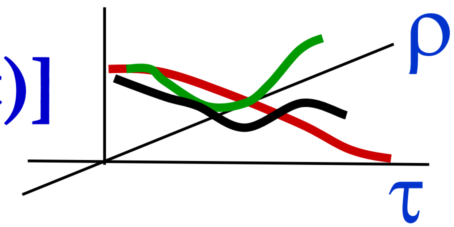
# EE359 – Lecture 7 Outline

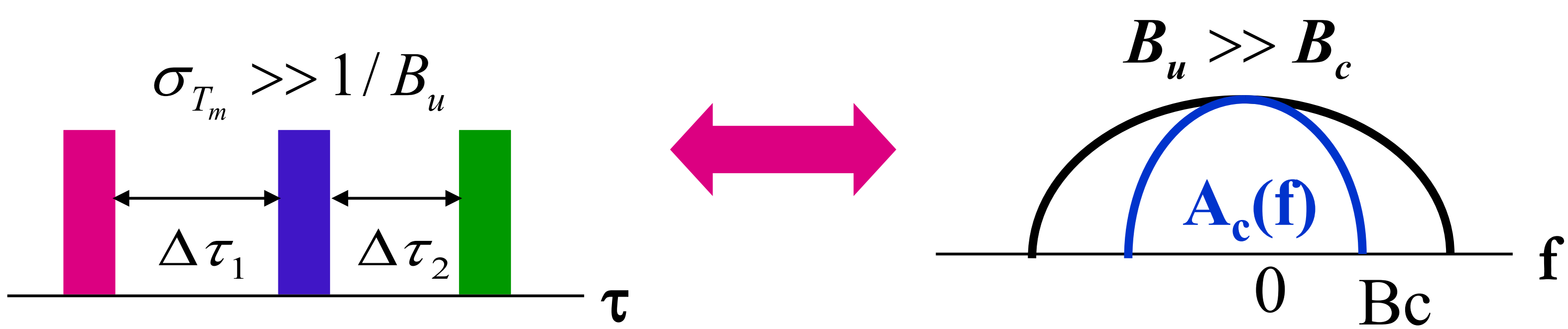
- Announcements:
  - Schedule changes this week
    - 10/18 – Discussion 5-6pm Pack 364, OHs 4:00PM - 5:00PM (3rd Floor Packard) and just after Discussion.
    - Makeup class Friday 10/20 (no lecture next Tues 10/24). Makeup class will be 10/20 at 10:30AM at Thornton 102 (with donuts)
    - Tom's OHs 10/20 will be 9:30-10:20 (3rd Floor Packard)
    - Email OHs 10/20 from 1-2pm
  - Project proposals due 10/28; I can provide early feedback
  - MT Nov. 7, 8, or 9 from 6-8pm (pizza after), poll today; details soon
- Shannon Capacity
- Capacity of Flat-Fading Channels
  - Fading Statistics Known
  - Fading Known at RX
  - Fading Known at TX and RX: water-filling
  - Channel Inversion
  - Truncated Channel Inversion

# Review of Last Lecture

- **Wideband channels: $B_u >> 1/\sigma_{T_m}$**
- **Scattering Function: $s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$**
  - **Used to characterize $c(\tau,t)$ statistically**




- **Multipath Intensity Profile:**
- **Determines average ($\mu_{T_m}$) and rms ($\sigma_{T_m}$) delay spread**
  - **Coherence bandwidth $B_c=1/\sigma_{T_m}$**

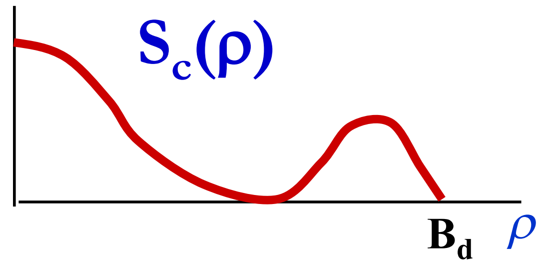


# Review Continued

**Scattering Function:** $s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$

- **Doppler Power Spectrum:** $S_c(\rho)= \mathcal{F}_{\Delta t}\,[A_c(\Delta f=0,\Delta t)\triangleq Ac(\Delta t)]$

$$A_c(\Delta f,\Delta t)=\mathcal{F}_{\tau}[A_c(\tau,\Delta t)]$$




  - Power of multipath at given Doppler
  - Doppler spread $B_d$: Max. doppler for which $S_c(\rho)>0$.
  - Coherence time $T_c=1/B_d$: Max time over which $A_c(\Delta t)>0$
    - $A_c(\Delta t)=0$ implies signals separated in time by $\Delta t$ uncorrelated at RX

- **Why do we look at Doppler w.r.t. $A_c(\Delta f=0,\Delta t)$?**
  - Captures Doppler associated with a narrowband signal
  - Autocorrelation over a narrow range of frequencies
  - Fully captures time-variations, multipath angles of arrival

# Shannon Capacity

- Defined as channel's maximum mutual information
- Shannon proved that capacity is the maximum error-free data rate a channel can support.
- Theoretical limit (not achievable)
- Channel characteristic
  - Not dependent on design techniques
- In AWGN, $C = B\log_2(1+\gamma)$ bps
  - $B$ is the signal bandwidth
  - $\gamma=P_r/(N_0B)$ is the received signal to noise power ratio

# Capacity of Flat-Fading Channels

- Capacity defines theoretical rate limit
  - Maximum error free rate a channel can support
- Depends on what is known about channel
- Fading Statistics Known
  - Hard to find capacity
- Fading Known at Receiver Only

$$C = \int_0^\infty B \log_2\left(1+\gamma\right) p(\gamma) d\gamma \le B \log_2(1+\bar{\gamma})$$

- Fading known at TX and RX
  - Multiplex optimal strategy over each channel state

# Capacity with Fading Known at Transmitter and Receiver

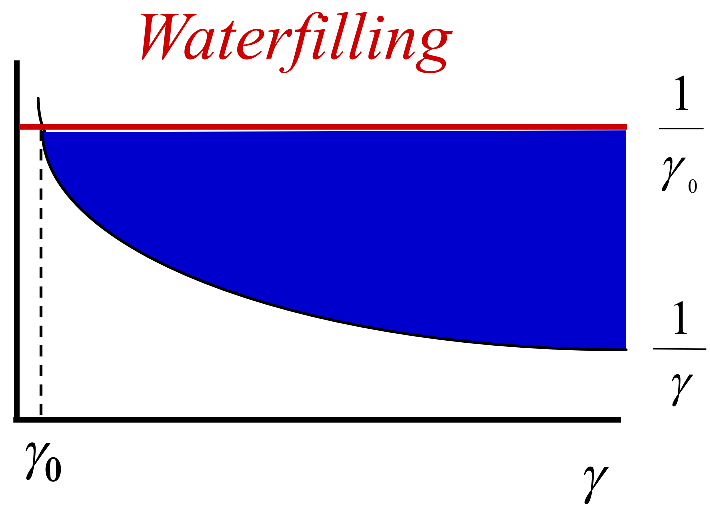
- For fixed transmit power, same as with only receiver knowledge of fading
- Transmit power *P(γ)* can also be adapted
- Leads to optimization problem

$$C = \max_{P(\gamma):E[P(\gamma)]=\overline{P}} \int_0^\infty B\log_2\left(1+\frac{\gamma P(\gamma)}{\overline{P}}\right)p(\gamma)d\gamma$$

# Optimal Adaptive Scheme

- **Power Adaptation**

$$\frac{P(\gamma)}{\overline{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \textbf{else} \end{cases}$$




- **Capacity**

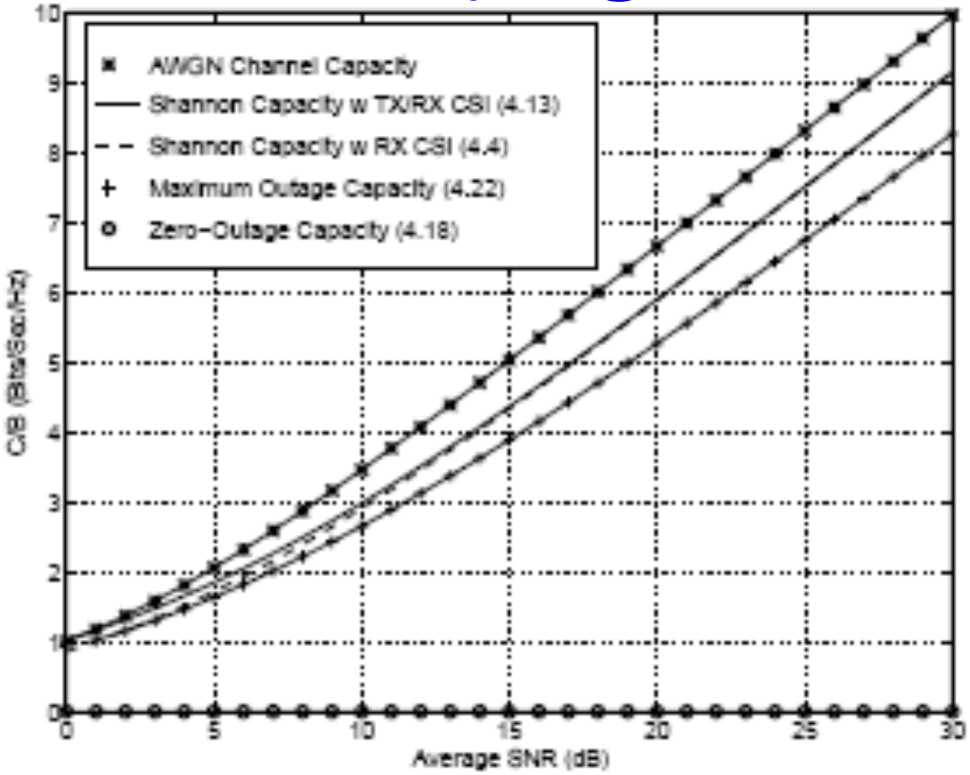
$$\frac{R}{B} = \int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) p(\gamma)\, d\gamma.$$

# Channel Inversion

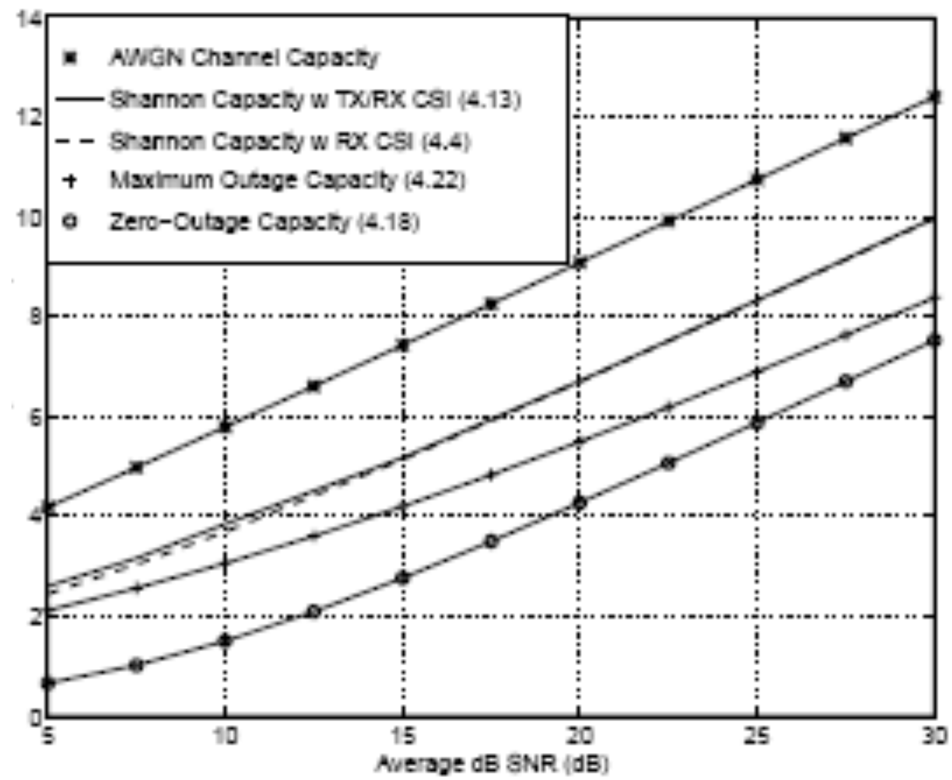
- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
  - Capacity is zero in Rayleigh fading
- Truncated inversion
  - Invert channel above cutoff fade depth
  - Constant SNR (fixed rate) above cutoff
  - Cutoff greatly increases capacity
    - Close to optimal

# Capacity in Flat-Fading

## Rayleigh




## Log-Normal



# Main Points

- Fundamental channel capacity defines maximum data rate that can be supported on a channel
- Capacity in fading depends what is known at TX/RX
- Capacity with RX CSI is average of AWGN capacity
- Capacity with TX/RX knowledge requires optimal adaptation based on current channel state
- Almost same capacity as with RX knowledge only
- Channel inversion has poor performance, but significantly improved with truncation