1. (10 pts) Outage probability and average probability of error (Problems 6-10, 6-11):
   (a) (5 pts) Consider a cellular system at 900 MHz with a transmission rate of 64 kbps and multipath fading. Explain which performance metric—average probability of error or outage probability—is more appropriate (and why) for user speeds of 1 mph, 10 mph, and 100 mph.
   (b) (5 pts) Derive the expression for the moment generating function for SNR in Rayleigh fading.

2. (15 pts) QoS in a cell (Problem 6-17): Consider a cellular system with circular cells of radius 100 meters. Assume that propagation follows the simplified path-loss model with $K = 1$, $d_0 = 1$ m, and $\gamma = 3$. Assume the signal experiences (in addition to path loss) log-normal shadowing with $\sigma_{\psi,B} = 4$ as well as Rayleigh fading. The transmit power at the base station is $P_t = 100$ mW, the system bandwidth is $B = 30$ kHz, and the noise PSD $N_0/2$ has $N_0 = 10^{-14}$ W/Hz. Assuming BPSK modulation, we want to find the cell coverage area (percentage of locations in the cell) where users have average $P_b$ of less than $10^{-4}$.
   (a) Find the received power due to path loss at the cell boundary.
   (b) Find the minimum average received power (due to path loss and shadowing) such that, with Rayleigh fading about this average, a BPSK modulated signal with this average received power at a given cell location has $\bar{P}_b < 10^{-4}$.
   (c) Given the propagation model for this system (simplified path loss, shadowing, and Rayleigh fading), find the percentage of locations in the cell where $\bar{P}_b < 10^{-4}$ under BPSK modulation.

3. (10 pts) Error floors (Problem 6-21*): Consider a wireless channel with average delay spread of 100 ns and a Doppler spread of 80 Hz. Given the error floors due to Doppler and ISI and assuming DPSK modulation in Rayleigh fading and uniform scattering—approximately what range of data rates can be transmitted over this channel with a BER of less than $10^{-4}$? Section 6.4 of the textbook has relevant content on error floors, equation (6.90) is especially useful.

4. (10 pts) Fading with diversity combining (Problem 7-1): Find the outage probability of QPSK modulation at $P_s = 10^{-3}$ for a Rayleigh fading channel with SC diversity for $M = 1$ (no diversity), $M = 2$, and $M = 3$. Assume branch SNRs of $\tilde{\gamma}_1 = 10$ dB, $\tilde{\gamma}_2 = 15$ dB, and $\tilde{\gamma}_3 = 20$ dB.

5. (10 pts) Average probability of error in fading (Problem 7-5): Derive the average probability of bit error for DPSK under SSC with i.i.d. Rayleigh fading on each branch as given by (7.16).
6. **MRC (Problems 7-7 & 7-16):**

(a) (5 pts) Show that the weights \(a_i\) maximizing \(\gamma \sum\) under receiver diversity with MRC are \(a_i^2 = r_i^2 / N_0\) for \(N_0/2\) the common noise PSD on each branch. Also show that, with these weights, \(\gamma \sum = \sum_i \gamma_i\).

(b) (10 pts) Consider a fading distribution \(p(\gamma)\) where \(\int_0^\infty p(\gamma)e^{-\gamma x} d\gamma = (1 + 2\gamma x)^{-1/2} \forall x > -1/2\gamma\). Find the average \(P_b\) for a BPSK modulated signal where the receiver has 2-branch diversity with MRC combining, and each branch has an average SNR of 10 dB and experiences independent fading with distribution \(p(\gamma)\).

7. **General diversity scheme (Midterm 2013):** Consider a diversity method which combines the low-complexity of SC with the performance benefits of MRC. The diversity combiner has \(N\) receiver branches with independent fading on each arm but only \(K\) receivers, whereby the combiner chooses the \(K\) strongest signals from the \(N\) branches and then combines them using MRC. The SNR on the \(i\)th branch is \(\gamma_i\) which is iid distributed uniformly between 0 and 1 in linear units. Denote the SNR for the combiner output as \(\gamma \Sigma\): the outage probability for threshold \(\gamma_0\) is \(P_{out} = p(\gamma \Sigma < \gamma_0)\). We will assume that there are \(N = 3\) branches and the threshold is \(\gamma_0 = 1\).

(a) Suppose \(K = 1\), state the form of diversity combining for this special case and compute \(P_{out}\).

(b) Repeat part (a) if \(K = 3\).

(c) Suppose \(K = 2\). Show that \(p(\gamma_{\text{max}} < \gamma_0/2) \leq P_{out} = p(\gamma \Sigma < \gamma_0) \leq p(\gamma_{\text{min}} < \gamma_0/2)\) for \(\gamma_{\text{min}} = \min_i \gamma_i, \gamma_{\text{max}} = \max_i \gamma_i\). Compute upper and lower bounds for \(P_{out}\).

8. **Diversity gain computation:** In this problem we explore the concept of diversity gain under both outage probability and average probability of error metrics, and show they are the same for extreme SNR values. For this question consider \(M\) independent Rayleigh fading random variables, corresponding to the channel between a single transmit antenna and \(M\) receiver antennas. The average SNR at each antenna is given by \(\text{SNR}\). You may assume DPSK transmission.

(a) (5 pts) Defining outage probability for \(M > 1\) as the probability of the instantaneous SNR at all the receiver antennas being less than a constant \(c\), compute \(-\frac{\log(\text{outage probability})}{\log(\text{SNR})}\) as \(\text{SNR} \to \infty\). Does it depend on \(c\) ?

(b) (5 pts) Now consider an alternative definition of diversity gain based on average error probability as

\[-\frac{\log(\text{average probability of error with MRC})}{\log(\text{SNR})}\]

For \(M > 1\), evaluate this as \(\text{SNR} \to \infty\).

(c) (5 pts) Now evaluate the diversity gain according to both the definitions as \(\text{SNR} \to 0\). Are these definitions of diversity gain more suitable at high \(\text{SNR}\) or low \(\text{SNR}\) to describe the gains from having \(M > 1\) independent fading paths?