Shadowing, Combined Path Loss/Shadowing, Data Model Parameters, Statistical Multipath Model

Lecture Outline

- Log Normal Shadowing
- Combined Path Loss and Shadowing
- Outage Probability
- Model Parameters from Empirical Data
- Statistical Multipath Model

1. Log-normal Shadowing:
   - Statistical model for variations in the received signal amplitude due to blockage.
   - The received signal power with the combined effect of path loss (power falloff model) and shadowing is, in dB, given by
     \[ P_r(\text{dB}) = P_t(\text{dB}) + 10 \log_{10} K - 10 \log_{10}(d/d_r) - \psi(\text{dB}). \]
   - Empirical measurements support the log-normal distribution for \( \psi \):
     \[ p(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi dB}} \exp \left[ -\frac{(10 \log_{10} \psi - \mu_{\psi dB})^2}{2\sigma_{\psi dB}^2} \right], \quad \psi > 0, \]
     where \( \xi = 10/\ln 10, \mu_{\psi dB} \) is the mean of \( \psi_{dB} = 10 \log_{10} \psi \) in dB and \( \sigma_{\psi dB} \) is the standard deviation of \( \psi_{dB} \), also in dB.
   - With a change of variables, setting \( \psi_{dB} = 10 \log_{10} \psi \), we get
     \[ p(\psi_{dB}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi dB}} \exp \left[ -\frac{(\psi_{dB} - \mu_{\psi dB})^2}{2\sigma_{\psi dB}^2} \right], \quad -\infty < \psi_{dB} < \infty. \]
   - This empirical distribution can be justified by a CLT argument.
   - The autocorrelation based on measurements follows an autoregressive model:
     \[ A_{\psi}(\delta) = \sigma_{\psi dB}^2 e^{-\delta/X_c} = \sigma_{\psi dB}^2 e^{-\mu_{\psi dB} e^{-\delta/X_c}}, \]
     where \( X_c \) is the decorrelation distance, which depends on the environment.

2. Combined Path Loss and Shadowing
   - Linear Model:
     \[ \frac{P_r}{P_t} = K \left( \frac{d}{d_r} \right) \gamma. \]
   - dB Model:
     \[ \frac{P_r}{P_t}(\text{dB}) = 10 \log_{10} K - 10 \gamma \log_{10}(d/d_r) - \psi_{dB}. \]
   - Average shadowing attenuation: when \( K_{dB} = 10 \log_{10} K \) captures average dB shadowing, \( \mu_{\psi dB} = 0 \), otherwise \( \mu_{\psi dB} > 0 \) since shadowing causes positive attenuation.
3. Outage Probability under Path Loss and Shadowing

- With path loss and shadowing, the received power at any given distance between transmitter and receiver is random.
- Leads to a non-circular coverage area around the transmitter, i.e. non-circular contours of constant power above which performance (e.g. in WiFi or cellular) is acceptable.
- Outage probability $P_{\text{out}}(P_{\text{min}}, d)$ is defined as the probability that the received power at a given distance $d$, $P_r(d)$, is below a target $P_{\text{min}}$: $P_{\text{out}}(P_{\text{min}}, d) = p(P_r(d) < P_{\text{min}})$.
- For the simplified path loss model and log normal shadowing this becomes
  
  $$p(P_r(d) \leq P_{\text{min}}) = 1 - Q\left(\frac{P_{\text{min}} - (P_t + K_{\text{dB}} - 10\gamma \log_{10}(d/d_r))}{\sigma_{\psi_{\text{dB}}}}\right).$$

4. Model Parameters from Empirical Data:

- Constant $K_{\text{dB}}$ typically obtained from measurement at distance $d_0$.
- Power falloff exponent $\gamma$ obtained by minimizing the MSE of the predicted model versus the data (assume $N$ samples):
  
  $$F(\gamma) = \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - M_{\text{model}}(d_i)]^2,$$

  where $M_{\text{measured}}(d_i)$ is the $i$th path loss measurement at distance $d_i$ and $M_{\text{model}}(d_i) = K_{\text{dB}} - 10\gamma \log_{10}(d_i)$. The minimizing $\gamma$ is obtained by differentiating with respect to $\gamma$, setting this derivative to zero, and solving for $\gamma$.
- The resulting path loss model will include average attenuation, so $\mu_{\psi_{\text{dB}}} = 0$.
- Can also solve simultaneously for $(K_{\text{dB}}, \gamma)$ via a least squares fit of both parameters to the data. Using the line equation for each data point $y_i$ that $y_i = mx_i + K_{\text{dB}}$ for $m = -10\gamma$ and $x_i = \log_{10}(d_i)$, the error of the straight line fit is
  
  $$F(K, \gamma) = \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - (mx_i + K_{\text{dB}})]^2,$$

  The shadowing variance $\sigma_{\psi_{\text{dB}}}^2$ is obtained by determining the MSE of the data versus the empirical path loss model with the minimizing $\gamma = \gamma_0$:
  
  $$\sigma_{\psi_{\text{dB}}}^2 = \frac{1}{N} \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - M_{\text{model}}(d_i)]^2; M_{\text{model}}(d_i) = K_{\text{dB}} - 10\gamma_0 \log_{10}(d_i).$$

5. Statistical Multipath Model:

- At each time instant there are a random number $N(t)$ of multipath signal components.
- At time $t$ the $i$th component has a random amplitude $\alpha_i(t)$, angle of arrival $\theta_i(t)$, Doppler shift $f_{D_i} = \frac{v}{c} \cos \theta_i(t)$ and associated phase shift $\phi_{D_i} = \int_t f_{D_i}(t) dt$, and path delay relative to the LOS component $\tau_i(t) = (x_i(t))/c$.
- Thus, the received signal is given by the following expression, which implies the channel has a time-varying impulse response.
  
  $$r(t) = \Re\left\{ \sum_{i=0}^{N(t)} \alpha_i(t) u(t - \tau_i(t)) e^{j(2\pi f_i(t - \tau_i(t)) + \phi_{D_i})} \right\}.$$
Main Points

- Shadowing decorrelates over its decorrelation distance, which is on the order of the size of shadowing objects.
- Combined path loss and shadowing leads to outage and non-circular coverage area (cells).
- Path loss and shadowing parameters are obtained from empirical measurements through a least-squares fit.
- Can find path loss exponent $\gamma$ by a 1-dimensional least-squares-error line fit assuming a fixed value of $K_{dB}$ from one far-field measurement (most common), or find path loss exponent $\gamma$ and $K_{dB}$ parameters simultaneously through a 2-dimensional least-squares-error line fit.
- Statistical multipath model leads to a time varying channel impulse response