EE359, Wireless Communications, Winter 2020
Homework 1: Due 4pm on Friday, January 17

Please refer to the homework page on the website (ee359.stanford.edu/homework) for guidelines.

1. (5 pts) **Wireless versus wired channel BER** (Problem 1-3): Fiber optic cable typically exhibits a probability of bit error of $P_b = 10^{-12}$. A form of wireless modulation, DPSK, has $P_b = \frac{1}{2\gamma}$ in some wireless channels, where γ is the average SNR. Find the average SNR required to achieve the same P_b in the wireless channel as in the fiber optic cable. Because of this extremely high required SNR, wireless channels typically have P_b much larger than 10^{-12}.

2. (15 pts) **Exploring the two-ray propagation model** (Problem 2-4): Consider Eq. 2.14 of the course reader
$$d_1 - d_0 = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}.$$
Assume that $G_{r_0} = G_{r_1} = 1$, $G_{t_0} = G_{t_1} = 1$, and that $R = -1$, that is, the reflected ray under goes a phase inversion upon reflection. Derive an approximate expression for the distance values below the critical distance d_c at which signal nulls occur.

3. (15 pts) **Narrowband approximation in two-ray model**: In this problem we examine the validity of the narrowband approximation in the two ray model. Consider the received signal for the two ray model (Equation 2.12), where $u(t) = e^{j2\pi f_s t}$, $G_{r_0} = G_{r_1} = 1$, $G_{t_0} = G_{t_1} = 1, d = 10$ km, $h_t = h_r = 50$ m.

   (a) (5 pts) Compute the delay spread τ. Assuming distance d is uncertain within $\pm 1\%$, what range does the delay spread lie in? (need not be exact, you can see this from a plot or by evaluating at some points)

   (b) (10 pts) Compute the phase difference $\Delta\phi$ between the signal components in the following cases:
      i. $f_s = \frac{1}{m}$ where m is the mid point of the range identified above
      ii. $f_s = \frac{1}{2m}$
      iii. $f_s = \frac{1}{100m}$

   In which of the above cases does the approximation $u(t) \approx u(t - \tau)$ hold best (why)?

   Hint: Phase "wraps" around at 2π and its integral multiples so the phase angles found in this problem should be reduced to lie in the range $(-\pi, \pi]$ — e.g. a phase difference of 4.5π should be reported as 0.5π.

4. (20 pts) **Estimating path loss parameters** (Problem 2-14): Table 1 lists a set of empirical path loss measurements. Assume $d_0 = 1$ m, and $f_c = 900$ MHz.

   (a) (10 pts) Find the parameters of a simplified path-loss model plus log-normal shadowing that best fit this data.

   Hint: The simplified path-loss model has two parameters that must be estimated based on the provided data: K and γ. Additionally, you must estimate the shadow-fading variance: $\sigma_{\phi_{dB}}$.

   (b) (5 pts) Find the path loss at 2 km based on this model.

Distance from transmitter (m)	$\frac{P_r}{P_t}$ (in dB)
5	-60
25	-80
65	-105
110	-115
400	-135
1000	-150

Table 1: Empirical Pathloss Measurements

(c) (5 pts) Find the outage probability at a distance d assuming the received power at d due to path loss alone is 10 dB above the required power for non-outage.

5. (15 pts) **Determining Cell Size** (Problem 2-15) Consider a cellular system operating at 900 MHz where propagation follows free space path loss with variations about this path from log-normal shadowing with $\sigma = 6$ dB. Suppose that for acceptable voice quality a signal-to-noise power ratio of 15 dB is required at the mobile. Assume the base station transmits at 1 W and its antenna has a 3 dB gain. There is no antenna gain at the mobile and the receiver noise in the bandwidth of interest is -40 dBm. Find the maximum cell size so that a mobile on the cell boundary will have acceptable voice quality 90% of the time.

6. (15 pts) **Ten-ray path loss model** (Problems 2-23, 2-24): Consider the ten-ray model described in Section 2.8.1 of the text.

   (a) (5 pts) What average power falloff with distance do you expect for the ten-ray model? Why?

   (b) (10 pts) For the ten-ray model, assume that the transmitter and receiver are at the same height in the middle of a street of width 20 m. The transmitter-receiver separation is 500 m. The transmitter is very close to the street (i.e. the height is very low). Find the delay spread for this model.

7. (15 pts) **Path loss with mmWave** (Problem 2-27) Consider a wireless link operating outdoors over 200m in the mmWave communication band. We consider the simplified path loss model with a path loss exponent $\gamma = 2$ and reference distance for antenna far field $d_0 = 1$m. Without any additional attenuation caused by atmospheric and rain conditions, $K = 1$. As described by Ostrometzky and Messer [1], attenuation due to rain is given by

$$K_{\text{rain}} = 0.95R^{0.77},$$

   where K_{rain} is the rain attenuation in dB per km and R is the rain-rate in mm/Hr. Fig. 1 on the next page from Rappaport et al [2], displays atmospheric attenuation.

   (a) (5 pts) What is the received signal power due to path loss and oxygen absorption when communication occurs at 60 and 80 GHz bands assuming there is no rain? The transmit power is $1W$.

   (b) (10 pts) Consider now only the 80 GHz link. Assume a day where it is dry at 8 am, there is a heavy drizzle of 2.5mm/Hr at 12 pm and a heavy downpour of 50mm/Hr at 5 pm. What is the

[1] Ostrometzky, Jonatan, and Hagit Messer. "Accumulated rainfall estimation using maximum attenuation of microwave radio signal." *Sensor Array and Multichannel Signal Processing Workshop (SAM), 2014 IEEE 8th.* IEEE, 2014.

[2] Rappaport, Theodore S., et al. "Millimeter wave mobile communications for 5G cellular: It will work!." *Access, IEEE* 1 (2013): 335-349.

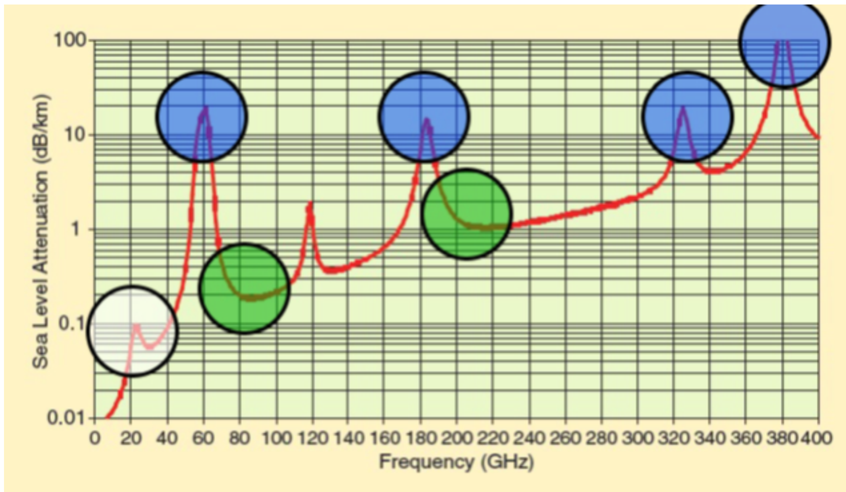

Figure 1: Atmospheric attenuation for question 7.

required transmit power at 8 am, 12 pm and 5 pm if we desire a received signal power of −50dBm at each of these time instances.