EE359, Wireless Communications, Fall 2017
Homework 8 (100 pts)

Due: Sunday (December 10), 4 pm

Please refer to the homework page on the website [ee359.stanford.edu/homework](http://ee359.stanford.edu/homework) for guidelines.
Please note that we have a hard deadline (no late submissions please) this time because we will be uploading solutions soon after.

1. (15 pts) **Multicarrier transmission scheme to mitigate ISI** (Problem 12-4): Consider a data signal with a bandwidth of .5 MHz and a data rate of .5 Mbps. The signal is transmitted over a wireless channel with a delay spread of 10 µs.

   (a) If multicarrier modulation with nonoverlapping subchannels is used to mitigate the effects of ISI, approximately how many subcarriers are needed? What is the data rate and symbol time on each subcarrier? (We do not need to eliminate the ISI completely, so $T_s = T_m$ is sufficient for ISI mitigation.)

   Assume for the remainder of the problem that the average received SNR ($\gamma_s$) on the $n$th subcarrier is $1000/n$ (linear units) and that each subcarrier experiences fast Rayleigh fading (so ISI is completely eliminated).

   (b) Suppose BPSK modulation is used for each subcarrier. If a repetition code is used across all subcarriers (i.e., if a copy of each bit is sent over each subcarrier), then what is the BER after majority decoding? What is the data rate of the system?

   (c) Suppose you use adaptive loading (i.e., different constellations on each subcarrier) such that the average BER on each subcarrier does not exceed $10^{-3}$ (this is averaged over the fading distribution; do not assume that the transmitter and receiver adapt power or rate to the instantaneous fade values). Find the MQAM constellation (constellation size is a power of 2) that can be transmitted over each subcarrier while meeting this average BER target using the bound $BER \leq 2e^{-1.5\gamma_s/(M-1)}$. What is the total data rate of the system with adaptive loading?

2. (10 pts) **On an equivalent OFDM symbol** (Problem 12-10): Show that appending the all-zero prefix to an OFDM symbol and then adding in the tail of the received sequence, as shown in Figure 12.8, results in the same received sequence as with a cyclic prefix. Why might this be more desirable than adding a cyclic prefix?

3. (15 pts) **Constructing circulant matrix from Toeplitz matrix** (Problem 12-11 with two additional questions): Consider a discrete-time FIR channel with

   $$h[n] = 0.7\delta[n] + 0.5\delta[n-1] + 0.3\delta[n-3].$$

   Consider an OFDM system with $N = 8$ subchannels. Material in section 12.4.4 will be useful for this question.

   (a) Find the matrix $H$ corresponding to the matrix representation of the $y = Hx + \nu$ given in (12.23).
(b) Find the circulant convolution matrix $\tilde{H}$ corresponding to the matrix representation in (12.25), as well as its eigenvalue decomposition $\tilde{H} = MAM^H$.

(c) What are the flat fading channel gains associated with each subchannel in the representation of part (b)?

(d) Consider now a point to point link with 2 Tx antennas and 2 Rx antennas. The discrete time FIR channels are as follows:

- From Tx 1 to Rx 1: $h[n] = 0.7\delta[n] + 0.5\delta[n - 1] + 0.3\delta[n - 3]$
- From Tx 2 to Rx 1: $h[n] = 0.1\delta[n] - 0.7\delta[n - 1] + 0.6\delta[n - 2]$
- From Tx 1 to Rx 2: $h[n] = 0.8\delta[n] - 0.2\delta[n - 1]$
- From Tx 2 to Rx 2: $h[n] = -0.8\delta[n] - 0.1\delta[n - 1]$

What are the channel matrices from the transmitters to the receivers (please refer to eq (12.23))?

(e) How are the eigenvectors of the circulant forms of these matrices related to the $M$ above? (Note that you need not compute the eigendecomposition to answer this. Another hint: the eigenvectors of a general circulant matrix are the columns of a DFT matrix.)

4. (10 pts) **SIR for a spread signal** (Problem 13-1): In this problem we derive the SIR ratio (13.6) for a randomly spread signal with interference. The correlator output of the $i$th receiver branch in this system is given by (13.5) as

$$x_i = \int_0^T x(t)s_i(t)dt = \sum_{j=1}^N (s_{ij}^2 + I_{ij}s_{ij})$$

(a) Show that the conditional expectation of $x_i$, conditioned on the fact that $s_i(t)$ is transmitted over the channel, is equal to $E_s$.

(b) Assuming equiprobable signaling ($p(s_i(t)) = 1/M$), show that $E[x_i] = E_s/M$.

(c) Show that $\text{Var}[x_i - \sum_{j=1}^N s_{ij}^2|s_i(t)] = E_sE_J/N$.

(d) Show that (again with equiprobable signaling) $\text{Var}[x_i - \sum_{j=1}^N s_{ij}^2|s_i(t)]p(s_i(t)) = E_sE_J/NM$.

(e) The SIR is given by

$$\text{SIR} = \frac{E[x_i]^2}{\text{Var}[x_i - \sum_{j=1}^N s_{ij}^2|s_i(t)]p(s_i(t))}.$$  

Show that

$$\text{SIR} = \frac{E_s}{E_J} \cdot \frac{N}{M}.$$  

5. (15 pts) **On RAKE receivers** (Problem 13-13): This problem illustrates the benefits of RAKE receivers and the optimal choice of multipath components for combining when the receiver complexity is limited. Consider a multipath channel with impulse response

$$h(t) = \alpha_0\delta(t) + \alpha_1\delta(t - \tau_1) + \alpha_2\delta(t - \tau_2).$$

The $\alpha_i$ are Rayleigh fading coefficients, but their expected power varies (because of shad- owing) such that $E[\alpha_0^2] = 5$ with probability .5 and 10 with probability .5, $E[\alpha_1^2] = 0$ with probability .5 and 20 with probability .5, and $E[\alpha_2^2] = 5$ with probability .75 and 10 with probability .25 (all units are linear). The transmit power and noise power are such that a spread-spectrum receiver locked to the $i$th multipath component will have an SNR of $\alpha_i^2$ in the absence of the other multipath components. Assume spreading codes with autocorrelation equal to a delta function are used which implies that there is no interference from other multipath components when a spread-spectrum receiver locks on to a particular multipath component.
(a) What is the outage probability of DPSK modulation at an instantaneous $P_b = 10^{-3}$ for a single-branch spread-spectrum receiver locked to the LOS path?

(b) Find the outage probability of DPSK modulation at an instantaneous $P_b = 10^{-3}$ for a three-branch RAKE receiver, where each branch is locked to one of the multipath components and SC is used to combine the paths.

(c) Suppose receiver complexity is limited such that only a two-branch RAKE with SC can be built. Find which two multipath components the RAKE should lock to in order to minimize the outage probability of DPSK modulation at $P_b = 10^{-3}$, and then find this minimum outage probability.

6. (10 pts) **Single cell capacity with TDMA, FDMA and CDMA:** Consider an uplink channel with $B$ MHz, shared by $K$ users. Consider a TDMA system, an FDMA system, and a spread spectrum system, with a spreading factor $G = 128$.

Assume that system uses DPSK modulation and the required average signal to interference noise ratio is such that in AWGN, the BER is $2.27 \times 10^{-5}$. The transmit power of each user is such that the received average SNR is 50 linear units (only with additive noise, without any interference).

(a) (1 pt) Compute the required average SNR to meet the performance requirements assuming no fading.

(b) (2 pts) Assume that we have FDMA. Each user has an information signal bandwidth of $B/128$ with zero guard band requirement. How many users can the uplink support? Is the number same as that with TDMA?

(c) (2 pts) In practice, both TDMA and FDMA systems require some “guard bands” to compensate for imperfect filters, adjacent channel interference or spectral spreading due to Doppler in FDMA systems; and synchronization and multipath effects in TDMA systems. You may want to review Sections 14.2.1 and 14.2.2 in the textbook for a detailed description. Considering guard band requirements of 10% (typical for GSM in 2G cellular, in LTE this number can be much lower), by how much does the number of supported users go down?

(d) (2 pts) How many users can be supported with CDMA? By what factor is it lower than the corresponding number for FDMA?

(e) (2 pts) Suppose you have an interfering user in a different cell arriving at the base station with a received average SNR of 5 dB. How will this impact the average SINR in each of the above systems (assume that all the systems support the largest number of users in the current cell)?

(f) (1 pt) Explain how this answer might have impacted the decision to use CDMA in 2nd generation (IS-95) and 3rd generation cellular (WCDMA) systems.

7. (15 pts) **Degradation of WiFi** In a coffee shop, there are $M$ users communicating with a WiFi access point. At every time slot, a user can communicate over any one of $N$ different frequency slots.

(a) If the access point performs centralized control, it can allocate frequency slots to users at each point in time. What is the expected number of users that are served at any time instant?

(b) WiFi uses decentralized random access protocols. Here, each user chooses a frequency slot at random. If a frequency slot is picked by a single user, it can be used for communication. Otherwise, there are collisions. Find the expected number of users that are served at any time instant. Evaluate this expression for $N = 10$ and $M = 3, 10$ and 30.

*Hint: Use linearity of expectations*

(c) What fraction of users are served when $M = kN$ for $k \geq 1$ and $N$ is large?

8. (10 pts) **Cellular network** (Problem 15-11) Consider a one-dimensional cellular system deployed along a highway. The system has square cells of length $2R = 2$ km as shown in Fig. 1. This problem focuses on downlink transmission from base stations to mobiles. Assume that each cell has two mobiles.
located as shown in the figure, so that the mobiles in each cell have the exact same location relative to their respective base stations. Assume a total transmit power at each base station of \( P_t = 5 \text{ W} \), which is evenly divided between the two users in its cell. The total system bandwidth is 100 kHz, and the noise power spectral density at each receiver is \( 10^{-16} \text{ W/Hz} \). The propagation follows the model \( P_r = 100 P_t / d^3 \). All interference should be treated as AWGN, and interference outside the first ring of interfering cells can be neglected. The system uses a frequency-division strategy, with the bandwidth allocated to each base station evenly divided between the two users in its cell. Neglecting and fading or shadowing, and assuming user rates are equal to the AWGN Shannon capacity of their links, compute the spectral efficiency of each cell when frequencies are reused every other cell (reuse distance 2).