume \( N = 3 \), and that the SNR on each of these 3 branches is i.i.d. and uniformly distributed between zero and ten (linear units, not dB).

a) Suppose \( K = 1 \). State the form of diversity-combining for this special case and find \( P_{out} \) for \( \gamma_0 = 1 \).

b) Suppose \( K = 3 \). State the form of diversity-combining for this special case and find \( P_{out} \) for \( \gamma_0 = 1 \).

c) Suppose \( K = 2 \). Show that \( p(\gamma_{max} < \gamma_0/2) \leq P_{out} = p(\gamma_\Sigma < \gamma_0) \leq p(\gamma_{min} < \gamma_0/2) \) for \( \gamma_{min} = \min_i(\gamma_i) \) and \( \gamma_{max} = \max_i(\gamma_i) \). Then find the \( P_{out} \) lower and upper bounds \( p(\gamma_{max} < \gamma_0/2) \) and \( p(\gamma_{min} < \gamma_0/2) \) for \( \gamma_0 = 1 \). Are these bounds tight or loose (justify your answer)?

d) Suppose graduate students complain that your phone display is too small to make out the video clearly. To make more room for a bigger screen, you remove one of the antennas at the receiver but you add another antenna at the transmitter (i.e. now you have 2 transmit and 2 receive antennas.) Using the Alamouti scheme for transmit diversity and selection combining at the receiver, if the channel gains \( |h_{ij}|^2 \) from transmitter antenna \( i \) to receiver antenna \( j \), \( i = 1, 2 \) are i.i.d. and uniformly distributed between zero and one (linear units) and \( E_s/N_0 = 20 \), find \( P_{out} \) for \( \gamma_0 = 1 \). Is this better or worse than when using MRC with 3 receive antennas?

**NOTE:** The exact value for \( P_{out} \) under general \( N \) and \( K \) values can be obtained using order statistics, but the calculations are rather messy, even for simple branch distributions such as the uniform distribution considered here. This form of diversity has several names, including generalized diversity combining, \( N \)-choose-\( K \) diversity combining, and hybrid selection/maximal-ratio combining.