Announcements:
- HW due Fri, new HW to be posted today
- End-of-Quarter schedule and possible bonus lecture
  - No lecture March 5, makeup lecture Monday March 2, 10:30-11:50, Gates B03
  - Advanced topics lecture? Could extend last class March 12 to 3:30 (i.e. 1:30-3:30)
  - Final exam: Tues March 17, 3:30-6:30pm, here. More details next week.

MIMO Receiver Design
- Linear Receivers, Sphere Decoder
- Other MIMO Design Issues
  - Space-time coding, adaptive techniques, limited feedback
- ISI Countermeasures
- Multicarrier Modulation

Review of Last Lecture (Cont’d)
- Beamforming: Scalar transmission
  - Principle vectors of $U$ and $V$ are weights: maximizes SNR
  - Diversity-Multiplexing Tradeoff: high SNR
    - Can use some antennas for diversity, some for capacity gain: $d'(r) = (M_r - r)(M_r - r)$
    - How antennas used depends on system metric
    - If requirements unmet, need more antennas

MIMO RX Design
- Optimal Receiver is ML: finds input vector $x$ most likely to have resulted in $y=Hx+n$, exponentially complex in $M_r$
- Linear Receivers: performs linear equalization $\hat{x} = A y$
  - Zero-Forcing ($A = H^H$: the Moore-Penrose pseudo inverse of $H$): 
    - If $H$ invertible, equals inverse, else $H^H = (H^H H)^{-1} H^H$; forces off-diagonal terms to zero
    - Minimum Mean Square Error ($A = H^H (H H^H + \lambda I)^{-1}$: $\lambda \propto 1/SNR$)
  - Sphere Decoder: Uses QR decomposition of $H$
    - Balances zero forcing against noise enhancement
- Sphere Decoder: Uses QR decomposition of $H$
  - Consider possibilities within sphere of transformed received symbol
    - If minimum distance symbol is within sphere, optimal, otherwise null is returned

Capacity of Fading & Massive MIMO Systems
- Static channel with perfect TX and RX CSI: waterfill over space
- Fading waterfill over space (based on instantaneous power constraint $P$)
- Fading over space-time (with average power constraint $P$)
- Without transmitter channel knowledge, capacity metric is based on an outage probability with respect to transmitted rate $C$
- Massive MIMO: $M \rightarrow \infty$ by random matrix theory
- $\lim_{M \rightarrow \infty} B \log_2 (\left| I_{M_r} + \frac{\rho}{M_r} H H^H \right|) = B \log_2 (\left| I_{M_r} + \rho I_{M_r} \right|) = M_r B \log_2 (1 + \rho)$

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Other MIMO Design Issues
Not covered in lecture/HW/exams

- **Space-time coding:**
  - Map symbols to both space and time via space-time block and convolutional codes.
  - For OFDM systems, codes are also mapped over frequency tones.

- **Adaptive techniques:**
  - Need fast and accurate channel estimation
  - Adapt the use of transmit/receive antennas
  - Adapting modulation and coding.

- **Limited feedback transmit precoding:**
  - Partial CSI introduces interference in parallel decomp: can use interference cancellation at RX
  - TX codebook design for quantized channel

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ISI Countermeasures

- **Equalization**
  - Signal processing at receiver to eliminate ISI
  - Complex at high data rates, performs poorly in fast-fading
  - Not used in state-of-the-art wireless systems

- **Multicarrier Modulation**
  - Break data stream into lower-rate substreams modulated onto narrowband flat-fading subchannels

- **Spread spectrum**
  - Superimpose a fast (wideband) spreading sequence on top of data sequence, allows resolution for combining or attenuation of multipath components.

- **Antenna techniques (Massive MIMO)**
  - (Highly) directional antennas reduce delay spread/ISI

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Multicarrier Modulation

- **Breaks data into N substreams**
- **Substream modulated onto separate carriers**
  - Substream passband BW is B/N for B total BW
  - B/N<Bc implies flat fading on each subcarrier (no ISI)

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Overlapping Substreams

- **Can have completely separate subchannels**
  - Required passband bandwidth is B.

- **OFDM overlaps sub streams**
  - Substreams (symbol time T_N) separated in RX
  - Minimum substream separation is 1/T_N for rectangular pulses
  - Total required bandwidth is B/2
Main Points

- MIMO RX design trades complexity for performance
  - ML detector optimal - exponentially complex
  - DF receivers prone to error propagation
  - Sphere decoders allow performance tradeoff via radius

- Other MIMO design issues include space-time coding, adaptation, codebooks for limited feedback

- ISI mitigated through equalization, multicarrier modulation (MCM) or spread spectrum
  - Today, equalizers often too complex or can't track channel.
  - MCM splits channel into NB flat fading subchannels
  - Can overlap subcarriers to preserve bandwidth