EE359 – Lecture 8 Outline

- **Announcements**
  - Schedule changes next week.
  - No lecture next Tues 2/3.
  - Makeup class: Wed 2/5 11:30-12:30pm w/lunch in Gates B03
  - Project proposals due 2/7; I can provide early feedback
  - MT week of 2/17, 6-8pm (pizza after), poll this week; details soon
  - New version of Reader with Chapters 1-7 available next week

- **Capacity of Fading channels**
  - Recap Optimal Rate/Power Adaptation with TX/RX CSI
  - Channel Inversion with Fixed Rate
  - Capacity of Freq. Selective Fading Channels
  - Linear Digital Modulation Review
  - Performance of Linear Modulation in AWGN

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Review of Last Lecture

- **Channel Capacity**
  - Maximum data rate that can be transmitted over a channel with arbitrarily small error
  - Capacity of AWGN Channel: $\log_2[1+\gamma]$ bps
  - $\gamma=P_r/(N_0B)$ is the receiver SNR

- **Capacity of Flat-Fading Channels**
  - Nothing known: capacity typically zero
  - Fading Statistics Known (few results)
  - Fading Known at RX (average capacity)
  
  $$C = \int_0^\infty B \log_2 \left(1 + \gamma \right) p(\gamma) d\gamma \\ \leq B \log_2 \left(1 + \gamma \right)$$

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**Capacity in Flat-Fading: $\gamma$ known at TX/RX**

$$C = \max_{P(\gamma)} \left[ \int \log_2 \left(1 + \frac{\gamma P(\gamma)}{\bar{P}} \right) p(\gamma) d\gamma \right]$$

**Optimal Rate and Power Adaptation**

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{\gamma}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \text{else} \end{cases}$$

$$C = \int_{\gamma_0}^{\gamma} \log_2 \left(\frac{\gamma}{\gamma_0} \right) p(\gamma) d\gamma.$$

- The instantaneous power/rate only depend on $p(\gamma)$ through $\gamma_0$

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Channel Inversion

- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
  - Capacity is zero in Rayleigh fading
- Truncated inversion
  - Invert channel above cutoff fade depth
  - Constant SNR (fixed rate) above cutoff
  - Cutoff greatly increases capacity
  - Close to optimal
Capacity in Flat-Fading

### Rayleigh
- AWGN Capacity
- Shannon Capacity of 600 Gb/s
- Maximum Output Power = 30 dBm

### Log-Normal
- AWGN Capacity
- Shannon Capacity of 600 Gb/s
- Maximum Output Power = 30 dBm

For time-invariant channels, capacity achieved by water-filling in frequency.
- Capacity of time-varying channel unknown
- Approximate by dividing into subbands
  - Each subband has width $B_c$ (like MCM/OFDM).
  - Independent fading in each subband
  - Capacity is the sum of subband capacities

Review of Linear Digital Modulation
- Signal over $i$th symbol period:
  \[ s(t) = s_{i1}g(t)\cos(2\pi f_c t + \phi_0) - s_{i2}g(t)\sin(2\pi f_c t + \phi_0) \]
  - Pulse shape $g(t)$ typically Nyquist
  - Signal constellation defined by $(s_{i1}, s_{i2})$ pairs
  - Can be differentially encoded
  - $M$ values for $(s_{i1}, s_{i2}) \rightarrow \log_2 M$ bits per symbol

- $P_s$ depends on
  - Minimum distance $d_{min}$ (depends on $\gamma$)
  - # of nearest neighbors $\alpha_M$
  - Approximate expression:
    - Standard/alternate Q function
      \[ P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right) \]

Main Points
- Channel inversion practical, but should truncate or get a large capacity loss
- Capacity of wideband channel obtained by breaking up channel into subbands
  - Similar to multicarrier modulation
  - Linear modulation dominant in high-rate wireless systems due to its spectral efficiency
- $P_s$ approximation in AWGN:
  \[ P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right) \]
- Alternate Q function useful in diversity analysis