EE359 – Lecture 5 Outline

- Announcements:
  - HW posted, due Friday 4pm
  - Background on random processes in Appendix B
- Review of Last Lecture: Narrowband Fading
- Auto and Cross Correlation of In-Phase and Quadrature Signal Components
- Correlation and PSD in uniform scattering
- Signal Envelope Distributions
- Wideband Channels and their Characterization

Review of Last Lecture

- Model Parameters from Measurements
- Random Multipath Model
- Channel Impulse Response
  \[ c(r, t) = \sum_{i=0}^{N} a_i(t)e^{-j\phi_i(t)} \delta(t - \tau_i(t)) \]
  - Many multipath components, Amplitudes change slowly, Phases change rapidly
- For delay spread max|\tau_n(t)-\tau_m(t)| << 1/B, \( u(t) = u(t)e^{j\omega_0 t} \).
- Received signal given by
  \[ r(t) = \Re \left[ u(t)e^{j\omega c t} \sum_{n=0}^{N} a_n(t)e^{-j\phi_n(t)} \right] \]
  - No signal distortion in time
  - Multipath yields complex scale factor in brackets

Cross Correlation

- Cross Correlation of inphase/quad signal is
  \[ A_{r_I, r_Q}(\tau) = -A_{Q,r_I}(\tau) = .5 \sum_i \phi_i^2 E_\theta \left[ \sin \left( \frac{2\pi \phi_i \tau}{\lambda \cos \theta} \right) \right] \]
  - Thus, \( A_{r_I, r_Q}(0) = 0 \), so \( r_I(t) \) and \( r_Q(t) \) independent
- Autocorrelation of received signal is
  \[ A_r(\tau) = A_{r_I}(\tau) \cos(2\pi f_c \tau) - A_{r_Q}(\tau) \sin(2\pi f_c \tau) \]
  - Thus, \( r(t) \) is stationary (WSS)

Review Continued: Narrowband Model

- For \( u(t) = e^{j\phi} \), \( r(t) = r_I(t)\cos(2\pi f_c t + \phi_I) - r_Q(t)\sin(2\pi f_c t + \phi_Q) \)
- In-phase and quadrature signal components:
  \[ r_I(t) = \sum_{i=0}^{N} a_i(t)\cos(2\pi f_i t), \quad r_Q(t) = \sum_{i=0}^{N} a_i(t)\sin(2\pi f_i t) \]
  \[ \phi_i(t) = 2\pi f_i t + \phi_i(0) - \phi_0 \]
- For \( N(t) \) large, \( r_I(t) \) & \( r_Q(t) \) jointly Gaussian by CLT
- Received signal characterized by its mean, autocorrelation, and cross correlation. Let \( \phi \sim U[0,2\pi] \)
  \[ A_{r_I}(\tau) = A_{r_Q}(\tau) = .5 \sum_i \phi_i^2 E_\theta \left[ \cos \left( \frac{2\pi \phi_i \tau}{\lambda \cos \theta} \right) \right] \]
- If \( \phi_i(t) \) uniform, in-phase/quad components are mean zero, independent, and stationary (WSS)
**Uniform Scattering**
- Multipath comes uniformly from all directions
- Power in each component is the same: $E[|\alpha_i|^2] = 2P_r/N$

**Autocorrelation and PSD under uniform scattering**
- Under uniform scattering, in phase and quad comps have no cross correlation and autocorrelation is
  \[ A_\tau (\tau) = A_0 (\tau) = P_r J_0(2\pi f_D \tau) \]
  Decorrelates over roughly half a wavelength
- The PSD of received signal is
  \[ S_r (f) = 0.25[S_n (f - f_c) + S_n (f + f_c)] \]
  \[ S_n (f) = q[P_r J_0(2\pi f_D \tau)] \]
  Used to generate simulation values

**Signal Envelope Distribution**
- CLT approx. leads to Rayleigh distribution (power is exponential)
- When LOS component present, Ricean distribution is used
- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models “worse than Rayleigh”
  - Lends itself better to closed form BER expressions

**Wideband Channels**
- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth
- Requires statistical characterization of $c(\tau, t)$
  - Assume CLT, stationarity and uncorrelated scattering
  - Leads to simplification of its autocorrelation function
  - $\Delta \tau < 1/B_w$
  - $\Delta \tau > 1/B_w$
Main Points

- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath
- Uniform scattering makes autocorrelation of inphase and quad comps of RX signal follow Bessel function
  - Signal components decorrelate over half wavelength
  - The PSD has a bowel shape centered at carrier frequency
- Fading distribution depends on environment
  - Rayleigh, Ricean, and Nakagami all common
- Wideband channels have resolvable multipath
  - Will statistically characterize c(τ,t) for WSSUS model