
Lecture Outline

- Nakagami Fading
- Wideband Channel Models
- Scattering Function
- Multipath Intensity Profile, Delay Spread, and Coherence Bandwidth
- Doppler Power Spectrum, Doppler Spread, and Coherence Time

1. Nakagami Fading Distribution
   - Experimental results support a Nakagami distribution for the signal envelope for some environments. Nakagami is similar to Rician, but can model “worse than Rayleigh.”
   - Model generally leads to closed-form expressions in BER and diversity analysis.
   - Distribution is $p_{Z}(z) = 2^{m} m z^{2m-1} \frac{\exp \left[ - \frac{mz^2}{P_r} \right]}{\Gamma(m)} P_r^m$, $m \geq .5$. By change of variables, power distribution is $p_{Z^2}(x) = \left( \frac{m}{P_r} \right)^{m} x^{m-1} \frac{1}{\Gamma(m)}$.

2. Wideband Channel Models
   - In wideband multipath channels the individual multipath components can be resolved by the receiver. True if $T_m > 1/B$.
   - If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).

3. Channel Scattering Function:
   - Typically time-varying channel impulse response $c(\tau, t)$ is unknown, so its wideband model must be characterized statistically.
   - Since under our random model with a large number of scatterers, $c(\tau, t)$ is Gaussian. We assume it is WSS, so we only need to characterize its mean and correlation, which is independent of time. Similar to narrowband model, for $\phi_n$ uniformly distributed, $c(\tau, t)$ has mean zero.
   - Autocorrelation of $c(\tau, t)$ is $A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1, \tau_2; \Delta t) \delta(\tau_1 - \tau_2) = A_c(\tau; \Delta t)$ since we assume channel response associated with different scatterers is uncorrelated.
   - Statistical scattering function defined as $S(\tau, \rho) = \mathcal{F}_{\Delta t}[A_c(\tau, \Delta t)]$. This function measures the average channel gain as a function of both delay $\tau$ and Doppler $\rho$.
   - $S(\tau, \rho)$ easy to measure empirically and is used to get average delay spread $T_M$, rms delay spread $\sigma_\tau$, and Doppler spread $B_d$ for empirical channel measurements.

4. Multipath Intensity Profile and Delay Spread
   - Multipath intensity profile (delay power spectrum) defined as $A_c(\tau; \Delta t = 0) \overset{\Delta}{=} A_c(\tau)$, i.e. the autocorrelation relative to delay $\tau$ at a fixed time.
• The average delay $\mu T_m$ and rms delay spread $\sigma T_m$ are defined relative to $A_c(\tau)$. These parameters approximate the maximum delay of nontrivial multipath components.

5. Coherence Bandwidth

• The coherence bandwidth is defined relative to the Fourier transform of $A_c(\tau)$, given by $A_C(\Delta f) = F\{A_c(\tau)\}$. Note that $A_C(\Delta f) = A_C(\Delta f, \Delta t = 0)$.

• Since $A_C(\Delta f)$ is the autocorrelation of a Gaussian process, multipath components separated by $\Delta f_0$ are independent if $A_C(\Delta f_0) \approx 0$.

• By the Fourier transform relationship, the bandwidth over which $A_C(\Delta f)$ is nonzero is roughly $B_c \approx 1/\sigma T_m$ or $B_c \approx 1/\sigma T_m$ (can also add constants to these denominators).

• $B_c$ defines the coherence bandwidth of the channel, i.e., the bandwidth over which fading is correlated.

• A signal experiences frequency selective fading or ISI if its bandwidth exceeds the coherence bandwidth of the channel.

6. Doppler Power Spectrum, Doppler Spread, and Coherence Time:

• Doppler power spectrum is defined with respect to $A_c(\Delta f; \Delta t) = F_\tau[A_c(\tau, \Delta t)]$.

• Specifically, the Doppler power spectrum is $S_c(\rho) = F_{\Delta t}[A_c(\Delta f = 0, \Delta t) \overset{\Delta}{=} A_c(\Delta t)]$, which measures channel intensity as a function of Doppler frequency.

• The maximum value of $\rho$ for which $|S_c(\rho)| > 0$ is called the channel Doppler spread, which is denoted by $B_d$.

• By the Fourier transform relationship, $A_c(\Delta t) \approx 0$ for $\Delta t > 1/B_d$. Thus, the channel becomes uncorrelated over a time of $1/B_d$ seconds.

• We define the channel coherence time as $T_c = 1/B_d$. A deep fade lasts approximately $T_c$ seconds. Hence, if the coherence time greatly exceeds a bit time, the signal experiences error bursts lasting $T_c$ seconds.

Main Points

• Wideband models characterized by scattering function, which measures average channel gain relative to delay and Doppler.

• Scattering function used to obtain key channel characteristics of rms delay spread and Doppler spread, which are important for system design.

• Multipath delay spread defines the maximum delay of significant multipath components. Its inverse is the channel coherence bandwidth. Signals separated in frequency by the coherence bandwidth have independent fading.

• Doppler spread defines the channel’s maximum nonzero Doppler. Its inverse is the channel coherence time. Signals separated in time by the coherence time have independent fading.