EE359 – Lecture 14 Outline

- Announcements:
  - MT today, 2-4pm, 103 Hewlett, pizza afterwards
  - HW posted today, due next Friday
- Discrete-Rate Adaptive Modulation
- Introduction to MIMO Communications
- MIMO Channel Decomposition
- MIMO Channel Capacity
- MIMO Beamforming

Review of Last Lecture

- Introduction to adaptive modulation
  - Vary different parameters of modulation relative to fading
- Variable-rate variable-power MQAM
  - Maximize average throughput by changing rate and power
  - Optimal power adaptation is water-filling
  - Optimal rate adaptation:
    \[
    R = \max \left\{ \sum_{j=0}^{N} p_j M_j \beta_j \log_2 (M_j) : M_j \leq M(\gamma) \right\}
    \]
    equals capacity with effective power loss \( K = 1.5 \ln(5 \text{BER}) \).

Review continued

Constellation Restriction

- Restrict \( M_D(\gamma) \) to \( \{M_0, \ldots, M_N\} \).
- Let \( M(\gamma) = \gamma / \gamma_K^* \), where \( \gamma_K^* \) is optimized for max rate
- Set \( M_D(\gamma) \) to max \( M_j : M_j \leq M(\gamma) \) (conservative)
- Region boundaries are \( \gamma_j = M_j / \gamma_K^* \), \( j = 0, \ldots, N \)
- Power control maintains target BER

Power Adaptation and Average Rate

- Power adaptation: Fixed BER within each region
  - \( E_s / N_0 = (M-1) / K \)
  - Channel inversion within a region
  - Requires power increase when increasing \( M(\gamma) \)
    \[
    P_j(\gamma) = \begin{cases} 
    \frac{(M_j - 1) / (j K)}{M_j}, & \gamma_j \leq \gamma < \gamma_{j+1}, j > 0 \\
    0, & \gamma < \gamma_j 
    \end{cases}
    \]
- Average Rate
    \[
    \frac{R}{B} = \sum_{j=1}^{N} \log_2 M_j p(\gamma_j \leq \gamma < \gamma_{j+1})
    \]
- Practical Considerations (not covered in lecture):
  - Cannot update more than every 10-100 symbols
  - Estimation error/delay leads to irreducible error floor
Efficiency in Rayleigh Fading

- Spectral Efficiency (bps/Hz)
- Average SNR (dB)

Multiple Input Multiple Output (MIMO) Systems

- MIMO systems have multiple transmit and receiver antennas ($M_t$ at TX, $M_r$ at RX)
- Input described by vector $x$ of dimension $M_t$
- Output described by vector $y$ of dimension $M_r$
- Channel described by $M_r \times M_t$ matrix
- Design and capacity analysis depends on what is known about channel $H$ at TX and RX
  - If $H$ unknown at TX, requires vector modulation/demodulation

MIMO Decomposition

- Decompose channel through transmit precoding ($x=V\tilde{x}$) and receiver shaping ($y=UH\tilde{y}$)
- Leads to $R_1 \leq \min(M_r, M_t)$ independent channels with gain $\sigma_i$ ($i^{th}$ singular value of $H$) and AWGN
- Independent channels lead to simple capacity analysis and modulation/demodulation design

MIMO Fading Channel Capacity

- If channel $H$ known, waterfill over space (fixed power at each time instant) or space-time
- Without transmitter channel knowledge, capacity is based on an outage probability
  - $P_{out}$ is probability that channel capacity given the channel realization is below the transmission rate $C$
    \[
    P_{out} = \Pr \left( H : B \log_2 \det \left[ I_{M_r} + \frac{P}{M_t} HH^H \right] < C \right)
    \]
- Massive MIMO: random channel gains converge to static values:
  \[
  \lim_{M_r \to \infty} B \log_2 \det \left[ I_{M_r} + \frac{P}{M_t} HH^H \right] = B \log_2 \det \left[ I_{M_t} + \rho I_{M_r} \right] = M_r \log_2(1 + \rho)
  \]
## Main Points

- Discretizing the constellation size in adaptive MQMA results in negligible performance loss.
  - Constellations cannot be updated faster than 10s to 100s of symbol times: OK for most dopplers.
  - Estimation error/delay causes error floor
- MIMO systems exploit multiple antennas at both TX and RX for capacity and/or diversity gain
- With TX/RX CSI, decomposes into independent channels
- Capacity of MIMO systems
  - Static channel with TX/RX CSI: sum of capacity on each spatial dimension
  - Static channel without TX CSI: capacity metric is outage.
  - Fading channel with TX/RX CSI: water-fill power over space or space-time to achieve capacity
  - With only RX CSI, capacity metric is outage.
  - Massive MIMO: \( C = \min(M_r, M_t) \log(1+\rho) \)