The filter outputs then correspond, respectively, to the in-phase and quadrature components of the narrowband fading process with PSDs $S_{r_I}(f)$ and $S_{r_Q}(f)$. A similar procedure using discrete filters can be used to generate discrete fading processes. Most communication simulation packages (e.g. the Matlab communications toolbox) have standard modules that simulate narrowband fading based on this method. More details on this simulation method, as well as alternative methods, can be found in [1, 7, 9, 10].

We have now completed our model for the three characteristics of power versus distance exhibited in narrowband wireless channels. These characteristics are illustrated in Figure 3.8, adding narrowband fading to the single-slope path loss and log-normal shadowing models developed in Chapter 2. In this figure we see the signal power due to path loss decreasing at a slope of -10γ relative to $\log_1 0d/d_0$ for γ the path-loss exponent and d_0 the reference distance at which the path loss equals the constant K. The more rapid variations due to shadowing change on the order of the decorrelation distance X_c, and the very rapid variations due to multipath fading change on the order of half the signal wavelength. If we blow up a small segment of this figure over distances where path loss and shadowing are constant we obtain Figure 3.9, which plots the dB value of P_r/P_t versus linear distance $d = vt$ (not $\log$ distance). In this figure the average value of P_r/P_t is normalized to 0 dB. A mobile receiver traveling at fixed velocity v would experience stationary and ergodic received power variations over time as illustrated in this figure.

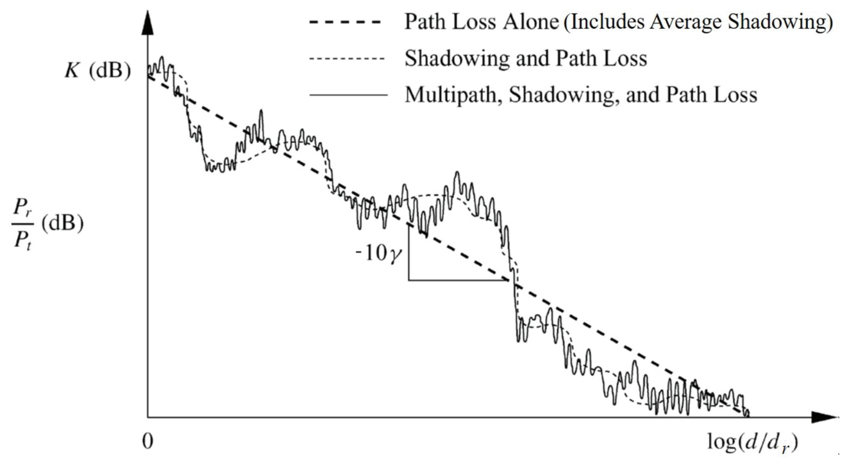

Figure 3.8: Combined single-slope path loss, log-normal shadowing, and narrowband fading.

3.2.2 Envelope and Power Distributions

We now consider the distribution of the envelope and power for the narrowband received signal $r(t) = r_I(t)\cos(2\pi f_c t) - r_Q(t)\sin(2\pi f_c t)$. It can be shown that, for any two Gaussian random variables X and Y, both with mean zero and equal variance σ^2, $Z = \sqrt{X^2 + Y^2}$ is Rayleigh distributed and Z^2 is exponentially distributed. We have seen that, for $\phi_i(t)$ uniformly distributed, r_I and r_Q are both zero-mean Gaussian random variables. If we assume a variance of $\sigma^2 = .5\sum_i \mathbf{E}[\alpha_i^2]$ for both in-phase and quadrature components, then the signal envelope $z(t) = |r(t)| = \sqrt{r_I^2(t) + r_Q^2(t)}$ is Rayleigh distributed with distribution

$$p_Z(z) = \frac{2z}{P_r}\exp\left[-\frac{z^2}{P_r}\right] = \frac{z}{\sigma^2}\exp\left[-\frac{z^2}{2\sigma^2}\right], \quad z \geq 0, \tag{3.33}$$

where $P_r = 2\sigma^2$ is the power of $z(t)$ which equals the power of $r(t)$.

We obtain the power distribution by making the change of variables $z^2(t) = |r(t)|^2$ in (3.33) to obtain

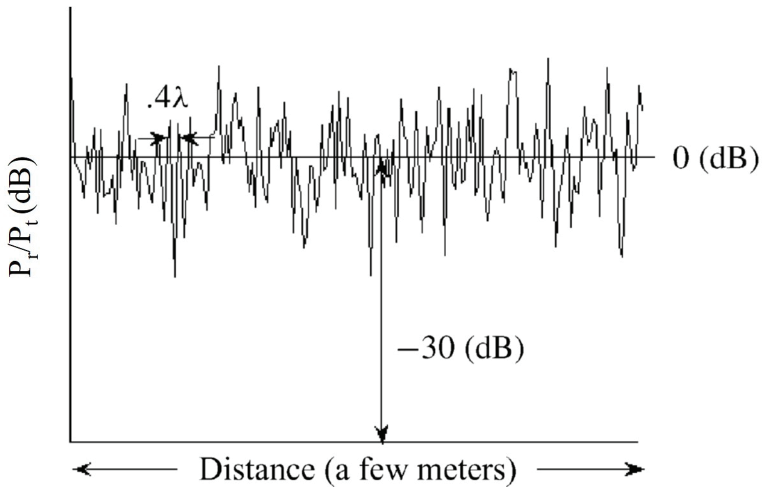

Figure 3.9: Narrowband fading.

$$p_{Z^2}(x) = \frac{1}{P_r} e^{-x/P_r} = \frac{1}{2\sigma^2} e^{-x/2\sigma^2}, \quad x \geq 0, \tag{3.34}$$

i.e. the power in $r(t)$ is exponentially distributed with mean $P_r = 2\sigma^2$. Thus, $r(t)$ has a Rayleigh-distributed amplitude and exponentially-distributed power with mean $P_r = 2\sigma^2$. The equivalent lowpass signal for $r(t)$ is given by $r_{LP}(t) = r_I(t) + jr_Q(t)$, which has the same power as $z(t)$ and phase $\theta = \arctan\,(r_Q(t)/r_I(t))$. For $r_I(t)$ and $r_Q(t)$ uncorrelated Gaussian random variables we can show that θ is uniformly distributed and independent of $|r_{LP}|$.

Example 3.9: Consider a channel with Rayleigh fading and average received power $P_r = 20$ dBm. Find the probability that the received power is below 10 dBm.

Solution: We have $P_r = 20\text{ dBm} = 100\text{ mW}$. We want to find the probability that $Z^2 < 10\text{ dBm} = 10\text{ mW}$. Thus

$$p(Z^2 < 10) = \int_0^{10} \frac{1}{100} e^{-x/100} dx = .095.$$

If the channel has a LOS component with a much larger signal power than the other multipath components, then $r_I(t)$ and $r_Q(t)$ are not zero-mean random processes. That is because the average signal power is dominated by the LOS component, with small fluctuations about this average due to constructive and destructive addition of the multipath components. In this scenario the received signal equals the superposition of a complex Gaussian component and a higher-power LOS component. The signal envelope in this case can be shown to have a Rician distribution [11] given by

$$p_Z(z) = \frac{z}{\sigma^2} \exp\left[\frac{-(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right), \quad z \geq 0, \tag{3.35}$$

where $2\sigma^2 = \sum_{i,i\neq 0} \mathbf{E}[\alpha_i^2]$ is the average power in the non-LOS multipath components and $s^2 = \alpha_0^2$ is the power in the LOS component. The function I_0 is the modified Bessel function of zeroth order. The average received power in the Rician fading is given by